Dr Oliver Mathematics
GCSE Mathematics
2014 November Paper 1H: Non-Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Using the information that
$$6.7 \times 52 = 348.4,$$
   find the value of

   (a) 6.7×520, (1)

   Solution
   $$6.7 \times 52 = 348.4 \Rightarrow 6.7 \times 520 = \underline{3\,484}.$$

   (b) 67×0.52, (1)

   Solution
   $$6.7 \times 52 = 348.4 \Rightarrow 6.7 \times 5.2 = 34.84$$
   $$\Rightarrow 6.7 \times 0.52 = 3.484$$
   $$\Rightarrow 67 \times 0.52 = \underline{34.84}.$$

   (c) $3\,484 \div 5.2$. (1)

   Solution
   $$6.7 \times 52 = 348.4 \Rightarrow 6.7 \times 5.2 = 34.84$$
   $$\Rightarrow 67 \times 5.2 = 348.4$$
   $$\Rightarrow 670 \times 5.2 = 3\,484$$
   $$\Rightarrow \frac{34.84}{5.2} = \underline{670}.$$

2. Karen got 32 out of 80 in a maths test. (2)
   She got 38% in an English test.

Karen wants to know if she got a higher percentage in maths or in English.

Did Karen get a higher percentage in maths or in English?

Solution

In maths test, she got

$$\frac{32}{80} \times 100\% = \frac{32}{4} \times 5\%$$
$$= 8 \times 5\%$$
$$= 40\%$$

so she did better on the <u>maths test</u>.

3. Here are the heights, in cm, of 18 children. (3)

$$
\begin{array}{cccccc}
98 & 90 & 84 & 102 & 115 & 91 \\
88 & 91 & 108 & 110 & 97 & 93 \\
90 & 89 & 103 & 95 & 92 & 106 \\
\end{array}
$$

Show this information in an ordered stem and leaf diagram.

Solution

$$
\begin{array}{c|ccccccccc}
11 & 0 & 5 \\
10 & 2 & 3 & 6 & 8 \\
9 & 0 & 0 & 1 & 1 & 2 & 3 & 5 & 7 & 8 \\
8 & 4 & 8 & 9 \\
\end{array}
$$

Key: 8|4 means 84 cm.

4. Kalinda buys x packs of currant buns and y boxes of iced buns. (3)

There are 6 currant buns in a pack of currant buns.
There are 8 iced buns in a box of iced buns.

Kalinda buys a total of T buns.
Write down a formula for T in terms of x and y.

5. (a) Solve the inequality (2)

$$6y + 5 > 8.$$

Solution

$$6y + 5 > 8 \Rightarrow 6y > 3$$
$$\Rightarrow y > \tfrac{1}{2}.$$

(b) Here is an inequality, in x, shown on a number line. (2)
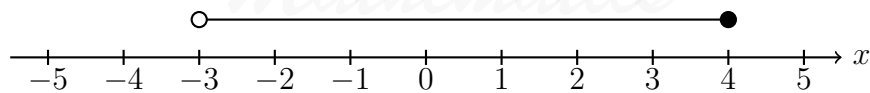
Write down the inequality.

Solution

$$-3 < x \leqslant 4.$$

6. Steve wants to put a hedge along one side of his garden. (4)

He needs to buy 27 plants for the hedge.
Each plant costs £5.54.

Steve has £150 to spend on plants for the hedge.

Does Steve have enough money to buy all the plants he needs?

Solution

$$\begin{array}{c|ccc} \times & 5 & 0.5 & 0.04 \\ \hline 20 & 100 & 10 & 0.8 \\ 7 & 35 & 3.5 & 0.28 \end{array}$$

He has

$$100 + 10 + 0.8 + 35 + 3.5 + 0.28 = 145 + 4.3 + 0.28$$
$$= 145 + 4.58$$
$$= \pounds 149.58$$

so, <u>yes</u>, he has enough money.

7. The diagram shows the plan of a floor. (4)

15 m

8 m

x m

3 m
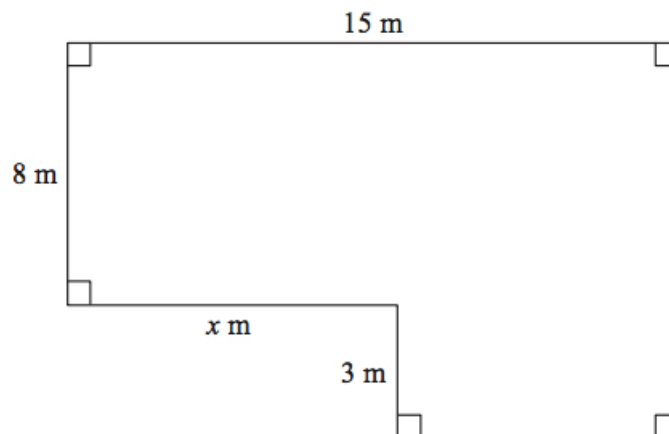
The area of the floor is 138 m².

Work out the value of x.

Solution

$$(8 \times 15) + [3 \times (15 - x)] = 138 \Rightarrow 120 + 3(15 - x) = 138$$
$$\Rightarrow 3(15 - x) = 18$$
$$\Rightarrow 15 - x = 6$$
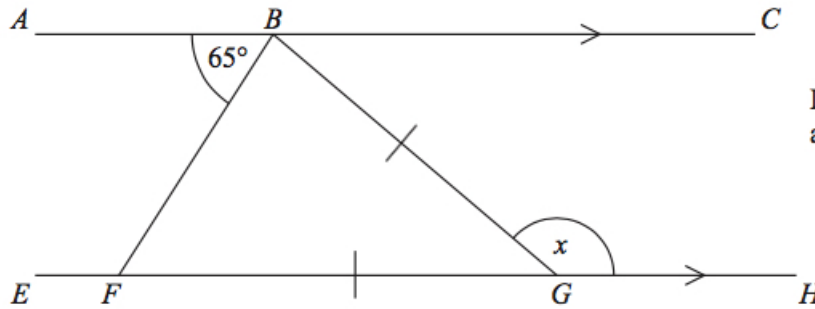$$\Rightarrow \underline{\underline{x = 9}}.$$

8. ABC is parallel to $EFGH$. (4)

Diagram **NOT** accurately drawn

$GB = GF$.
Angle $ABF = 65°$.

Work out the size of the angle marked x.
Give reasons for your answer.

> **Solution**
>
> $\angle BFG = 65°$ (alternate angles)
> $\angle FBG = 65°$ (base angles in an isosceles triangle)
> $x° = 65 + 65 = \underline{130°}$ (exterior angles)

9. Jack wants to find out how far people live from their nearest supermarket.

He uses this question on a questionnaire.

> How far do you live from your nearest supermarket?
>
> ☐   ☐   ☐   ☐
>
> 0 to 2   2 to 3   3 to 4   5 to 6

(a) Write down **two** things wrong with this question. (2)

> **Solution**
>
> E.g., There is an overlap on 2, there is an overlap on 3, there is a gap between '4 to 5,' no space to write down '6 or more,' no units (metres, yards, furlong, etc).

Jack also wants to find out how often people go shopping.

(b) Write a question Jack could use on his questionnaire to find out how often people go shopping. (2)
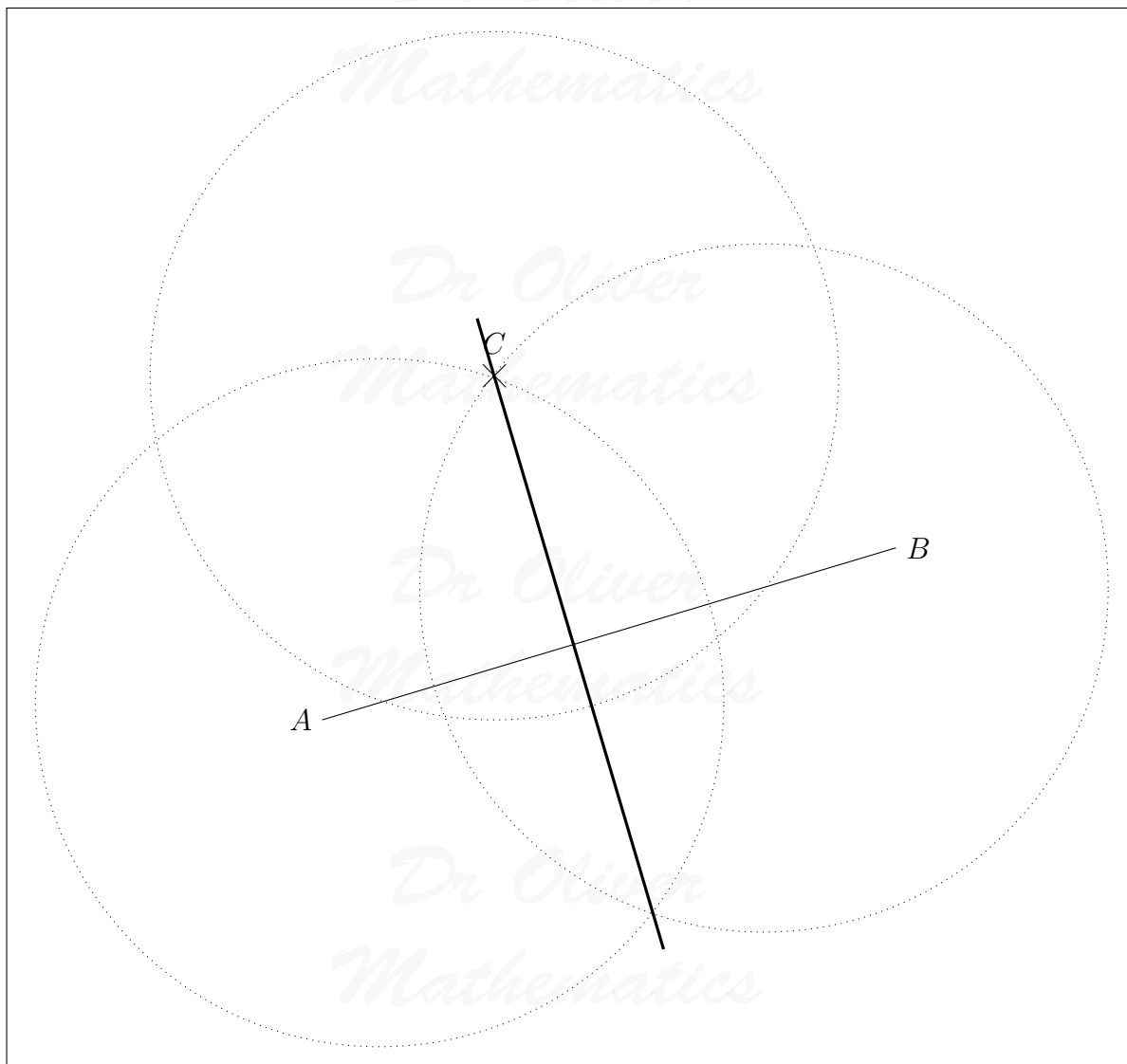
> **Solution**
>
> A suitable question with a time frame, e.g., "How far do you from your nearest supermarket? Tick the appropriate box."
> At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, 0 metres $< x < 300$ metres, 300 metres $\leqslant x < 600$ metres, 600 metres $\leqslant x < 1$ km, $x \geqslant 1$ km.

10. Use ruler and compasses to construct the perpendicular from point C to the line AB. You must show all your construction lines. (2)

C
$\times$

B

A
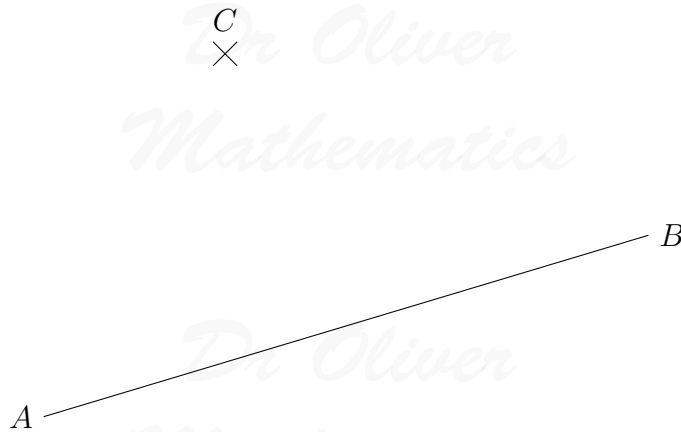
> **Solution**

11. Ria is going to buy a caravan. (4)
    The total cost of the caravan is £7 000 **plus** VAT at 20%.

    Ria pays a deposit of £3 000.
    She pays the rest of the total cost in 6 equal monthly payments.

    Work out the amount of each monthly payment.

Solution
Originally, he owes
$$7\,000 \times 1.2 = \text{£}8\,400.$$

Once the deposit has gone, Ria owes

$$8\,400 - 3\,000 = \text{\pounds}5\,400.$$

Finally, he amount of each monthly payment is

$$\frac{5\,400}{6} = \underline{\underline{\text{\pounds}900}}.$$

12. (a) Factorise   (1)

$$3e^2 + 5e.$$

Solution

$$3e^2 + 5e = \underline{\underline{e(3e + 5)}}.$$

(b) Solve   (3)

$$7(k - 3) = 3k - 5.$$

Solution

$$7(k - 3) = 3k - 5 \Rightarrow 7k - 21 = 3k - 5$$
$$\Rightarrow 4k = 16$$
$$\Rightarrow \underline{\underline{k = 4}}.$$

(c) Expand and simplify   (2)

$$(2x + 3)(x - 8).$$

Solution

$\times$	$2x$	$+3$
x	$2x^2$	$+3x$
-8	$-16x$	-24

$$(2x + 3)(x - 8) = \underline{\underline{2x^2 - 13x - 24}}.$$

(d) Solve (3)

$$\frac{7 - 3f}{4} = 2.$$

Solution

$$\frac{7 - 3f}{4} = 2 \Rightarrow 7 - 3f = 8$$
$$\Rightarrow 3f = -1$$
$$\Rightarrow \underline{\underline{f = -\tfrac{1}{3}}}.$$

13. (a) Express 180 as a product of its prime factors. (3)

Solution

$$
\begin{array}{c|c}
 & 180 \\
\hline
2 & 90 \\
\hline
2 & 45 \\
\hline
3 & 15 \\
\hline
3 & 5 \\
\hline
5 & 1 \\
\hline
\end{array}
$$

So

$$180 = 2 \times 2 \times 3 \times 3 \times 5 = \underline{\underline{2^2 \times 3^2 \times 5}}.$$

Martin thinks of two numbers.

He says, "The Highest Common Factor (HCF) of my two numbers is 6. The Lowest Common Multiple (LCM) of my two numbers is a multiple of 15."

(b) Write down **two** possible numbers that Martin is thinking of. (2)

Solution
E.g., $\underline{\underline{6}}$ and $2 \times 3 \times 5 = \underline{\underline{30}}$.

14. Suha has a full 600 ml bottle of wallpaper remover. (4)
    She is going to mix some of the wallpaper remover with water.

    Here is the information on the label of the bottle.

    > **Wallpaper remover**
    > 600 m*l*
    >
    > Mix $\dfrac{1}{4}$ of the wallpaper remover
    >
    > with 4500 m*l* of water

    Suha is going to use 750 ml of water.

    How many millilitres of wallpaper remover should Suha use?
    You must show your working.

    > **Solution**
    >
    > $$150 \text{ wallpaper} \leftrightarrow 4\,500 \text{ water} \Rightarrow 1 \text{ wallpaper} \leftrightarrow 30 \text{ water}$$
   > $$\Rightarrow 25 \text{ wallpaper} \leftrightarrow 750 \text{ water};$$
    >
    > hence, she should use 25 wallpaper remover.

15. The students in a class kept a record of the amount of time, in minutes, they spent doing homework last week.

    The table shows information about the amount of time the girls spent doing homework last week.

	Minutes
Least amount of time	60
Range	230
Median	170
Lower quartile	100
Upper quartile	220

    (a) On the grid, draw a box plot for the information in the table. (2)

Amount of time (minutes)

Solution

Amount of time (minutes)
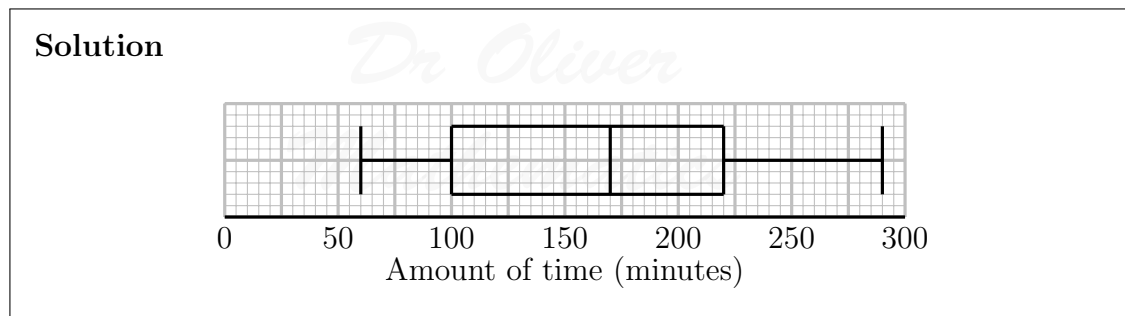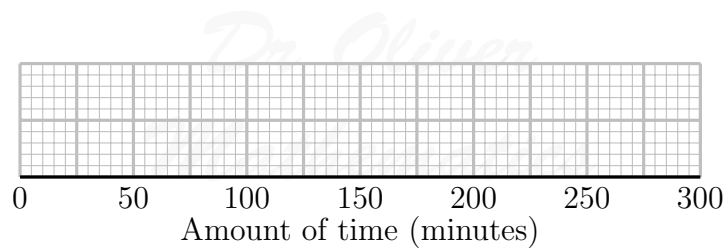
The box plot below shows information about the amount of time the boys spent doing homework last week.

Amount of time (minutes)
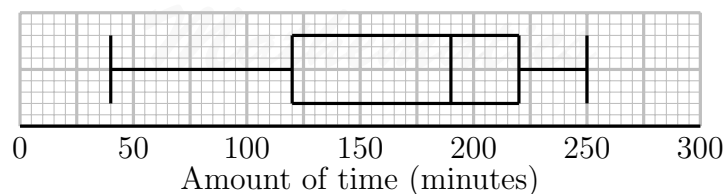
(b) Compare the amount of time the girls spent doing homework with the amount of time the boys spent doing homework.   (2)

> **Solution**
> **Average**
> Since the median for the boys (190) is higher than the median for the girls (170), the boys did more revision on average.
>
> **Spread**
> Since the range for the boys ($250 - 40 = 210$) is smaller than the range for the girls ($290 - 60 = 230$), the revision were more consistent in the boys.
> **OR**
> Since the IQR for the boys ($220 - 120 = 100$) is smaller than the IQR for the girls ($220 - 100 = 120$), the revision were more consistent in the boys.
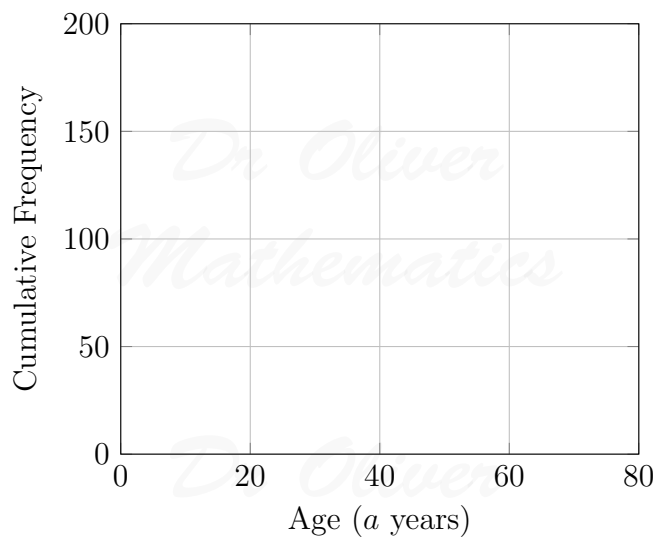
16. There are 200 workers at a factory.

   The cumulative frequency table gives information about their ages.
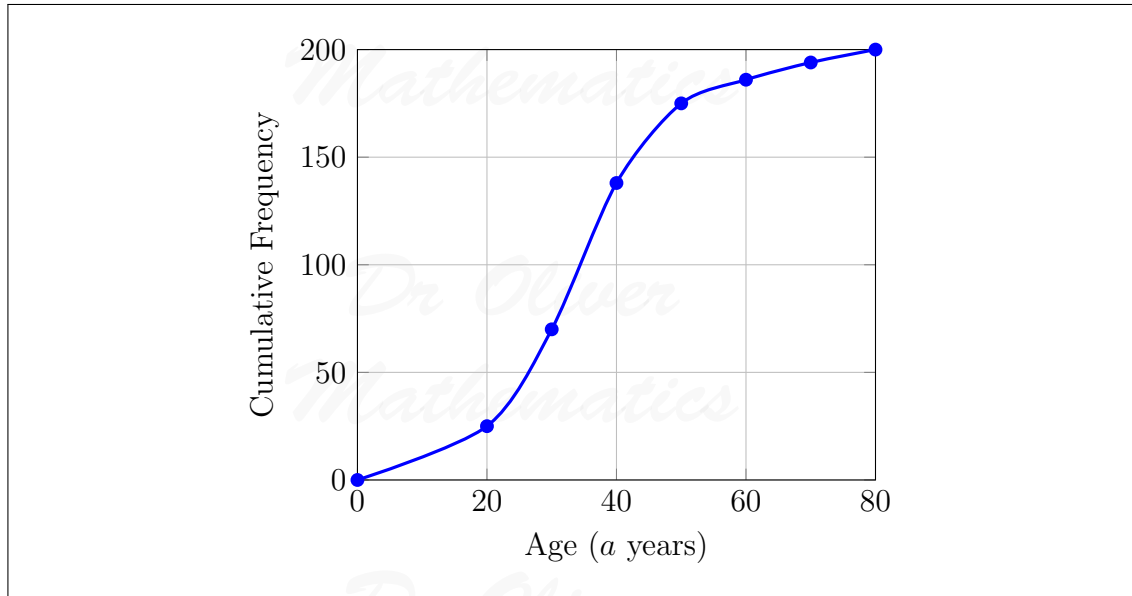
11

Age (a years)	Cumulative frequency
$0 < a \leqslant 20$	25
$0 < a \leqslant 30$	70
$0 < a \leqslant 40$	138
$0 < a \leqslant 50$	175
$0 < a \leqslant 60$	186
$0 < a \leqslant 70$	194
$0 < a \leqslant 80$	200

(a) On the grid, draw a cumulative frequency graph for this information. (2)
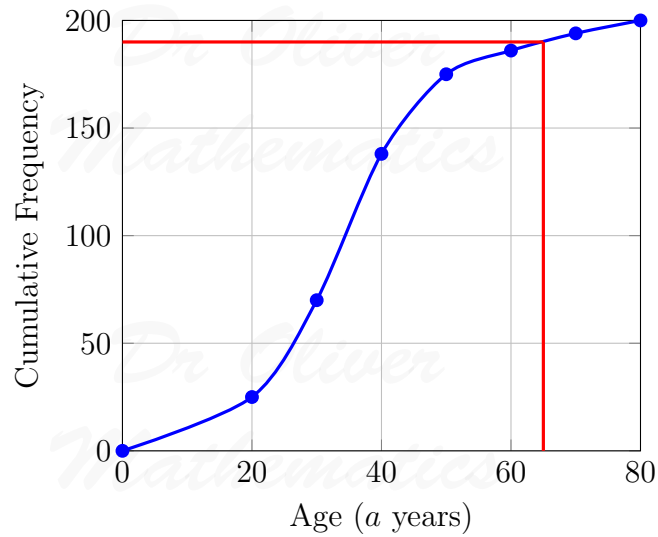
Solution

(b) Graham says, "10% of workers at the factory are older than 65." (2)

Is Graham correct?
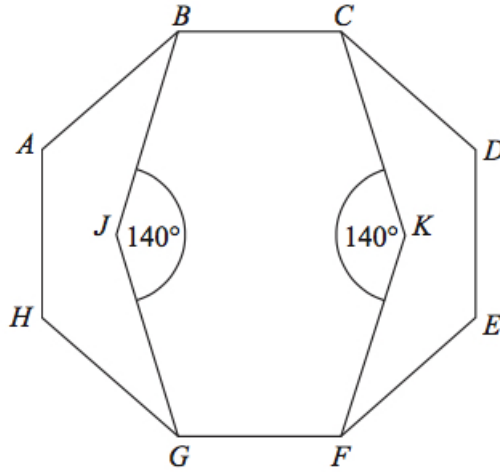You must show how you get your answer.

Solution

Correct read-off: approximately

$$\frac{10}{200} \times 100\% = 5\%$$

17. $ABCDEFGH$ is a regular octagon. (4)
    $BCKFGJ$ is a hexagon.

JK is a line of symmetry of the hexagon.
Angle BJG = angle CKF = 140°.

Work out the size of angle KFE.
You must show all your working.

Solution

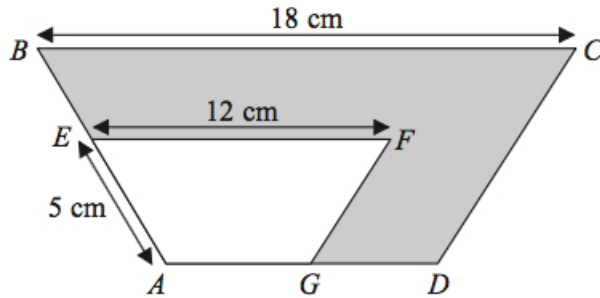The exterior angle in a regular octagon is

$$\frac{6}{8} \times 180 = 135°$$

and

$$\angle KFC = \angle KCF = \tfrac{1}{2}(180 - 140) = 20°.$$

Finally,

$$\angle KFE = 135 - 90 - 20 = \underline{\underline{25}}°.$$

18. $ABCD$ and $AEFG$ are mathematically similar trapeziums.

$AE = 5$ cm.
$EF = 12$ cm.
$BC = 18$ cm.

(a) Work out the length of AB. (2)

Solution
$$AB = \frac{18}{12} \times 5 = \underline{\underline{7.5 \text{ cm}}}.$$

Trapezium $AEFG$ has an area of 36 cm².

(b) Work out the area of the shaded region. (3)

Solution
The length scale factor (LSF) is

$$18 : 12 = 3 : 2$$

and so area scale factor (ASF) is

$$3^2 : 2^2 = 9 : 4.$$

Finally,

$$\text{area of the shaded region} = \left(\frac{9}{4} \times 36\right) - 36$$
$$= (9 \times 9) - 36$$
$$= 81 - 36$$
$$= \underline{\underline{45 \text{ cm}^2}}.$$

19. (a) Complete the table of values for $y = \frac{4}{x}$. (2)

x	0.5	1	2	4	5	8
y			4	2		

(b) On the grid, draw the graph of (2)
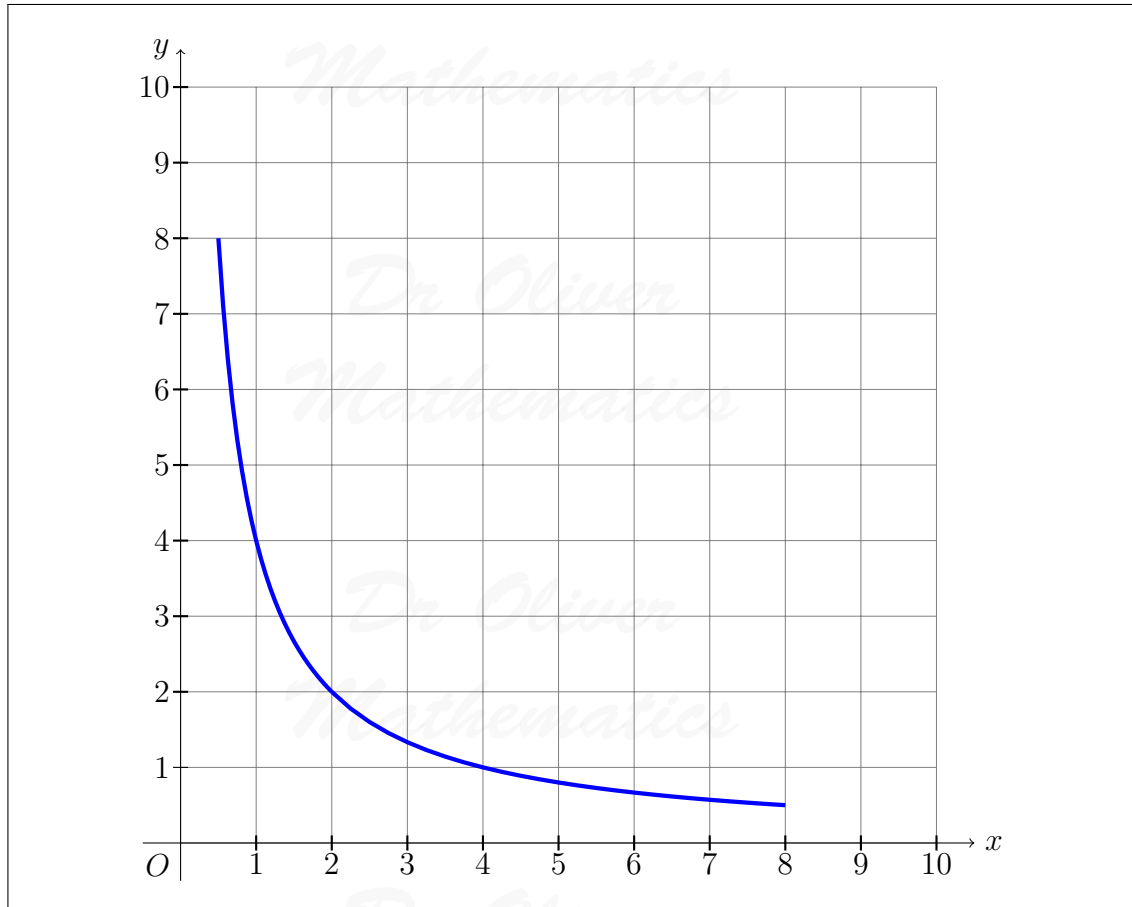
$$y = \frac{4}{x}$$

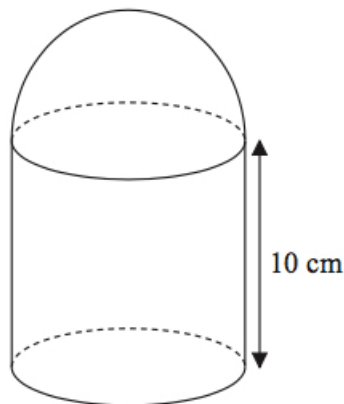for $0.5 \leqslant x \leqslant 8$.
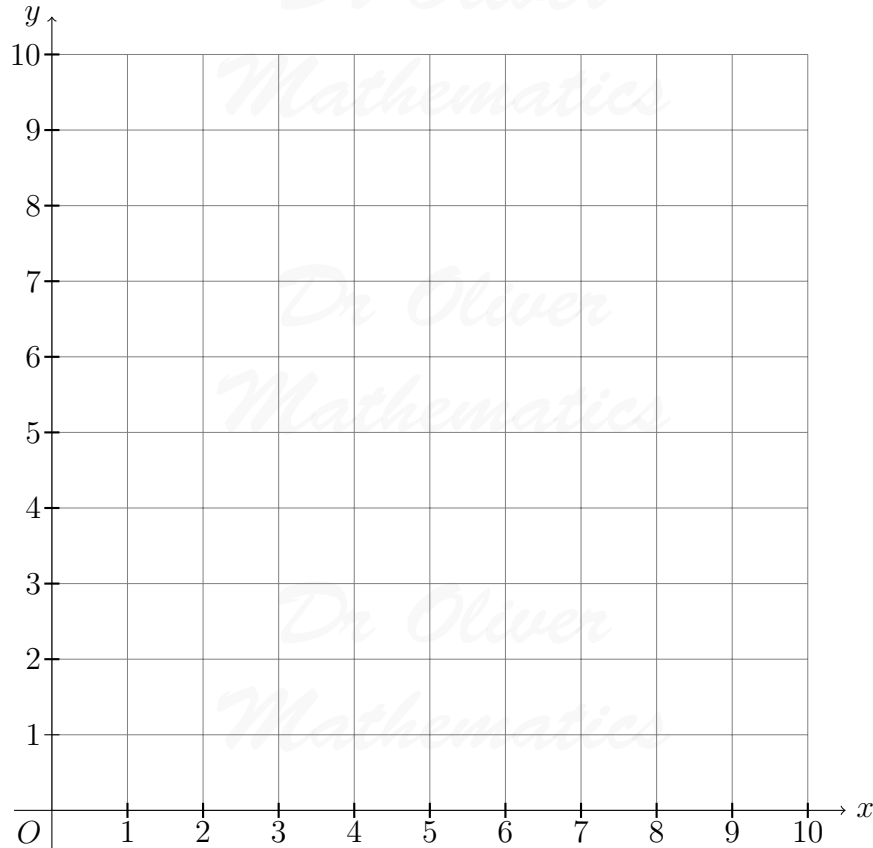
Solution

20. The diagram shows a solid shape. (5)

10 cm

The solid shape is made from a cylinder and a hemisphere.
The radius of the cylinder is equal to the radius of the hemisphere.

17

The cylinder has a height of 10 cm.
The curved surface area of the hemisphere is 32π cm^2.

Work out the total surface area of the solid shape.
Give your answer in terms of π.

Solution

Let r cm be the radius of the base. Now,

$$2 \times \pi \times r^2 = 32\pi \Rightarrow r^2 = 16$$
$$\Rightarrow r = 4$$

and the total surface area of the solid shape is

$$(\pi \times 4^2) + (2 \times \pi \times 4 \times 10) + 32\pi = (16 + 80 + 32)\pi$$
$$= \underline{\underline{128\pi}}.$$

21. Expand (2)
$$(1 + \sqrt{2})(3 - \sqrt{2}).$$
Give your answer in the form $a + b\sqrt{2}$ where a and b are integers.

Solution

$\times$	1	$+\sqrt{2}$
3	3	$+3\sqrt{2}$
$-\sqrt{2}$	$-\sqrt{2}$	-2

$$(1 + \sqrt{2})(3 - \sqrt{2}) = \underline{\underline{1 + 2\sqrt{2}}}.$$

22. (a) Simplify fully (1)
$$(3e)^0.$$

Solution

$$(3e)^0 = \underline{\underline{1}}$$

(unless $e = 0$.)

(b) Simplify fully (2)

$$\left(\frac{64x^6}{25y^2}\right)^{-\frac{1}{2}}.$$

Solution

$$\left(\frac{64x^6}{25y^2}\right)^{-\frac{1}{2}} = \left(\frac{25y^2}{64x^6}\right)^{\frac{1}{2}}$$

$$= \underline{\underline{\frac{5y}{8x^3}}}.$$

(c) Write (3)

$$\frac{5}{x-3} - \frac{4}{x+3}$$

as a single fraction in its simplest form.

Solution

$$\frac{5}{x-3} - \frac{4}{x+3} = \frac{5(x+3)}{(x-3)(x+3)} - \frac{4(x-3)}{(x-3)(x+3)}$$

$$= \frac{5(x+3) - 4(x-3)}{(x-3)(x+3)}$$

$$= \frac{5x + 15 - 4x + 12}{(x-3)(x+3)}$$

$$= \underline{\underline{\frac{x+27}{(x-3)(x+3)}}}.$$

23. Paul has 8 cards. (4)

There is a number on each card.

| 2 | 3 | 3 | 4 | 5 | 5 | 5 | 5 |

Paul takes at random 3 of the cards.
He adds together the 3 numbers on the cards to get a total T.

Work out the probability that T is an odd number.

> **Solution**
>
> $$\text{P(odd number)} = \text{P(2, 4, and an odd card)} + \text{P(three odd cards)}$$
> $$= \left(6 \times \tfrac{1}{8} \times \tfrac{1}{7} \times \tfrac{6}{6}\right) + \left(\tfrac{6}{8} \times \tfrac{5}{7} \times \tfrac{4}{6}\right)$$
> $$= \tfrac{36}{336} + \tfrac{120}{336}$$
> $$= \tfrac{156}{336}$$
> $$= \tfrac{78}{168}$$
> $$= \tfrac{39}{84}$$
> $$= \underline{\underline{\tfrac{13}{28}}}.$$

24. A is the point with coordinates $(1, 3)$.   (4)
    B is the point with coordinates $(4, -1)$.
    The straight line L goes through both A and B.

    Is the line with equation $2y = 3x - 4$ perpendicular to line L?
    You must show how you got your answer.

> **Solution**
>
> For L,
>
> $$\text{gradient} = \frac{3 - (-1)}{1 - 4}$$
> $$= -\tfrac{4}{3}$$
>
> and the line perpendicular to it has
>
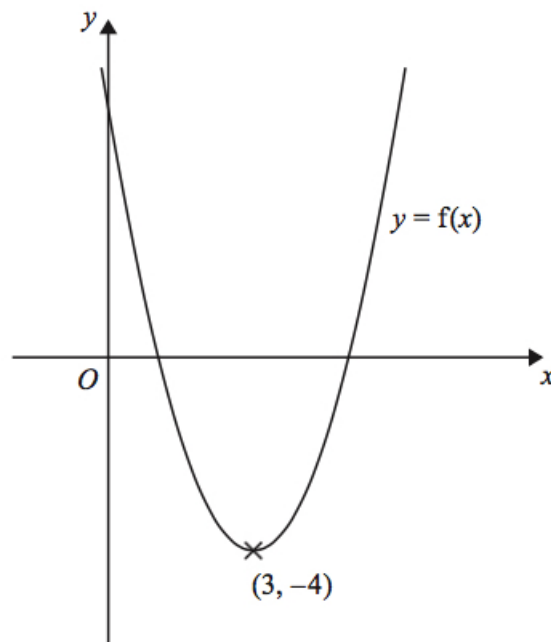> $$\text{gradient} = \tfrac{3}{4}.$$

Now,
$$2y = 3x - 4 \Rightarrow y = \tfrac{3}{2}x - 2;$$
clearly, $2y = 3x - 4$ and L <u>are not perpendicular</u> because
$$-\tfrac{4}{3} \times \tfrac{3}{2} \neq -1.$$

25. The diagram shows part of the curve with equation $y = \mathrm{f}(x)$.
    The coordinates of the minimum point of this curve are $(3, -4)$.

Write down the coordinates of the minimum point of the curve with equation

(a) $y = \mathrm{f}(x) + 3$,   (1)

Solution
$(3, -1)$.

(b) $y = \mathrm{f}(2x)$,   (1)

Solution
$(\tfrac{3}{2}, -4)$.

(c) $y = \mathrm{f}(-x)$. (1)

Solution

$(-3, -4)$.