Dr Oliver Mathematics
Mathematics
Simultaneous Equations
Past Examination Questions

This booklet consists of 10 questions across a variety of examination topics. The total number of marks available is 62.

1. Solve the simultaneous equations (6)

$$\begin{aligned} x + y &= 2 \\ x^2 + 2y &= 12. \end{aligned}$$

Solution

$$\begin{aligned} y = 2 - x &\Rightarrow x^2 + 2(2 - x) = 12 \\ &\Rightarrow x^2 + (4 - 2x) = 12 \\ &\Rightarrow x^2 - 2x - 8 = 0 \\ &\Rightarrow (x - 4)(x + 2) = 0 \\ &\Rightarrow x = 4 \text{ or } x = -2 \\ &\Rightarrow y = -2 \text{ or } y = 4; \end{aligned}$$

hence,

$$\underline{\underline{x = 4, y = -2}} \text{ or } \underline{\underline{x = -2, y = 4}}.$$

2. Solve the simultaneous equations (6)

$$\begin{aligned} x - 2y &= 1 \\ x^2 + y^2 &= 29. \end{aligned}$$

Solution

$$\begin{aligned} x = 2y + 1 &\Rightarrow (2y+1)^2 + y^2 = 29 \\ &\Rightarrow (4y^2 + 4y + 1) + y^2 = 29 \\ &\Rightarrow 5y^2 + 4y - 28 = 0 \\ &\Rightarrow (5y + 14)(y - 2) = 0 \\ &\Rightarrow y = -\tfrac{14}{5} \text{ or } y = 2 \\ &\Rightarrow x = -\tfrac{23}{5} \text{ or } x = 5; \end{aligned}$$

hence,

$$\underline{\underline{x = -\tfrac{23}{5}, y = -\tfrac{14}{5}}} \text{ or } \underline{\underline{x = 5, y = 2}}.$$

3. Solve the simultaneous equations (7)

$$\begin{aligned} y &= x - 2 \\ y^2 + x^2 &= 10. \end{aligned}$$

Solution

$$\begin{aligned} y = x - 2 &\Rightarrow (x-2)^2 + x^2 = 10 \\ &\Rightarrow (x^2 - 4x + 4) + x^2 = 10 \\ &\Rightarrow 2x^2 - 4x - 6 = 0 \\ &\Rightarrow 2(x^2 - 2x - 3) = 0 \\ &\Rightarrow 2(x - 3)(x + 1) = 0 \\ &\Rightarrow x = -1 \text{ or } x = 3 \\ &\Rightarrow y = -3 \text{ or } y = 1; \end{aligned}$$

hence,

$$\underline{\underline{x = -1, y = -3}} \text{ or } \underline{\underline{x = 3, y = 1}}.$$

4. Solve the simultaneous equations (5)

$$\begin{aligned} y &= x - 4 \\ 2x^2 - xy &= 8, \end{aligned}$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

Solution

$$\begin{aligned} y = x - 4 &\Rightarrow 2x^2 - x(x-4) = 8 \\ &\Rightarrow x^2 + 4x = 8 \\ &\Rightarrow x^2 + 4x + 4 = 12 \\ &\Rightarrow (x+2)^2 = 12 \\ &\Rightarrow x + 2 = \pm 2\sqrt{3} \\ &\Rightarrow x = -2 - 2\sqrt{3} \text{ or } x = -2 + 2\sqrt{3} \\ &\Rightarrow y = -6 + 2\sqrt{3} \text{ or } y = -6 - 2\sqrt{3}; \end{aligned}$$

hence,

$$\underline{\underline{x = -2 + 2\sqrt{3}, y = -6 + 2\sqrt{3}}} \text{ or } \underline{\underline{x = -2 - 2\sqrt{3}, y = -6 - 2\sqrt{3}}}.$$

5. Solve the simultaneous equations (7)

$$\begin{aligned} y - 3x + 2 &= 0 \\ y^2 - x - 6x^2 &= 0. \end{aligned}$$

Solution

$$\begin{aligned} y = 3x - 2 &\Rightarrow (3x-2)^2 - x - 6x^2 = 0 \\ &\Rightarrow (9x^2 - 12x + 4) - x - 6x^2 = 0 \\ &\Rightarrow 3x^2 - 13x + 4 = 0 \\ &\Rightarrow (3x-1)(x-4) = 0 \\ &\Rightarrow x = \tfrac{1}{3} \text{ or } x = 4 \\ &\Rightarrow y = -1 \text{ or } y = 10; \end{aligned}$$

hence,

$$\underline{\underline{x = \tfrac{1}{3}, y = -1}} \text{ or } \underline{\underline{x = 4, y = 10}}.$$

6. Solve the simultaneous equations (7)

$$\begin{aligned} x + y &= 2 \\ 4y^2 - x^2 &= 11. \end{aligned}$$

Solution

$$\begin{aligned} y = 2 - x &\Rightarrow 4(2-x)^2 - x^2 = 11 \\ &\Rightarrow 4(4 - 4x + x^2) - x^2 - 11 = 0 \\ &\Rightarrow 3x^2 - 16x + 5 = 0 \\ &\Rightarrow (3x-1)(x-5) = 0 \\ &\Rightarrow x = \tfrac{1}{3} \text{ or } x = 5 \\ &\Rightarrow y = \tfrac{5}{3} \text{ or } y = -3; \end{aligned}$$

hence,

$$\underline{\underline{x = \tfrac{1}{3}, y = \tfrac{5}{3}}} \text{ or } \underline{\underline{x = 5, y = -3}}.$$

7. Solve the simultaneous equations (5)

$$\begin{aligned} 2x + y &= 1 \\ x^2 - \tfrac{1}{4}y + \tfrac{5}{16} &= 0. \end{aligned}$$

Solution

$$\begin{aligned} y = 1 - 2x &\Rightarrow x^2 - \tfrac{1}{4}(1-2x) + \tfrac{5}{16} = 0 \\ &\Rightarrow x^2 + \tfrac{1}{2}x + \tfrac{1}{16} = 0 \\ &\Rightarrow (x + \tfrac{1}{4})^2 = 0 \\ &\Rightarrow x + \tfrac{1}{4} = 0 \\ &\Rightarrow x = -\tfrac{1}{4} \\ &\Rightarrow y = \tfrac{3}{2}; \end{aligned}$$

hence,

$$\underline{\underline{x = -\tfrac{1}{4}, y = \tfrac{3}{2}}}.$$

8. Solve the simultaneous equations (6)

$$\begin{aligned} y &= x + 2 \\ x^2 + 4y^2 - 2x &= 35. \end{aligned}$$

Solution

$$\begin{aligned} x^2 + 4(x+2)^2 - 2x = 35 &\Rightarrow x^2 + 4(x^2 + 4x + 4) - 2x - 35 = 0 \\ &\Rightarrow 5x^2 + 14x - 19 = 0 \\ &\Rightarrow (5x + 19)(x - 1) = 0 \\ &\Rightarrow x = -\tfrac{19}{5} \text{ or } x = 1 \\ &\Rightarrow y = -\tfrac{9}{5} \text{ or } y = 3; \end{aligned}$$

hence,

$$\underline{\underline{x = -\tfrac{19}{5}, y = \tfrac{9}{5}}} \text{ or } \underline{\underline{x = 1, y = 3}}.$$

9. Solve the simultaneous equations (7)

$$\begin{aligned} y - 2x - 4 &= 0 \\ 4x^2 + y^2 + 20x &= 0. \end{aligned}$$

Solution

$$\begin{aligned} y = 2x + 4 &\Rightarrow 4x^2 + (2x + 4)^2 + 20x = 0 \\ &\Rightarrow 4x^2 + (4x^2 + 16x + 16) + 20x = 0 \\ &\Rightarrow 8x^2 + 36x + 16 = 0 \\ &\Rightarrow 4(2x^2 + 9x + 4) = 0 \\ &\Rightarrow 4(2x + 1)(x + 4) = 0 \\ &\Rightarrow x = -4 \text{ or } x = -\tfrac{1}{2} \\ &\Rightarrow y = -4 \text{ or } y = 3; \end{aligned}$$

hence,

$$\underline{\underline{x = -4, y = -4}} \text{ or } \underline{\underline{x = -\tfrac{1}{2}, y = 3}}.$$

10. Solve the simultaneous equations (6)

$$\begin{aligned} y + 4x + 1 &= 0 \\ y^2 + 5x^2 + 2x &= 0. \end{aligned}$$

Solution

$$\begin{aligned} y = -4x - 1 &\Rightarrow (-4x-1)^2 + 5x^2 + 2x = 0 \\ &\Rightarrow (16x^2 + 8x + 1) + 5x^2 + 2x = 0 \\ &\Rightarrow 21x^2 + 10x + 1 = 0 \\ &\Rightarrow (7x+1)(3x+1) = 0 \\ &\Rightarrow x = -\tfrac{1}{3} \text{ or } x = -\tfrac{1}{7} \\ &\Rightarrow y = \tfrac{1}{3} \text{ or } y = -\tfrac{3}{7}; \end{aligned}$$

hence,

$$\underline{\underline{x = -\tfrac{1}{3}, y = \tfrac{1}{3}}} \text{ or } \underline{\underline{x = -\tfrac{1}{7}, y = \tfrac{3}{7}}}.$$