Dr Oliver Mathematics
Mathematics: Advanced Higher
2023 Paper 1: Non-Calculator
1 hour

The total number of marks available is 35.
You must write down all the stages in your working.

1. Given (2)
$$y = 7x \tan 2x,$$
find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

2. Express (3)
$$\frac{3x^2 - x - 14}{(x+3)(x-1)^2}$$
in partial fractions.

3. A system of equations is defined by (3)
$$\begin{aligned} x - 3y + z &= -1 \\ 3x - 2y + 4z &= 11 \\ x + 4y + 2z &= 15. \end{aligned}$$
Use Gaussian elimination to determine whether the system shows redundancy, inconsistency, or has a unique solution.

4. Use integration by parts to find (3)
$$\int x^4 \ln x \,\mathrm{d}x,\ x > 0.$$

5. Find the particular solution of the differential equation (9)
$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 4\frac{\mathrm{d}y}{\mathrm{d}x} - 5y = 10x^2 + 11x - 23,$$
given that $y = 2$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 14$ when $x = 0$.

6. (a) Express (2)
$$z = 1 + \mathrm{i}\sqrt{3}$$
in polar form.

(b) Hence, or otherwise, show that z^3 is real. (2)

7. (a) Find an expression for (2)

$$\sum_{r=1}^{n}(r^2 + 3r)$$

in terms of n.

Express your answer in the form

$$\tfrac{1}{3}n(n+a)(n+b).$$

(b) Hence, or otherwise, find (2)

$$\sum_{r=11}^{20}(r^2 + 3r).$$

8. (a) Consider the statement: (1)

$$\text{For all integers } a \text{ and } b, \text{if } a < b \text{ then } a^2 < b^2.$$

Find a counterexample to show that the statement is false.

Let n be an odd integer.

(b) Prove directly that $(n^2 - 1)$ is divisible by 4. (2)

9. (a) State the matrix $\mathbf{A}$, associated with an anti-clockwise rotation of $\frac{1}{2}\pi$ radians about the origin. (1)

The matrix $\mathbf{B}$ is given by

$$\mathbf{B} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}.$$

The matrix given by $\mathbf{AB}$ is associated with an anti-clockwise rotation of α radians about the origin.

(b) (i) Determine $\mathbf{AB}$. (1)

(ii) Find the value of α. (1)

(c) Determine the least positive integer value of n such that $(\mathbf{AB})^n = \mathbf{I}$, where $\mathbf{AB}$ is the 2×2 identity matrix. (1)