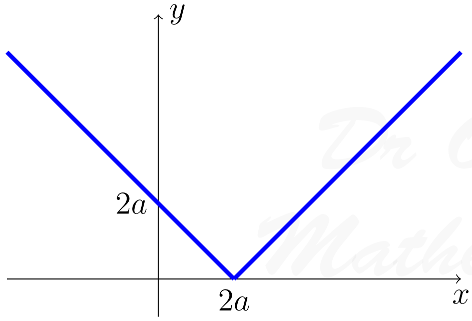
Dr Oliver Mathematics
Further Mathematics
Rational Inequalities
Past Examination Questions

This booklet consists of 20 questions across a variety of examination topics.
The total number of marks available is 162.

1. (a) Sketch the graph of $y = |x - 2a|$, given that $a > 0$. (2)

Solution



(b) Solve $|x - 2a| > 2x + a$, where $a > 0$. (3)

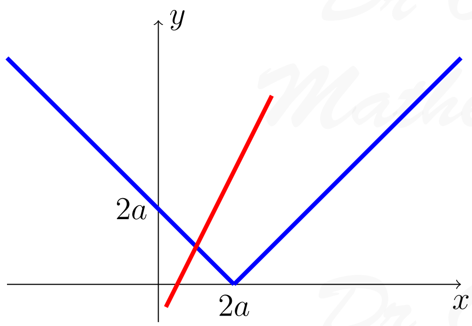
Solution

$\underline{x - 2a > 2x + a}$:

$$x - 2a > 2x + a \Rightarrow x < -3a.$$

$\underline{-(x - 2a) > 2x + a}$:

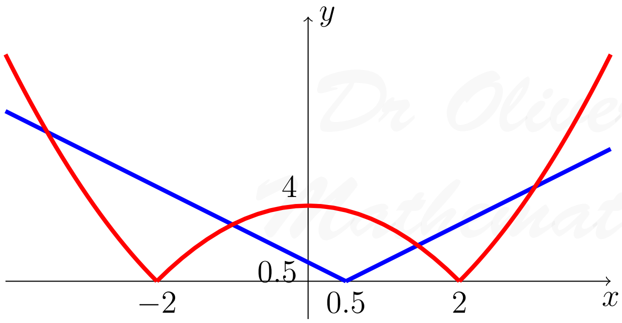
$$-x + 2a > 2x + a \Rightarrow a > 3x \Rightarrow x < \tfrac{1}{3}a.$$



Hence $\underline{\underline{x < \tfrac{1}{3}a}}$.

2. (a) On the same diagram, sketch the curve with equation $y = |x^2 - 4|$ and the line with equation $y = |2x - 1|$, showing the coordinates of the points where the curve meets the axes. (4)

Solution



(b) Solve $|x^2 - 4| = |2x - 1|$, giving your answer in surd form as appropriate. (5)

Solution

$\underline{x^2 - 1 = 2x - 1}$:

$$\begin{aligned} x^2 - 4 = 2x - 1 &\Rightarrow x^2 - 2x - 3 = 0 \\ &\Rightarrow (x - 3)(x + 1) = 0 \\ &\Rightarrow \underline{\underline{x = -1 \text{ or } x = 3}}. \end{aligned}$$

$\underline{x^2 - 1 = -(2x - 1)}$:

$$\begin{aligned} x^2 - 4 = -2x + 1 &\Rightarrow x^2 + 2x = 5 \\ &\Rightarrow x^2 + 2x + 1 = 6 \\ &\Rightarrow (x + 1)^2 = 6 \\ &\Rightarrow x + 1 = \pm\sqrt{6} \\ &\Rightarrow \underline{\underline{x = -1 \pm \sqrt{6}}}. \end{aligned}$$

(c) Hence, or otherwise, find the set of values of x for which $|x^2 - 4| > |2x - 1|$. (3)

Solution

Hence $\underline{\underline{x < -1 - \sqrt{6},\ -1 < x < -1 + \sqrt{6},\ \text{ or } x > 3}}$.

3. (a) Use algebra to find the exact solutions of the equation (6)

$$|2x^2 + x - 6| = 6 - 3x.$$

Solution

$\underline{2x^2 + x - 6 = 6 - 3x}$:

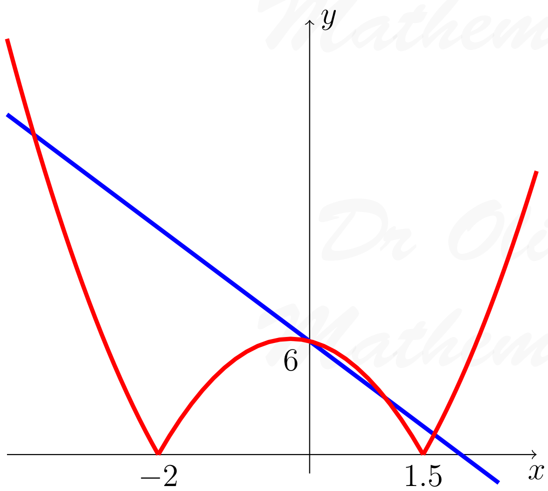
$$\begin{aligned} 2x^2 + x - 6 = 6 - 3x &\Rightarrow 2x^2 + 4x - 12 = 0 \\ &\Rightarrow 2(x^2 + 2x - 6) = 0 \\ &\Rightarrow x^2 + 2x - 6 = 0 \\ &\Rightarrow x^2 + 2x + 1 = 7 \\ &\Rightarrow (x+1)^2 = 7 \\ &\Rightarrow x + 1 = \pm\sqrt{7} \\ &\Rightarrow \underline{\underline{x = -1 \pm \sqrt{7}}}. \end{aligned}$$

$\underline{-(2x^2 + x - 6) = 6 - 3x}$:

$$\begin{aligned} -2x^2 - x + 6 = 6 - 3x &\Rightarrow 0 = 2x^2 - 2x \\ &\Rightarrow 0 = 2x(x - 1) \\ &\Rightarrow \underline{\underline{x = 0 \text{ or } x = 1.}} \end{aligned}$$

(b) On the same diagram, sketch the curve with equation $y = |2x^2 + x - 6|$ and the line with equation $y = 6 - 3x$. (3)

Solution



(c) Find the set of values of x for which (3)

$$|2x^2 + x - 6| > 6 - 3x.$$

Solution

$x < -1 - \sqrt{7}, 0 < x < 1, \text{ or } x > -1 + \sqrt{7}.$

4. Figure 1 shows a sketch of the curve with equation

$$y = \frac{x^2 - 1}{|x + 2|}, \, x \neq -2.$$

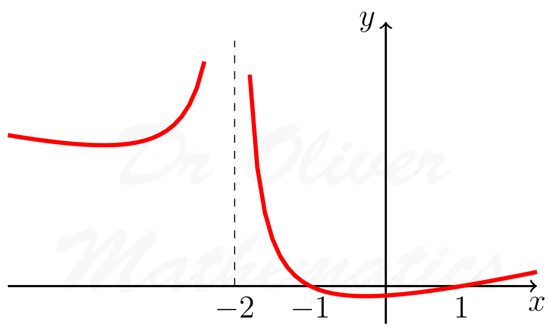


Figure 1: $y = \dfrac{x^2 - 1}{|x + 2|}$

The curve crosses the x-axis at $x = 1$ and $x = -1$ and the line $x = -2$ is an asymptote of the curve.

(a) Use algebra to solve the equation (6)

$$\frac{x^2 - 1}{|x + 2|} = 3(1 - x).$$

Solution

$\frac{x^2-1}{x+2} = 3(1-x)$:

$$\begin{aligned}\frac{x^2-1}{x+2} = 3(1-x) &\Rightarrow x^2 - 1 = 3(1-x)(x+2)\\ &\Rightarrow x^2 - 1 = 3(2 - x - x^2)\\ &\Rightarrow x^2 - 1 = 6 - 3x - 3x^2\\ &\Rightarrow 4x^2 + 3x - 7 = 0\\ &\Rightarrow (4x+7)(x-1) = 0\\ &\Rightarrow x = -\tfrac{7}{4} \text{ or } x = 1.\end{aligned}$$

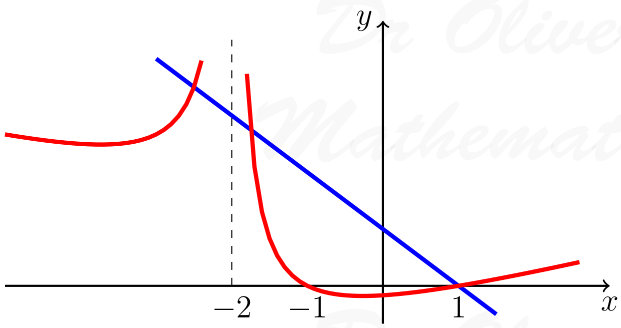
$\frac{x^2-1}{-(x+2)} = 3(1-x)$:

$$\begin{aligned}\frac{x^2-1}{-(x+2)} = 3(1-x) &\Rightarrow x^2 - 1 = -3(1-x)(x+2)\\ &\Rightarrow x^2 - 1 = -3(2 - x - x^2)\\ &\Rightarrow x^2 - 1 = 3x^2 + 3x - 6\\ &\Rightarrow 0 = 2x^2 + 3x - 5\\ &\Rightarrow 0 = (2x+5)(x-1)\\ &\Rightarrow x = -\tfrac{5}{2}. \qquad (x = 1 \text{ is not a solution})\end{aligned}$$

(b) Hence, or otherwise, find the set of values of x for which (3)

$$\frac{x^2-1}{|x+2|} < 3(1-x).$$

Solution



Hence $x < -\frac{5}{2}$ or $-\frac{7}{4} < x < 1$.

5. Find the set of values of x for which (7)

$$\frac{x+1}{2x-3} < \frac{1}{x-3}.$$

Solution

$$\begin{aligned}\frac{x+1}{2x-3} < \frac{1}{x-3} &\Rightarrow \frac{x+1}{2x-3} - \frac{1}{x-3} < 0 \\ &\Rightarrow \frac{(x+1)(x-3)-(2x-3)}{(x-3)(2x-3)} < 0 \\ &\Rightarrow \frac{(x^2-2x-3)-2x+3}{(x-3)(2x-3)} < 0 \\ &\Rightarrow \frac{x^2-4x}{(x-3)(2x-3)} < 0 \\ &\Rightarrow \frac{x(x-4)}{(x-3)(2x-3)} < 0,\end{aligned}$$

and so the four critical values are 0, $\frac{3}{2}$, 3, and 4.

	$x<0$	$0<x<\frac{3}{2}$	$\frac{3}{2}<x<3$	$3<x<4$	$x>4$
x	$-$	$+$	$+$	$+$	$+$
$x-\frac{3}{2}$	$-$	$-$	$+$	$+$	$+$
$x-3$	$-$	$-$	$-$	$+$	$+$
$x-4$	$-$	$-$	$-$	$-$	$+$
$\frac{x(x-4)}{(x-3)(2x-3)}$	$+$	$-$	$+$	$-$	$+$

Hence $\underline{\underline{0 < x < \frac{3}{2} \text{ or } 3 < x < 4}}$.

6. (a) Simplify the expression (4)

$$\frac{(x+3)(x+9)}{x-1} - (3x-5),$$

giving your answer in the form

$$\frac{a(x+b)(x+c)}{x-1},$$

where a, b, c are integers.

Solution

$$\begin{aligned}\frac{(x+3)(x+9)}{x-1}-(3x-5) &= \frac{(x+3)(x+9)-(3x-5)(x-1)}{x-1}\\ &= \frac{(x^2+12x+27)-(3x^2-8x+5)}{x-1}\\ &= \frac{-2x^2+20x+22}{x-1}\\ &= \frac{-2(x^2-10x-11)}{x-1}\\ &= \underline{\underline{\frac{-2(x-11)(x+1)}{x-1}}};\end{aligned}$$

hence, $\underline{\underline{a=-2}}$, $\underline{\underline{b=-11}}$, and $\underline{\underline{c=1}}$.

(b) Hence, or otherwise, solve the inequality (4)

$$\frac{(x+3)(x+9)}{x-1} > (3x-5).$$

Solution

So

$$\frac{(x+3)(x+9)}{x-1} > (3x-5) \Rightarrow \frac{-2(x-11)(x+1)}{x-1} > 0.$$

	$x<-1$	$-1<x<1$	$1<x<11$	$x>11$
$x+1$	$-$	$+$	$+$	$+$
$x-1$	$-$	$-$	$+$	$+$
$x-11$	$-$	$-$	$-$	$+$
$\frac{-2(x-11)(x+1)}{x-1}$	$+$	$-$	$+$	$-$

Hence $\underline{\underline{x<-1 \text{ or } 1<x<11}}$.

7. (a) Find, in the simplest surd form where appropriate, the exact values of x for which (5)

$$\frac{x}{2}+3=\left|\frac{4}{x}\right|.$$

Solution

$\underline{\frac{x}{2}+3=\frac{4}{x}}$:

$$\begin{aligned}\frac{x}{2}+3=\frac{4}{x} &\Rightarrow x^2+6x=8\\ &\Rightarrow x^2+6x-8=0\\ &\Rightarrow (x^2+6x+9)-17=0\\ &\Rightarrow (x+3)^2=17\\ &\Rightarrow x=-3\pm\sqrt{17}.\end{aligned}$$

But $x=-3-\sqrt{17}$ is **not** a solution (because we've flipped the left-hand part of the graph).

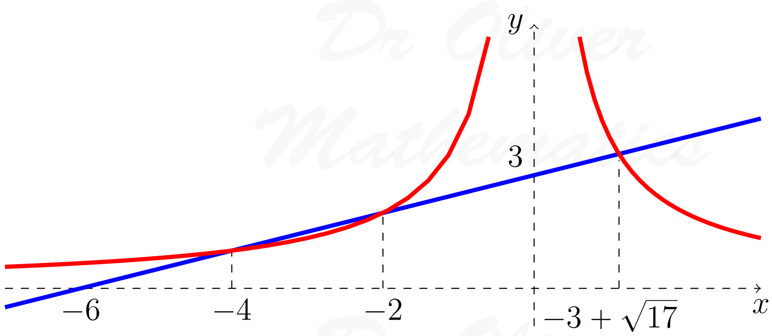
$\underline{\frac{x}{2}+3=-\frac{4}{x}}$:

$$\begin{aligned}\frac{x}{2}+3=-\frac{4}{x} &\Rightarrow x^2+6x=-8\\ &\Rightarrow x^2+6x+8=0\\ &\Rightarrow (x+2)(x+4)=0\\ &\Rightarrow x=-4 \text{ or } x=-2.\end{aligned}$$

The three correct solutions are $\underline{\underline{x=-4,-2,\text{ or } -3+\sqrt{17}}}$.

(b) Sketch, on the same axes, the line with equation $y=\frac{x}{2}+3$ and the graph of $y=\left|\frac{4}{x}\right|$, $x\neq 0$. (3)

Solution



(c) Find the set of values of x for which (2)

$$\frac{x}{2} + 3 > \left|\frac{4}{x}\right|.$$

Solution

$\underline{\underline{-4 < x < -2}}$ or $\underline{\underline{x > -3 + \sqrt{17}}}$

8. Find the set of values of x for which (6)

$$\frac{x^3 + 5x - 12}{x - 3} > 4.$$

Solution

$$\begin{aligned} \frac{x^3 + 5x - 12}{x - 3} > 4 &\Rightarrow \frac{x^3 + 5x - 12}{x - 3} - 4 > 0 \\ &\Rightarrow \frac{x^3 + 5x - 12 - 4(x - 3)}{x - 3} > 0 \\ &\Rightarrow \frac{x^3 + x}{x - 3} > 0 \\ &\Rightarrow \frac{x(x^2 + 1)}{x - 3} > 0, \end{aligned}$$

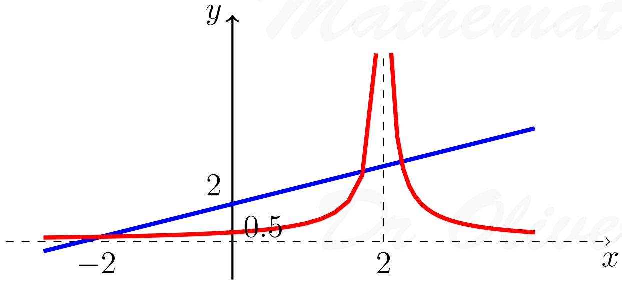
and so the two critical values are $x = 0$ and 3.

	$x < 0$	$0 < x < 3$	$x > 3$
x	$-$	$+$	$+$
$x - 3$	$-$	$-$	$+$
$\frac{x(x^2 + 1)}{x - 3}$	$+$	$-$	$+$

Hence $\underline{\underline{x < 0 \text{ or } x > 3}}$.

9. (a) On the same diagram, sketch the graph of $y = x + 2$ and the graph of $y = \left|\frac{1}{x-2}\right|$. Indicate on your sketch the coordinates of any points at which the graphs cross the axes, and state the equations of any asymptotes. (6)

Solution



(b) Find the set of values of x for which (6)

$$x + 2 < \left| \frac{1}{x-2} \right|.$$

Solution

$$-\frac{1}{x-2} < x + 2 < \frac{1}{x-2}$$

$-\frac{1}{x-2} < x + 2$:

$$\begin{aligned} -\frac{1}{x-2} < x + 2 &\Rightarrow -1 < (x+2)(x-2) \\ &\Rightarrow -1 < x^2 - 4 \\ &\Rightarrow 3 < x^2 \\ &\Rightarrow -\sqrt{3} < x < \sqrt{3}. \end{aligned}$$

$x + 2 < \frac{1}{x-2}$:

$$\begin{aligned} x + 2 < \frac{1}{x-2} &\Rightarrow (x+2)(x-2) < 1 \\ &\Rightarrow x^2 - 4 < 1 \\ &\Rightarrow x^2 < 5 \\ &\Rightarrow -\sqrt{5} < x \text{ or } x > \sqrt{5}. \end{aligned}$$

By taking a look at part (a), we can see that $-\sqrt{3} < x$ or $\sqrt{3} < x < \sqrt{5}$.

10. Figure 2 shows the graph of $y = 10 + 3x - x^2$ and the graph of $y = |3x - 1|$.

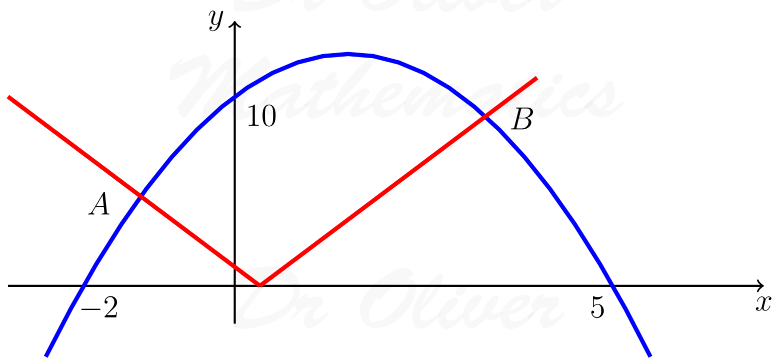


Figure 2: $y = 10 + 3x - x^2$ and $y = |3x - 1|$

The graphs intersect at the points A and B.

(a) Use algebra to find the exact x-coordinates of A and B. (5)

Solution

$\underline{10 + 3x - x^2 = 3x - 1}$:

$$\begin{aligned} 10 + 3x - x^2 = 3x - 1 &\Rightarrow 11 = x^2 \\ &\Rightarrow x = \pm\sqrt{11}; \end{aligned}$$

thus, $\underline{\underline{B = \sqrt{11}}}$.

$\underline{10 + 3x - x^2 = -(3x - 1)}$:

$$\begin{aligned} 10 + 3x - x^2 = -3x + 1 &\Rightarrow 0 = x^2 - 6x - 9 \\ &\Rightarrow 0 = (x^2 - 6x + 9) - 18 \\ &\Rightarrow 0 = (x - 3)^2 - 18 \\ &\Rightarrow x - 3 = \pm 3\sqrt{2} \\ &\Rightarrow x = 3 \pm 3\sqrt{2}; \end{aligned}$$

thus, $\underline{\underline{A = 3 - 3\sqrt{2}}}$.

(b) Find the set of values of x for which (2)

$$10 + 3x - x^2 > |3x - 1|.$$

Solution

$\underline{\underline{3 - 3\sqrt{2} < x < \sqrt{11}}}$.

(c) Find the set of values of x for which $|10 + 3x - x^2| < |3x - 1|$. (3)

Solution

Figure 3 shows the graph of $y = |10 + 3x - x^2|$ and the graph of $y = |3x - 1|$.

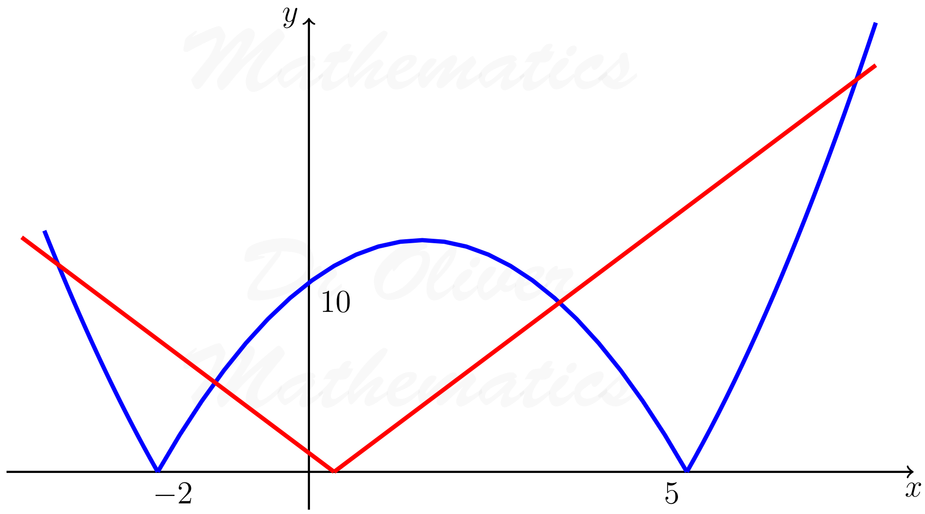


Figure 3: $y = |10 + 3x - x^2|$ and $y = |3x - 1|$

So, to take the four values of x,

$$\underline{\underline{-\sqrt{11} < x < 3 - 3\sqrt{2} \text{ and } \sqrt{11} < x < 3 + 3\sqrt{2}}}.$$

11. (a) Find the set of values of x for which (6)

$$x + 4 > \frac{2}{x + 3}.$$

Solution

$$
\begin{aligned}
x + 4 > \frac{2}{x+3} &\Rightarrow x + 4 - \frac{2}{x+3} > 0 \\
&\Rightarrow \frac{(x+3)(x+4) - 2}{x+3} > 0 \\
&\Rightarrow \frac{(x^2 + 7x + 12) - 2}{x+3} > 0 \\
&\Rightarrow \frac{x^2 + 7x + 10}{x+3} > 0 \\
&\Rightarrow \frac{(x+2)(x+5)}{x+3} > 0,
\end{aligned}
$$

and so the three critical values are $x = -5$, -3, and -2.

	$x < -5$	$-5 < x < -3$	$-3 < x < -2$	$x > -2$
$x + 5$	$-$	$+$	$+$	$+$
$x + 3$	$-$	$-$	$+$	$+$
$x + 2$	$-$	$-$	$-$	$+$
$\dfrac{(x+2)(x+5)}{x+3}$	$-$	$+$	$-$	$+$

Hence

$$\underline{\underline{-5 < x < -3 \text{ or } x > -2}}.$$

(b) Deduce, or otherwise find, the set of values of x for which (1)

$$x + 4 > \frac{2}{|x+3|}.$$

Solution

Consider the geometry of the problem. Nothing changes for $x > -2$ since both sides of the original inequality were already positive. For $-5 < x < -3$, both sides were negative so the introduction of the modulus on the right means that we lose these possible solutions. Hence $\underline{\underline{x > -2}}$.

12. Find the set of values of x for which (7)

$$\frac{3}{x+3} > \frac{x-4}{x}.$$

Solution

$$\begin{aligned}\frac{3}{x+3} > \frac{x-4}{x} &\Rightarrow \frac{3}{x+3} - \frac{x-4}{x} > 0\\ &\Rightarrow \frac{3x-(x+3)(x-4)}{x(x+3)} > 0\\ &\Rightarrow \frac{3x-(x^2-x-12)}{x(x+3)} > 0\\ &\Rightarrow \frac{12+4x-x^2}{x(x+3)} > 0\\ &\Rightarrow \frac{(6-x)(2+x)}{x(x+3)} > 0,\end{aligned}$$

and so the four critical values are $x = -3$, -2, 0, and 6.

	$x < -3$	$-3 < x < -2$	$-2 < x < 4$	$0 < x < 6$	$x > 6$
$x+3$	$-$	$+$	$+$	$+$	$+$
$x+2$	$-$	$-$	$+$	$+$	$+$
x	$-$	$-$	$-$	$+$	$+$
$6-x$	$+$	$+$	$+$	$+$	$-$
$\frac{(6-x)(2+x)}{x(x+3)}$	$-$	$+$	$-$	$+$	$-$

Hence $\underline{\underline{-3 < x < -2 \text{ or } 0 < x < 6}}$.

13. Find the set of values of x for which (5)

$$\left|x^2 - 4\right| > 3x.$$

Solution

$\underline{x^2 - 4 > 3x}$:

$$\begin{aligned}x^2 - 4 > 3x &\Rightarrow x^2 - 3x - 4 > 0\\ &\Rightarrow (x+1)(x-4) > 0\\ &\Rightarrow x < -1 \text{ or } x > 4.\end{aligned}$$

$\underline{-(x^2-4) > 3x}$:

$$\begin{aligned} -x^2+4 > 3x &\Rightarrow 0 > x^2+3x-4 \\ &\Rightarrow 0 > (x+4)(x-1) \\ &\Rightarrow -4 < x < 1. \end{aligned}$$

Hence

$$\underline{\underline{x < 1 \text{ or } x > 4}}.$$

14. Use algebra to find the set of values for which (7)

$$\frac{6x}{3-x} > \frac{1}{x+1}.$$

Solution

$$\begin{aligned} \frac{6x}{3-x} > \frac{1}{x+1} &\Rightarrow \frac{6x}{3-x} - \frac{1}{x+1} > 0 \\ &\Rightarrow \frac{6x(x+1)-(3-x)}{(x+1)(3-x)} > 0 \\ &\Rightarrow \frac{6x^x+7x-3}{(x+1)(3-x)} > 0 \\ &\Rightarrow \frac{(3x-2)(2x+3)}{(x+1)(3-x)} > 0 \end{aligned}$$

and so the four critical values are $x = -\frac{3}{2}$, -1, $\frac{2}{3}$, and 3.

	$x < -\frac{3}{2}$	$-\frac{3}{2} < x < -1$	$-1 < x < \frac{2}{3}$	$\frac{2}{3} < x < 3$	$x > 3$
$2x+3$	$-$	$+$	$+$	$+$	$+$
$x+1$	$-$	$-$	$+$	$+$	$+$
$3x-2$	$-$	$-$	$-$	$+$	$+$
$3-x$	$+$	$+$	$+$	$+$	$-$
$\frac{(3x-2)(2x+3)}{(x+1)(3-x)}$	$-$	$+$	$-$	$+$	$-$

Hence $\underline{\underline{-\frac{3}{2} < x < -1 \text{ or } \frac{2}{3} < x < 3}}$.

15. (a) Use algebra to find the exact solutions of the equation (6)

$$\left|2x^2 + 6x - 5\right| = 5 - 2x.$$

Solution

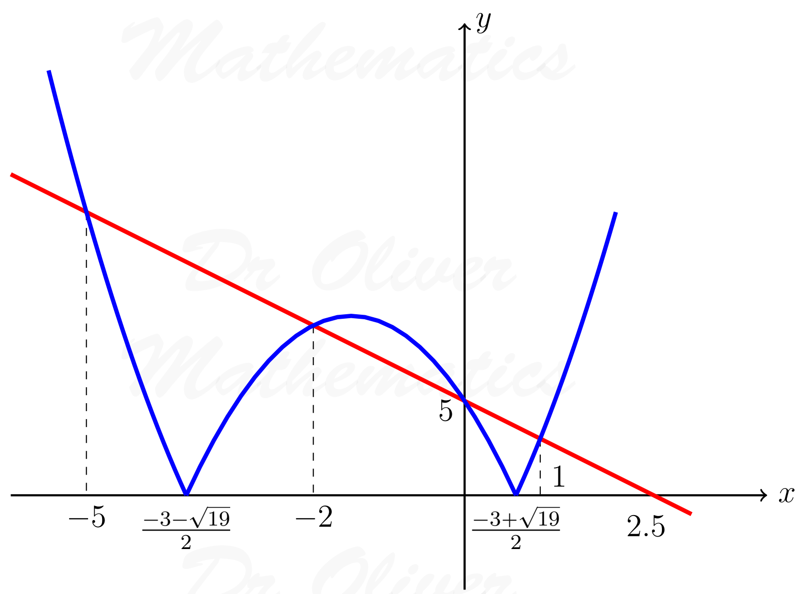
$$\begin{aligned}\left|2x^2 + 6x - 5\right| = 5 - 2x &\Rightarrow (2x^2 + 6x - 5)^2 = (5 - 2x)^2 \\ &\Rightarrow 4x^4 + 24x^3 + 16x^2 - 60x + 25 = 4x^2 - 20x + 25 \\ &\Rightarrow 4x^4 + 24x^3 + 12x^2 - 40x = 0 \\ &\Rightarrow 4x(x^3 + 6x^2 + 3x - 10) = 0 \\ &\Rightarrow 4x(x - 1)(x^2 + 7x + 10) = 0 \\ &\Rightarrow 4x(x - 1)(x + 2)(x + 5) = 0\end{aligned}$$

and hence

$$\underline{\underline{x = -5, -2, 0, \text{ or } 1}}.$$

(b) On the same diagram, sketch the curve with equation $y = |2x^2 + 6x - 5|$ and the line with equation $y = 5 - 2x$, showing the x-coordinate of the points where the line crosses the curve. (3)

Solution



(c) Find the set of values of x for which (3)

$$\left|2x^2 + 6x - 5\right| > 5 - 2x.$$

Solution

Using (a) and (b), $\underline{\underline{x < -5, -2 < x < 0, \text{ or } x > 1}}$.

16. Using algebra, find the set of values of x for which (5)

$$3x - 5 < \frac{2}{x}.$$

Solution

$$\begin{aligned} 3x - 5 < \frac{2}{x} &\Rightarrow 3x - 5 - \frac{2}{x} < 0 \\ &\Rightarrow \frac{x(3x-5) - 2}{x} < 0 \\ &\Rightarrow \frac{3x^2 - 5x - 2}{x} < 0 \\ &\Rightarrow \frac{(3x+1)(x-2)}{x} < 0 \end{aligned}$$

and so the three critical values are $x = -\frac{1}{3}$, 0, and 2.

	$x < -\frac{1}{3}$	$-\frac{1}{3} < x < 0$	$0 < x < 2$	$x > 2$
$3x + 1$	$-$	$+$	$+$	$+$
x	$-$	$-$	$+$	$+$
$x - 2$	$-$	$-$	$-$	$+$
$\frac{(3x+1)(x-2)}{x}$	$-$	$+$	$-$	$+$

Hence $\underline{\underline{x < -\frac{1}{3} \text{ or } 0 < x < 2}}$.

17. Use algebra to find the set of values of x for which (6)

$$\left|3x^2 - 19x + 20\right| < 2x + 2.$$

Solution

$3x^2 - 19x + 20 = (3x-4)(x-5)$ and so we can consider two cases.

$\underline{x \leqslant \frac{4}{3} \text{ or } x \geqslant 5}$: $3x^2 - 19x + 20 \geqslant 0$ and hence the inequality becomes

$$\begin{aligned} 3x^2 - 19x + 20 < 2x + 2 &\Rightarrow 3x^2 - 21x + 18 < 0 \\ &\Rightarrow x^2 - 7x + 6 < 0 \\ &\Rightarrow (x-1)(x-6) < 0 \\ &\Rightarrow 1 < x < 6 \end{aligned}$$

and hence $1 < x \leqslant \frac{4}{3}$ or $5 \leqslant x < 6$.

$\underline{\frac{4}{3} < x < 5}$: $3x^2 - 19x + 20 < 0$ and hence the inequality becomes

$$\begin{aligned} -(3x^2 - 19x + 20) < 2x + 2 &\Rightarrow 3x^2 - 17x + 22 > 0 \\ &\Rightarrow (3x-11)(x-2) > 0 \\ &\Rightarrow x < 2 \text{ or } x > \tfrac{11}{3} \end{aligned}$$

and hence $\frac{4}{3} < x < 2$ or $\frac{11}{3} < x < 5$. So the full solution is

$$\underline{\underline{1 < x < 2 \text{ or } \tfrac{11}{3} < x < 6.}}$$

18. (a) Use algebra to find the set of values of x for which (6)

$$x + 2 > \frac{12}{x+3}.$$

Solution

$$\begin{aligned} x + 2 > \frac{12}{x+3} &\Rightarrow x + 2 - \frac{12}{x+3} > 0 \\ &\Rightarrow \frac{(x+2)(x+3) - 12}{x+3} > 0 \\ &\Rightarrow \frac{(x^2+5x+6) - 12}{x+3} > 0 \\ &\Rightarrow \frac{x^2+5x-6}{x+3} > 0 \\ &\Rightarrow \frac{(x+6)(x-1)}{x+3} > 0, \end{aligned}$$

and hence the critical values are -6, -3, and 1.

	$x < -6$	$-6 < x < -3$	$-3 < x < 1$	$x > 1$
$x+6$	$-$	$+$	$+$	$+$
$x+3$	$-$	$-$	$+$	$+$
$x-1$	$-$	$-$	$-$	$+$
$\frac{(x+6)(x-1)}{x+3}$	$-$	$+$	$-$	$+$

Hence $\underline{\underline{-6 < x < -3 \text{ or } x > 1}}$.

(b) Hence, or otherwise, find the set of values of x for which (1)

$$x + 2 > \frac{12}{|x+3|}.$$

Solution

Consider the geometry of the problem. Nothing changes for $x > -3$ since both sides of the original inequality were already positive. For $x < -3$, both sides were negative so the introduction of the modulus on the right means that we lose these possible solutions. Hence $\underline{\underline{x > 1}}$.

19. Use algebra to find the set of values of x for which (6)

$$\frac{x}{x+1} < \frac{2}{x+2}.$$

Solution

$$\begin{aligned}
\frac{x}{x+1} < \frac{2}{x+2} &\Rightarrow \frac{x}{x+1} - \frac{2}{x+2} < 0 \\
&\Rightarrow \frac{x(x+2) - 2(x+1)}{(x+1)(x+2)} < 0 \\
&\Rightarrow \frac{x^2 - 2}{(x+1)(x+2)} < 0 \\
&\Rightarrow \frac{(x-\sqrt{2})(x+\sqrt{2})}{(x+1)(x+2)} < 0,
\end{aligned}$$

and hence the critical values are -2, $-\sqrt{2}$, -1, and $\sqrt{2}$. (Table 1 on page 21 shows the details.) Hence

$$\underline{\underline{-2 < x < -\sqrt{2} \text{ or } -1 < x < \sqrt{2}}}.$$

20. Use algebra to find the set of values of x for which (9)

$$\frac{x-2}{2(x+2)} \leqslant \frac{12}{x(x+2)}.$$

Solution

$$\begin{aligned}\frac{x-2}{2(x+2)} \leqslant \frac{12}{x(x+2)} &\Rightarrow \frac{x(x-2)-24}{2x(x+2)} \leqslant 0\\ &\Rightarrow \frac{x^2-2x-24}{2x(x+2)} \leqslant 0\\ &\Rightarrow \frac{(x-6)(x+4)}{2x(x+2)} \leqslant 0,\end{aligned}$$

and the critical values are -4, -2, 0, and 6.

	$x < -4$	$-4 < x < -2$	$-2 < x < 0$	$0 < x < 6$	$x > 6$
$x+4$	$-$	$+$	$+$	$+$	$+$
$x+2$	$-$	$-$	$+$	$+$	$+$
x	$-$	$-$	$-$	$+$	$+$
$x-6$	$-$	$-$	$-$	$-$	$+$
$\frac{(x-6)(x+4)}{2x(x+2)}$	$+$	$-$	$+$	$-$	$+$

	$x = -4$	$x = -2$	$x = 0$	$x = 6$
$\frac{x-2}{2(x+2)}$	$\frac{3}{2}$	undefined	$-\frac{1}{2}$	$\frac{1}{4}$
$\frac{12}{x(x+2)}$	$\frac{3}{2}$	undefined	undefined	$\frac{1}{4}$
Same?	the same	undefined	undefined	the same

So the full solution is

$$\underline{\underline{-4 \leqslant x < -2 \text{ or } 0 < x \leqslant 6}}.$$

	$x < -2$	$-2 < x < -\sqrt{2}$	$-\sqrt{2} < x < -1$	$-1 < x < \sqrt{2}$	$x > \sqrt{2}$
$x+2$	$-$	$+$	$+$	$+$	$+$
$x+\sqrt{2}$	$-$	$-$	$+$	$+$	$+$
$x+1$	$-$	$-$	$-$	$+$	$+$
$x-\sqrt{2}$	$-$	$-$	$-$	$-$	$+$
$\dfrac{x^2-2}{(x+1)(x+2)}$	$+$	$-$	$+$	$-$	$+$

Table 1: $\dfrac{x^2-2}{(x+1)(x+2)}$