Dr Oliver Mathematics
Applied Mathematics: Partial Fractions

The total number of marks available is 20.
You must write down all the stages in your working.

1. Express (3)

$$\frac{x^2+3}{x(1+x^2)}$$

in partial fractions.

Solution

$$\begin{aligned}\frac{x^2+3}{x(1+x^2)} &\equiv \frac{A}{x} + \frac{B+Cx}{1+x^2} \\ &\equiv \frac{A(1+x^2)+x(B+Cx)}{x(1+x^2)}\end{aligned}$$

and so

$$x^2+3 \equiv A(1+x^2)+x(B+Cx).$$

$\underline{x=0}$: $3 = A$.
$\underline{x=1}$: $4 = 2A+B+C \Rightarrow B+C=-2 \quad (1)$.
$\underline{x=-1}$: $4 = 2A-B+C \Rightarrow -B+C=-2 \quad (2)$.
Add (1) + (2):

$$\begin{aligned}2C = -4 &\Rightarrow C = -2 \\ &\Rightarrow B = 0.\end{aligned}$$

Hence,

$$\frac{x^2+3}{x(1+x^2)} \equiv \underline{\underline{\frac{3}{x} - \frac{2x}{1+x^2}}}.$$

2. Express (4)

$$\frac{8}{x(x+2)(x+4)}$$

in partial fractions.

Solution

$$\frac{8}{x(x+2)(x+4)} \equiv \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x+4}$$
$$\equiv \frac{A(x+2)(x+4) + Bx(x+4) + Cx(x+2)}{x(x+2)(x+4)}$$

and so

$$8 \equiv A(x+2)(x+4) + Bx(x+4) + Cx(x+2).$$

$\underline{x=0}$: $8 = 8A \Rightarrow A = 1$.
$\underline{x=-2}$: $8 = -4B \Rightarrow B = -2$.
$\underline{x=-4}$: $8 = 8C \Rightarrow B = 1$.
Hence,

$$\frac{8}{x(x+2)(x+4)} \equiv \underline{\underline{\frac{1}{x} - \frac{2}{x+2} + \frac{1}{x+4}}}.$$

3. Express (4)

$$y = \frac{4x-3}{x(x^2+3)},\ x \neq 0,$$

in partial fractions.

Solution

$$\frac{4x-3}{x(x^2+3)} \equiv \frac{A}{x} + \frac{B+Cx}{x^2+3}$$
$$\equiv \frac{A(x^2+3) + (B+Cx)x}{x(x^2+3)}$$

which means

$$4x - 3 \equiv A(x^2+3) + (B+Cx)x.$$

$\underline{x=0}$: $-3 = 3A \Rightarrow A = -1$.
$\underline{x=1}$: $1 = 4A + B + C \Rightarrow B + C = 5 \quad (2)$.
$\underline{x=-1}$: $-7 = 4A - B + C \Rightarrow -B + C = -3 \quad (3)$.
Now, (1) + (2):

$$2C = 2 \Rightarrow C = 1$$
$$\Rightarrow B = 4.$$

Finally,

$$y = \underline{\underline{-\frac{1}{x} + \frac{4+x}{x^2+3}}}.$$

4. Express (3)

$$\frac{3x}{(x+1)^2}$$

in partial fractions.

Solution

$$\begin{aligned}\frac{3x}{(x+1)^2} &\equiv \frac{A}{(x+1)} + \frac{B}{(x+1)^2} \\ &\equiv \frac{A(x+1)+B}{(x+1)^2}\end{aligned}$$

and so

$$3x \equiv A(x+1) + B.$$

$\underline{x = -1}$: $-3 = B$
$\underline{x = 0}$: $0 = A - 3 \Rightarrow A = 3$
Hence,

$$\frac{3x}{(x+1)^2} \equiv \underline{\underline{\frac{3}{(x+1)} - \frac{3}{(x+1)^2}}}.$$

5. Express (3)

$$\frac{1}{x^2+x}$$

in partial fractions, where x is neither 0 nor -1.

Solution

$$\begin{aligned}\frac{1}{x^2+x} &\equiv \frac{1}{x(x+1)} \\ &\equiv \frac{A}{x} + \frac{B}{x+1} \\ &\equiv \frac{A(x+1)+Bx}{x(x+1)}\end{aligned}$$

and so

$$1 \equiv A(x+1) + Bx.$$

$\underline{x=0}$: $1 = A$.
$\underline{x=-1}$: $1 = -B \Rightarrow B = -1$.
Hence,

$$\frac{1}{x^2+x} \equiv \underline{\underline{\frac{1}{x} - \frac{1}{x+1}}}.$$

6. Express (3)

$$\frac{1}{1-y^2}$$

in partial fractions.

Solution

$$\left.\begin{array}{ll}\text{add to:} & 0 \\ \text{multiply to:} & -1\end{array}\right\} -1, +1$$

$$\begin{aligned}\frac{1}{1-y^2} &\equiv \frac{1}{(1+y)(1-y)} \\ &\equiv \frac{A}{1+y} + \frac{B}{1-y} \\ &\equiv \frac{A(1-y) + B(1+y)}{(1+y)(1-y)}\end{aligned}$$

which means

$$1 \equiv A(1-y) + B(1+y).$$

$\underline{y=1}$: $1 = 2B \Rightarrow B = \frac{1}{2}$.
$\underline{y=-1}$: $1 = 2A \Rightarrow A = \frac{1}{2}$.
Hence,

$$\frac{1}{1-y^2} \equiv \underline{\underline{\frac{\frac{1}{2}}{1+y} + \frac{\frac{1}{2}}{1-y}}}.$$