# Dr Oliver Mathematics <br> GCSE Mathematics 2005 June Paper 5H: Non-Calculator 2 hours 

The total number of marks available is 100 .
You must write down all the stages in your working.

1. (a) Expand and simplify

$$
\begin{equation*}
(x+7)(x-4) \tag{2}
\end{equation*}
$$

(b) Expand

$$
\begin{equation*}
y\left(y^{3}+2 y\right) . \tag{2}
\end{equation*}
$$

(c) Factorise

$$
p^{2}+6 p
$$

(d) Factorise completely

$$
\begin{equation*}
6 x^{2}-9 x y \tag{2}
\end{equation*}
$$ get.

Design a suitable question she could use.
3. Triangle A and triangle $\mathbf{B}$ have been drawn on the grid.


Describe fully the single transformation which will map triangle $\mathbf{A}$ onto triangle $\mathbf{B}$.
4. (a) Solve

$$
\begin{equation*}
5-3 x=2(x+1) \tag{3}
\end{equation*}
$$

(b) $-3 \leqslant y<3$.
$y$ is an integer.
Write down all the possible values of $y$.
5. (a) Work out the value of

$$
\begin{equation*}
\frac{2}{3} \times \frac{3}{4} . \tag{2}
\end{equation*}
$$

Give your answer as a fraction in its simplest form.
(b) Work out the value of

$$
\begin{equation*}
1 \frac{2}{3}+2 \frac{3}{4} . \tag{3}
\end{equation*}
$$

Give your answer as a fraction in its simplest form.
6. (a) Write as a power of 5
(i) $5^{4} \times 5^{2}$,
(ii) $5^{9} \div 5^{6}$.
(b)

$$
\begin{equation*}
2^{x} \times 2^{y}=2^{10} \tag{3}
\end{equation*}
$$

and

$$
2^{x} \div 2^{y}=2^{4}
$$

Work out the value of $x$ and the value of $y$.
7. Work out the surface area of the triangular prism.


## Diagram NOT <br> accurately drawn

State the units with your answer.
8. The table shows some expressions. $a, b, c$, and $d$ represent lengths. $\pi$ and 3 are numbers which have no dimensions.

| $3 a^{2}$ | $\frac{\pi a b^{3}}{3 d}$ | $\pi b c$ | $a c+b d$ | $\pi(a+b)$ | $3(c+d)^{3}$ | $3 \pi b c^{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Tick $(\checkmark)$ the boxes underneath the three expressions which could represent volumes.
9. Here is a 4 -sided spinner.


The sides of the spinner are labelled $1,2,3$, and 4.
The spinner is biased.
The probability that the spinner will land on each of the numbers 2 and 3 is given in the table. The probability that the spinner will land on 1 is equal to the probability that it will land on 4.

| Number | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| Probability | $x$ | 0.3 | 0.2 | $x$ |

(a) Work out the value of $x$.

Sarah is going to spin the spinner 200 times.
(b) Work out an estimate for the number of times it will land on 2.
10. (a) Complete this table of values for $y=x^{3}+x-2$.

| $x$ | -2 | -1 | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -12 |  |  | 0 |  |

(b) Draw the graph of $y=x^{3}+x-2$.

11. The number 40 can be written as $2^{m} \times n$, where $m$ and $n$ are prime numbers.

Find the value of $m$ and the value of $n$.
12. The diagram shows three points $A(-1,5), B(2,-1)$, and $C(0,5)$.


The line $\mathbf{L}$ is parallel to $A B$ and passes through $C$.
Find the equation of the line $\mathbf{L}$.
13. Amy is going to play one game of snooker and one game of billiards.

The probability that she will win the game of snooker is $\frac{3}{4}$.
The probability that she will win the game of billiards is $\frac{1}{3}$.
(a) Complete the probability tree diagram.

(b) Work out the probability that Amy will win exactly one game.

Amy played one game of snooker and one game of billiards on a number of Fridays.
She won at both snooker and billiards on 21 Fridays.
(c) Work out an estimate for the number of Fridays on which Amy did not win either game.
14. In the diagram, $A, B$, and $C$ are points on the circumference of a circle, centre $O$.


Diagram NOT
accurately drawn
$P A$ and $P B$ are tangents to the circle.
Angle $A C B=75^{\circ}$.
(a) (i) Work out the size of angle $A O B$.
(ii) Give a reason for your answer.
(b) Work out the size of angle $A P B$.
15. (a) Change $\frac{3}{11}$ to a decimal.
(b) Prove that the recurring decimal

$$
\begin{equation*}
0 . \dot{3} \dot{9}=\frac{13}{33} \tag{3}
\end{equation*}
$$

16. $d$ is directly proportional to the square of $t$.
$d=80$ when $t=4$.
(a) Express $d$ in terms of $t$.
(b) Work out the value of $d$ when $t=7$.
(c) Work out the positive value of $t$ when $d=45$.
17. Here is a sketch of the graph of

$$
y=25-\frac{(x-8)^{2}}{4} \text { for } 0 \leqslant x \leqslant 12
$$


$P$ and $Q$ are points on the graph.
$P$ is the point at which the graph meets the $y$-axis.
$Q$ is the point at which $y$ has its maximum value.
(a) Find the coordinates of
(i) $P$,
(ii) $Q$.
(b) Show that

$$
25-\frac{(x-8)^{2}}{4}=\frac{(2+x)(18-x)}{4} .
$$

18. Two cylinders, $\mathbf{P}$ and $\mathbf{Q}$, are mathematically similar.

Diagram NOT


4 cm

accurately drawn

The total surface area of cylinder $\mathbf{P}$ is $90 \pi \mathrm{~cm}^{2}$.
The total surface area of cylinder $\mathbf{Q}$ is $810 \pi \mathrm{~cm}^{2}$.
The length of cylinder $\mathbf{P}$ is 4 cm .
(a) Work out the length of cylinder $\mathbf{Q}$.

The volume of cylinder $\mathbf{P}$ is $100 \pi \mathrm{~cm}^{3}$.
(b) Work out the volume of cylinder $\mathbf{Q}$.

Give your answer as a multiple of $\pi$.
19. (a) Find the value of
(i) $64^{0}$,
(ii) $64^{\frac{1}{2}}$,
(iii) $64^{-\frac{2}{3}}$.
(b) $3 \times \sqrt{27}=3^{n}$.

Find the value of $n$.
20. Diagram 1 is a sketch of part of the graph of $y=\sin x^{\circ}$.


Diagram 1
(a) Write down the coordinates of
(i) $P$,
(ii) $Q$.

Diagram 2 is a sketch of part of the graph of $y=3 \cos 2 x^{\circ}$.


## Diagram 2

(b) Write down the coordinates of
(i) $R$,
(ii) $S$.
21. The radius of the base of a cone is $x \mathrm{~cm}$ and its height is $h \mathrm{~cm}$.


Diagram NOT
accurately drawn

The radius of a sphere is $2 x \mathrm{~cm}$.
The volume of the cone and the volume of the sphere are equal.
Express $h$ in terms of $x$.
Give your answer in its simplest form.
22. $O P Q R$ is a trapezium with $P Q$ parallel to $O R$.

$\overrightarrow{O P}=2 \mathbf{b}$.
$\overrightarrow{P Q}=2 \mathbf{a}$.
$\overrightarrow{O R}=6 \mathbf{a}$.
$M$ is the midpoint of $P Q$ and $N$ is the midpoint of $O R$.
(a) Find the vector $\overrightarrow{M N}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
$X$ is the midpoint of $M N$ and $Y$ is the midpoint of $Q R$.
(b) Prove that $X Y$ is parallel to $O R$.

