Dr Oliver Mathematics
Finding the Perpendicular Line Through A Given Point

We will investigate finding the perpendicular line through a given point. We will do a series of examples and I am sure that you will get the hang of it.

Example 1
Finding the perpendicular line to

$$4x + 3y + 19 = 0$$

through $(1, 1)$.

Solution 1
We need five steps.
Step 1: Discard any constant, so it becomes

$$4x + 3y =$$

Step 2: *Swap* the multiples of x and y, so it becomes

$$3x + 4y =$$

Step 3: Change **one** sign, so it becomes

$$3x - 4y = \quad \textbf{or} \quad -3x + 4y =$$

Step 4: Use the point to evaluate the constant, so it becomes

$$3x - 4y = 3(1) - 4(1) = -1 \ \textbf{or} \ -3x + 4y = -3(1) + 4(1) = 1.$$

Step 5: And that's it — unless you need to tidy it up, for example, taking the constant term over to the LHS as in $ax + bx + c = 0$:

$$3x - 4y + 1 = 0 \ \textbf{or} \ -3x + 4y - 1 = 0.$$

Why? Well,

$$\begin{aligned} 4x + 3y + 19 &\Rightarrow 3y = -4x - 19 \\ &\Rightarrow y = -\tfrac{4}{3}x - \tfrac{19}{3}. \end{aligned}$$

So, the gradient of the line is

$$m = -\tfrac{4}{3}$$

and the perpendicular line has gradient

$$m_{\text{normal}} = -\frac{1}{-\frac{4}{3}} = \tfrac{3}{4}.$$

Finally, the equation of the perpendicular line is

$$\begin{aligned} y-1 = \tfrac{3}{4}(x-1) &\Rightarrow 4(y-1) = 3(x-1) \\ &\Rightarrow 4y-4 = 3x-3 \\ &\Rightarrow 3x-4y+1 = 0 \text{ or } -3x+4y-1 = 0. \end{aligned}$$

Example 2
Finding the perpendicular line to

$$y = x + 2$$

through $(3, -1)$.

Solution 2
Step 1: Discard any constant, so it becomes

$$y = x+$$

Step 2: *Swap* the multiples of x and y, so it becomes

$$x = y+$$

Step 3: Change **one** sign, so it becomes

$$x = -y + \text{ or } -x = y+$$

Step 4: Use the point to evaluate the constant, so it becomes either

$$\begin{aligned} 3 = -(-1) + c &\Rightarrow 3 = 1 + c \\ &\Rightarrow c = 2 \end{aligned}$$

or

$$-3 = -1 + c \Rightarrow c = -2;$$

hence,

$$x = -y + 2 \text{ or } -x = y - 2.$$

Step 5: And that's it — unless you need to tidy it up, for example,

$$x + y - 2 = 0 \text{ or } y = -x - 2.$$

You should be able to go from step 1 to step 3 instantly.

Example 3
Finding the perpendicular line to

$$x + 4y = 3$$

through $(7, -2)$.

Solution 3
So it becomes

$$4x - y =$$

Step 4:

$$4(7) - 1(-2) = 30$$

and the equation of the perpendicular line is

$$4x - y = 30.$$

Example 4
Finding the perpendicular line to

$$y = 2$$

through $(3, 4)$.

Solution 4
It does not involve x! So let us put that right:
Step 1: Discard any constant, so it becomes

$$0x + 1y =$$

Step 2: *Swap* the multiples of x and y, so it becomes

$$1x + 0y =$$

Step 3: Change **one** sign, so it becomes

$$1x - 0y = \quad \textbf{or} \quad -1x + 0y =$$

Of course, $0y$ is just *zero* ...
Step 4: Use the point to evaluate the constant, hence,

$$x = 3.$$