Dr Oliver Mathematics
Mathematics Standard Grade: Credit Level
2010 Paper 2: Calculator
1 hour 20 minutes

The total number of marks available is 52.
You must write down all the stages in your working.

1. It is estimated that an iceberg weighs 84 000 tonnes. (4)
   As the iceberg moves into warmer water, its weight decreases by 25% each day.
   What will the iceberg weigh after 3 days in the warmer water?
   Give your answer **correct to three significant figures**.

Solution

$$\begin{aligned}\text{Mass} &= 84\,000 \times (1 - 0.25)^3 \\ &= 84\,000 \times (0.75)^3 \\ &= 35\,437.5 \text{ (exact)} \\ &= \underline{\underline{35\,400 \text{ tonnes (3 sf)}}}.\end{aligned}$$

2. Expand fully and simplify (2)

$$x(x-1)^2.$$

Solution

$\times$	x	-1
x	x^2	$-x$
-1	$-x$	$+1$

$$\begin{aligned}x(x-1)^2 &= x(x^2 - 2x + 1) \\ &= \underline{\underline{x^3 - 2x^2 + x}}.\end{aligned}$$

3. A machine is used to put drawing pins into boxes.
A sample of 8 boxes is taken and the number of drawing pins in each is counted.
The results are shown below:

$$102, \quad 102, \quad 101, \quad 98, \quad 99, \quad 101, \quad 103, \quad 102.$$

(a) Calculate the mean and standard deviation of this sample. (3)

Solution

x	x^2
102	10 404
102	10 404
101	10 201
98	9 604
99	9 801
101	10 201
103	10 609
102	10 404
$\sum x = 808$	$\sum x^2 = 81\,628$

$$\begin{aligned}\text{Mean} &= \frac{\sum x}{n} \\ &= \frac{808}{8} \\ &= \underline{\underline{101 \text{ pins}}}\end{aligned}$$

and

$$\begin{aligned}\text{standard deviation} &= \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} \\ &= \sqrt{\frac{81\,628 - \frac{808^2}{8}}{7}} \\ &= 1.690\,308\,509 \text{ (FCD)} \\ &= \underline{\underline{1.69 \text{ pins (3 sf)}}}.\end{aligned}$$

A sample of 8 boxes is taken from another machine.
This sample has a mean of 103 and a standard deviation of 2.1.

(b) Write down two valid comparisons between the samples. (2)

Solution

Average: Since the mean for the second box (103) is lower than the mean for the first box (101), the second box has more pins on average.

Spread: Since the standard deviation for the second box (2.1) is larger than the range for the first box (1.69), the second box varies more.

4. Use the quadratic formula to solve the equation (4)

$$3x^2 + 5x - 7 = 0.$$

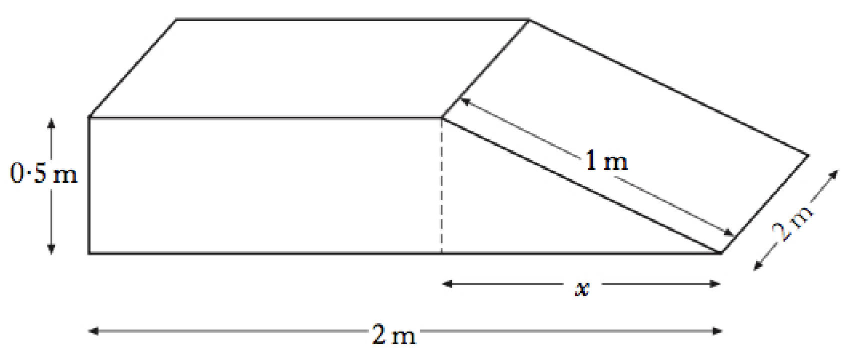
Give your answers correct to **1 decimal place**.

Solution

$a = 3$, $b = 5$, and $c = -7$:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-5 \pm \sqrt{5^2 - 4 \times 3 \times (-7)}}{2 \times 3} \\ &= \frac{-5 \pm \sqrt{109}}{6} \\ &= -2.573\,384\,418, 0.906\,717\,751\,5 \text{ (FCD)} \\ &= \underline{\underline{-2.6, 0.9 \text{ (1 dp)}}}. \end{aligned}$$

5. A concrete ramp is to be built.
The ramp is in the shape of a cuboid and a triangular prism with dimensions as shown.



(a) Calculate the value of x. (2)

Solution
Pythagoras' theorem:

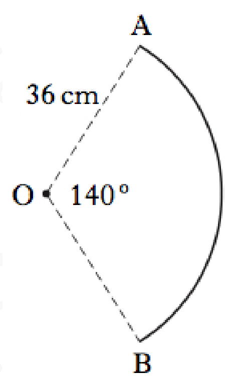
$$x^2 + 0.5^2 = 1^2 \Rightarrow x^2 = 0.75$$
$$\Rightarrow \underline{\underline{x = \frac{\sqrt{3}}{2} \text{ or } 0.866 \text{ m (3 sf)}}}.$$

(b) Calculate the volume of concrete required to build the ramp. (3)

Solution

$$\text{Volume} = [(0.5 \times (2 - x)) + (\tfrac{1}{2} \times x \times 0.5)] \times 2$$
$$= \underline{\underline{\frac{8 - \sqrt{3}}{4} \text{ or } 1.57 \text{ m}^3 \text{ (3 sf)}}}.$$

6. A circle, centre O, has radius 36 centimetres. (3)
Part of this circle is shown.
Angle $AOB = 140^\circ$.

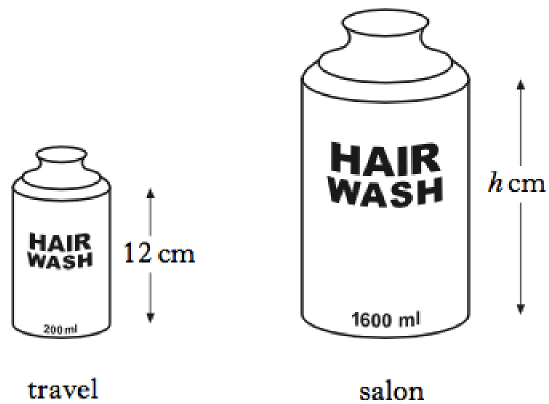


Calculate the length of arc AB.

Solution

$$
\begin{aligned}
\text{Arc} &= \frac{140}{360} \times 2 \times \pi \times 36 \\
&= 87.964\,594\,3 \text{ (FCD)} \\
&= \underline{\underline{88.0 \text{ cm (3 sf)}}}.
\end{aligned}
$$

7. Shampoo is available in travel size and salon size bottles. (3)
The bottles are mathematically similar.



The travel size contains 200 millilitres and is 12 centimetres in height.
The salon size contains 1 600 millilitres.
Calculate the height of the salon size bottle.

Solution

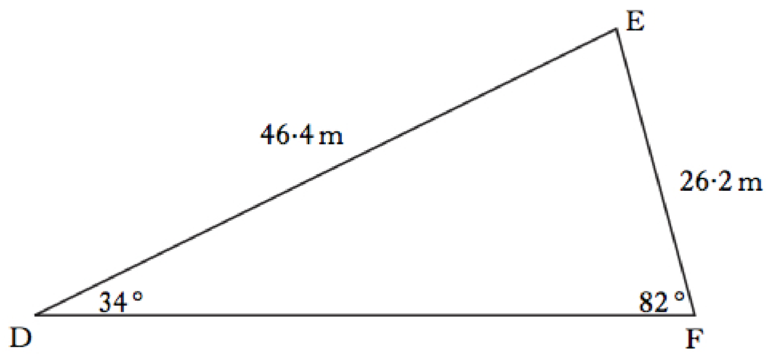
Let h cm be the height of the salon size bottle. The volume scale ratio (VSR) is

$$\frac{1\,600}{200} = 8 = 2^3$$

and so the length scale ratio (LSR) is 2. Finally,

$$h = 12 \times 2 = \underline{\underline{24 \text{ cm}}}.$$

8. As part of their training, footballers run around a triangular circuit DEF. (4)



$\angle EDF = 34°$.
$\angle DFE = 82°$.
$DE = 46.4$ metres.
$EF = 26.2$ metres.

How many complete circuits must they run to cover at least 1 000 metres?

Solution

$$\angle DEF = 180 - 34 - 82 = 64°$$

and

$$\begin{aligned} DF &= \sqrt{DE^2 + EF^2 - 2 \cdot DE \cdot EF \cdot \cos DEF} \\ &= \sqrt{46.4^2 + 26.2^2 - 2 \times 46.4 \times 26.2 \times \cos 64°} \\ &= 42.113\,678\,64 \text{ (FCD).} \end{aligned}$$

Now,

$$\frac{1\,000}{46.4 + 26.2 + 42.113\ldots} = 8.717\,356\,622 \text{ (FCD);}$$

hence, the football players must run $\underline{\underline{\text{9 laps}}}$.

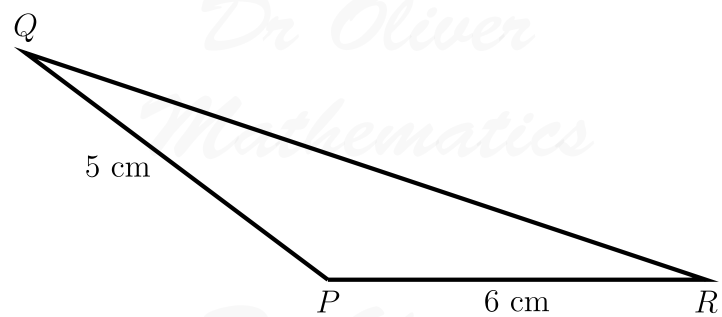
9. The ratio of sugar to fruit in a particular jam is 5 : 4. (4)
It is decided to **decrease** the sugar content by 20% and **increase** the fruit content by 20%.
Calculate the new ratio of sugar to fruit.
Give your answer in its simplest form.

Solution

The new ratio is

$$\begin{aligned}(5-20\%):(4+20\%) &\Rightarrow (5-1):(4+0.8)\\ &\Rightarrow 4:4.8\\ &\Rightarrow 1:\frac{6}{5}\\ &\Rightarrow \underline{\underline{5:6}}.\end{aligned}$$

10. In triangle PQR: $PQ = 5$ centimetres, $PR = 6$ centimetres, area of triangle $PQR =$ 12 square centimetres, and angle QPR is **obtuse**. (4)



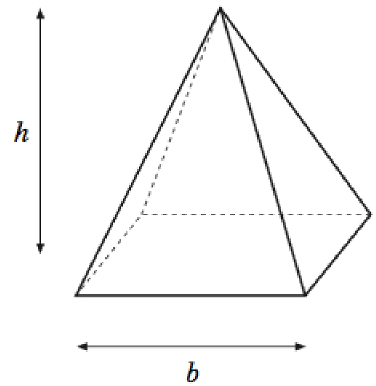
Calculate the size of angle QPR.

Solution

$$\begin{aligned}\tfrac{1}{2}\times 5\times 6\times \sin QPR = 12 &\Rightarrow \sin QPR = \tfrac{4}{5}\\ &\Rightarrow \angle QPR = 53.130\,102\,35, 126.869\,897\,6 \text{ (FCD)};\end{aligned}$$

hence, $\underline{\underline{\angle QPR = 126.9^\circ \text{ (1 dp)}}}$.

11. The height, h, of a square-based pyramid varies directly as its volume, V, and inversely as the square of the length of the base, b.



(a) Write down an equation connecting h, V, and b. (2)

Solution

$$h \propto \frac{V}{b^2} \Rightarrow \underline{\underline{h = \frac{kV}{b^2}}},$$

for some constant k.

A square-based pyramid of height 12 centimetres has a volume of 256 cubic centimetres and length of base 8 centimetres.

(b) Calculate the height of a square-based pyramid of volume 600 cubic centimetres and length of base 10 centimetres. (3)

Solution

$$12 = \frac{k \times 256}{8^2} \Rightarrow k = 3$$

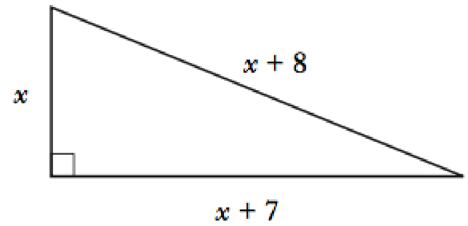
and

$$h = \frac{3V}{b^2}.$$

Finally,

$$\begin{aligned} h &= \frac{3 \times 600}{10^2} \\ &= \underline{\underline{18 \text{ cm}}}. \end{aligned}$$

12. A right-angled triangle has dimensions, in centimetres, as shown. (5)



Calculate the value of x.

Solution

$\times$	x	$+7$
x	x^2	$+7x$
$+7$	$+7x$	$+49$

$\times$	x	$+8$
x	x^2	$+8x$
$+8$	$+8x$	$+64$

$$\begin{aligned} x^2 + (x+7)^2 = (x+8)^2 &\Rightarrow x^2 + (x^2 + 14x + 49) = (x^2 + 16x + 64) \\ &\Rightarrow 2x^2 + 14x + 49 = x^2 + 16x + 64 \\ &\Rightarrow x^2 - 2x - 15 = 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & -2 \\ \text{multiply to:} & -15 \end{array}\right\} -5,\ +3$$

$$\begin{aligned} &\Rightarrow (x+3)(x-5) = 0 \\ &\Rightarrow x + 3 = 0 \text{ or } x - 5 = 0 \\ &\Rightarrow x = -3 \text{ or } x = 5; \end{aligned}$$

hence, $x \neq -3$ and $\underline{\underline{x = 5}}$.

13. The depth of water, D metres, in a harbour is given by the formula

$$D = 3 + 1.75\sin(30h^\circ),$$

where h is the number of hours after midnight.

(a) Calculate the depth of water at 5 am. (2)

Solution

$$\begin{aligned} D &= 3 + 1.75\sin(30 \times 5^\circ) \\ &= 3 + 1.75\sin(150^\circ) \\ &= \underline{\underline{3\tfrac{7}{8}\text{ m}}}. \end{aligned}$$

(b) Calculate the maximum difference in depth of the water in the harbour. (2)
Do not use a trial and improvement method.

Solution

First,

$$\sin(30 \times h^\circ) = \sin 90^\circ \Rightarrow h = 3$$

and so

$$D = 3 + 1.75 = 4.75\text{ m}.$$

Second,

$$\sin(30 \times h^\circ) = \sin 270^\circ \Rightarrow h = 9$$

and so

$$D = 3 - 1.75 = 1.25\text{ m}.$$

Hence, the maximum difference in depth of the water in the harbour is

$$4.75 - 1.25 = \underline{\underline{3.5\text{ m}}}.$$