Dr Oliver Mathematics
GCSE Mathematics
2023 June Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Work out (3)

$$8.46 \div 0.15.$$

Solution

$$\begin{aligned} 8.46 \div 0.15 &= 8.46 \div \tfrac{3}{20} \\ &= 8.46 \times \tfrac{20}{3} \\ &= 2.82 \times 20 \\ &= \underline{\underline{56.4}}. \end{aligned}$$

2. Work out (3)

$$7\tfrac{3}{8} - 2\tfrac{1}{2}.$$

Give your answer as a mixed number.

Solution

$$\begin{aligned} 7\tfrac{3}{8} - 2\tfrac{1}{2} &= (7-2) + (\tfrac{3}{8} - \tfrac{1}{2}) \\ &= 5 + (\tfrac{3}{8} - \tfrac{4}{8}) \\ &= 5 - \tfrac{1}{8} \\ &= \underline{\underline{4\tfrac{7}{8}}}. \end{aligned}$$

3. A cube has a total surface area of 150 cm^2. (4)

Work out the volume of the cube.

Solution

The cube has 6 faces so each face has an area of

$$\frac{150}{6} = 25 \text{ cm}^2.$$

Square root:

$$\sqrt{25} = 5 \text{ cm}.$$

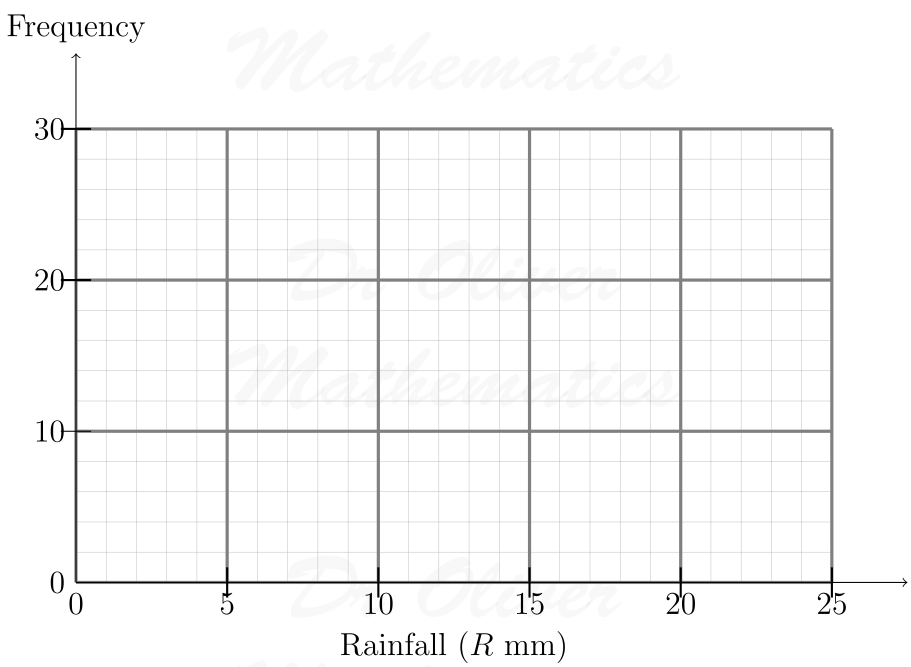
And cube the answer:

$$5^3 = \underline{\underline{125 \text{ cm}^3}}.$$

4. The table shows information about the daily rainfall in a town for 60 days. (2)

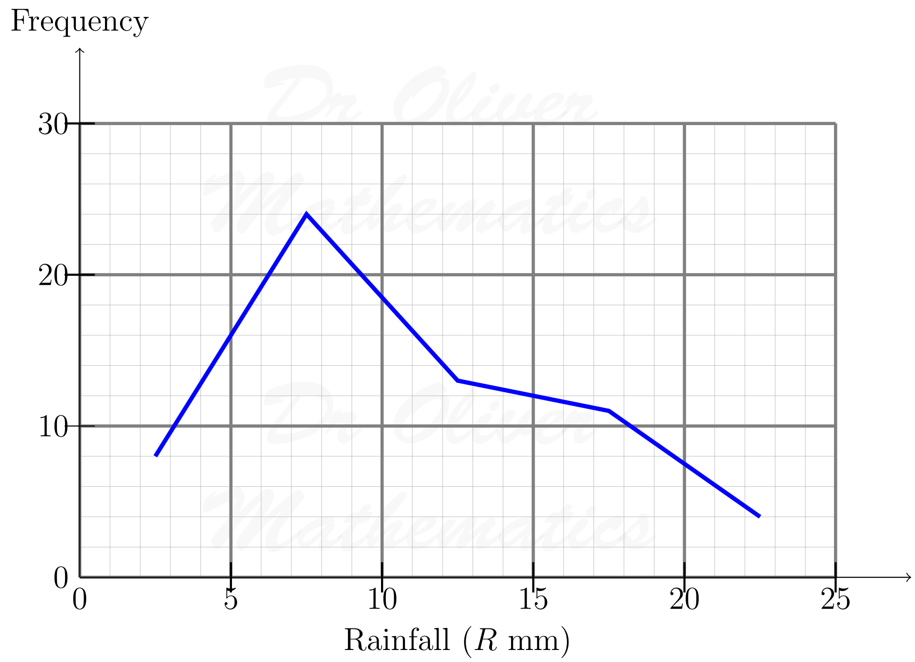
Rainfall (R mm)	Frequency
$0 \leqslant R < 5$	8
$5 \leqslant R < 10$	24
$10 \leqslant R < 15$	13
$15 \leqslant R < 20$	11
$20 \leqslant R < 25$	4

Draw a frequency polygon for this information.



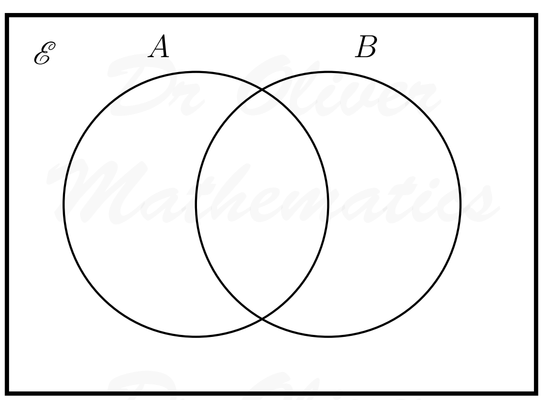
Solution

We plot (2.5, 8), (7.5, 24), (12.5, 13), (17.5, 11), and (22.5, 4) and join them with piece-wise line:

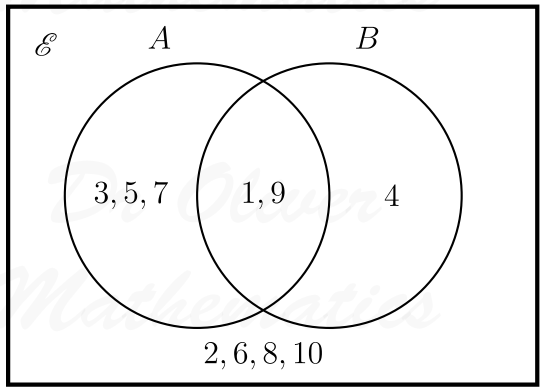


5.
- $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
- $A = \{\text{odd numbers}\}$.
- $B = \{\text{square numbers}\}$.

(a) Complete the Venn diagram for this information. (3)



Solution



A number is chosen at random from the universal set $\mathscr{E}$.

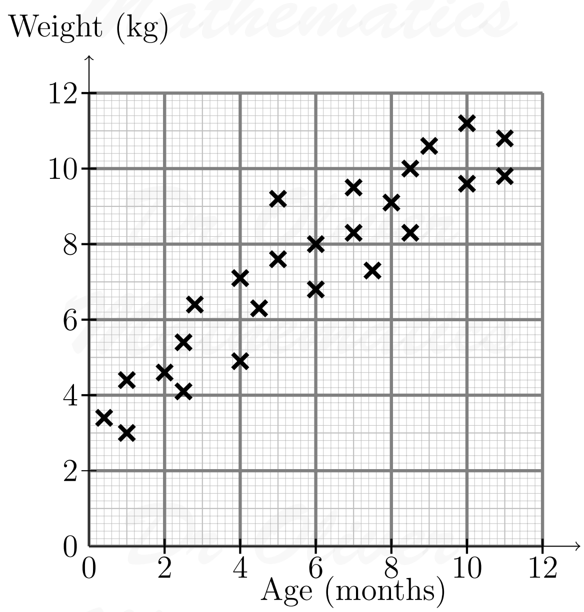
(b) Find the probability that this number is in the set B'. (2)

Solution

Well, there are 7 members of the set B' and so

$$\mathrm{P}(B') = \underline{\underline{\tfrac{7}{10}}}.$$

6. The scatter graph shows information about the ages and weights of some babies.



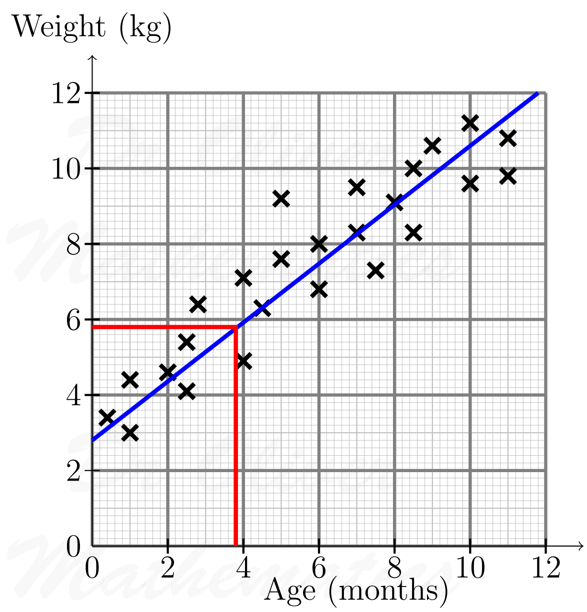
(a) Describe the relationship between the age and the weight of the babies. (1)

Solution
E.g., positive correlation: as the age increases, the weight increases, etc.

Another baby has a weight of 5.8 kg.

(b) Using the scatter graph, find an estimate for the age of this baby. (2)

Solution
Draw a line of best fit and the correct read-off:



approximately 3.8 months.

7. The price of a holiday increases by 20%. (2)
This 20% increase adds £240 to the price of the holiday.

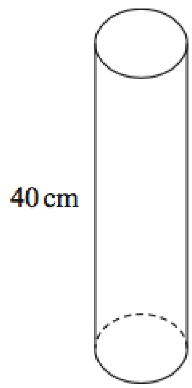
Work out the price of the holiday before the increase.

Solution

Let £x be the price of the holiday before the increase. Now,

$$\begin{aligned} 1.2x = x + 240 &\Rightarrow 0.2x = 240 \\ &\Rightarrow x = 5 \times 240 \\ &\Rightarrow \underline{\underline{x = 1\,200}}. \end{aligned}$$

8. The diagram shows a solid cylinder on a horizontal floor. (3)



$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

The cylinder has a

- volume of $1\,200$ cm^3 and
- height of 40 cm.

The cylinder exerts a force of 90 newtons on the floor.

Work out the pressure on the floor due to the cylinder.

Solution

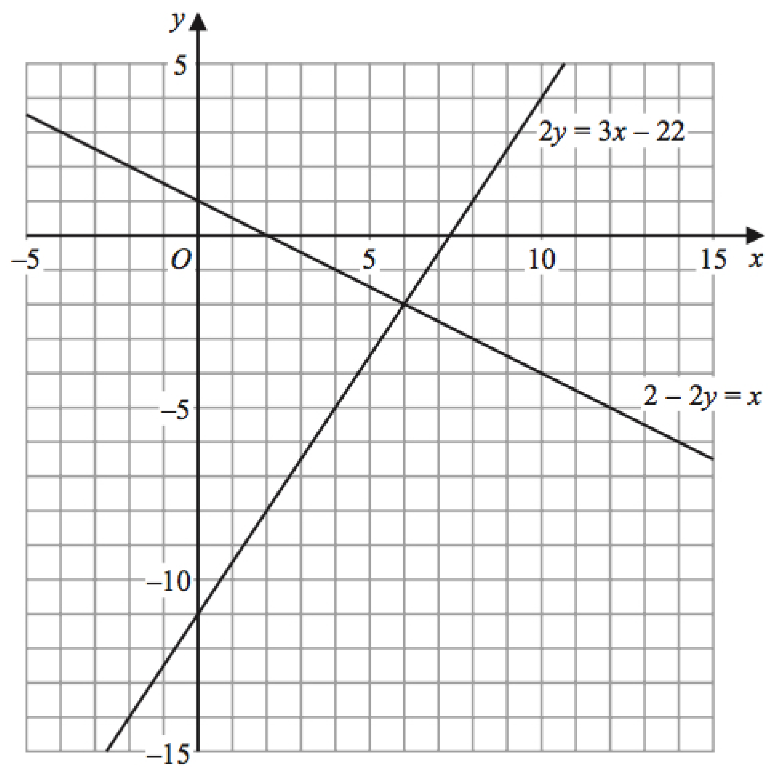
Well,

$$\begin{aligned} \text{area} &= \frac{\text{force}}{\text{pressure}} \\ &= \frac{1\,200}{40} \\ &= 30 \end{aligned}$$

and

$$\text{pressure} = \frac{90}{30} = \underline{\underline{3 \text{ Pa}}}.$$

9. Use these graphs to solve the simultaneous equations (1)



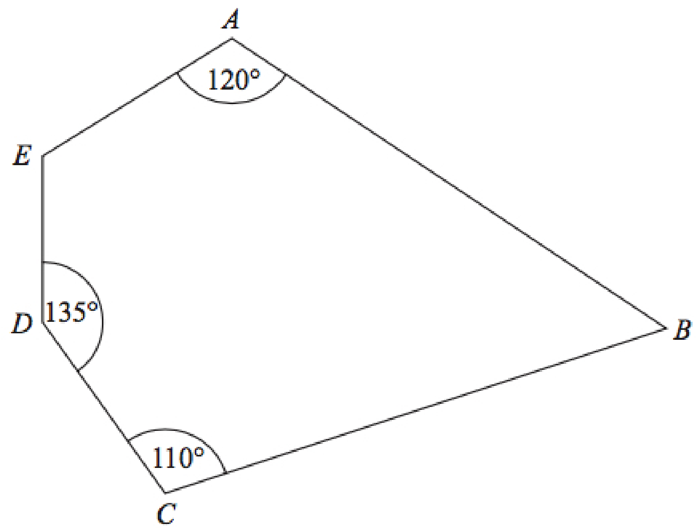
$$2 - 2y = x$$
$$2y = 3x - 22.$$

Solution

They intersect at

$$\underline{\underline{x = 6,\ y = -2}}.$$

10. Here is a pentagon (4)



Angle $AED = 4 \times$ angle ABC.

Work out the size of angle AED.
You must show all your working.

Solution

The pentagon's five angles add up to

$$(5-2) \times 180 = 540^\circ$$

so

$$\begin{aligned} 120 + \angle ABC + 110 + 135 + \angle AED = 540 &\Rightarrow \angle ABC + 4\angle ABC = 175 \\ &\Rightarrow 5\angle ABC = 175 \\ &\Rightarrow \angle ABC = 35 \\ &\Rightarrow \angle AED = 4 \times 35 \\ &\Rightarrow \underline{\underline{\angle AED = 140^\circ}}. \end{aligned}$$

11. Write (3)

$$\frac{(6x^5y^3)^2}{3x^2y^7 \times 4xy^{-3}}$$

in the form

$$ax^b y^c,$$

where a, b, and c are integers.

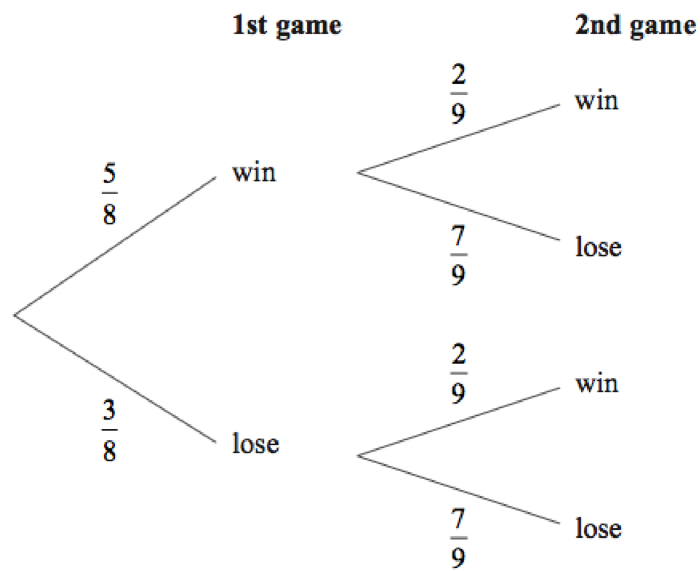
Solution

$$\begin{aligned}\frac{(6x^5y^3)^2}{3x^2y^7 \times 4xy^{-3}} &= \frac{36x^{10}y^6}{12x^3y^4} \\ &= \underline{\underline{3x^7y^2}};\end{aligned}$$

hence, $\underline{\underline{a = 3}}$, $\underline{\underline{b = 7}}$, and $\underline{\underline{c = 2}}$.

12. Martha plays a game twice. (3)

The probability tree diagram shows the probabilities that Martha will win or lose each game.



Find the probability that Martha will lose at least one game.

Solution

$$\begin{aligned}
\text{P(lose at least one game)} &= 1 - \text{P}(WW) \\
&= 1 - \left(\tfrac{5}{8} \times \tfrac{2}{9}\right) \\
&= 1 - \tfrac{10}{72} \\
&= 1 - \tfrac{5}{36} \\
&= \underline{\underline{\tfrac{31}{36}}}.
\end{aligned}$$

13. y is directly proportional to x. (3)

$y = 24$ when $x = 1.5$.

Work out the value of y when $x = 5$.

Solution

Well,

$$y \propto x \Rightarrow y = kx,$$

for some constant k. Now,

$$\begin{aligned}
x = 1.5, y = 24 &\Rightarrow 24 = 1.5k \\
&\Rightarrow k = 16
\end{aligned}$$

and so

$$y = 16x.$$

Finally,

$$x = 5 \Rightarrow y = 16 \times 5 = \underline{\underline{80}}.$$

14. (a) Write $\frac{1}{16}$ in the form 4^n where n is an integer. (1)

Solution

$$\begin{aligned}
\tfrac{1}{16} &= \tfrac{1}{4^2} \\
&= \underline{\underline{4^{-2}}}.
\end{aligned}$$

(b) Work out the value of (3)

$$8^{\frac{5}{3}} - 9^{\frac{3}{2}}.$$

Solution

$$\begin{aligned} 8^{\frac{5}{3}} - 9^{\frac{2}{3}} &= (8^{\frac{1}{3}})^5 - (9^{\frac{1}{2}})^3 \\ &= 2^5 - 3^3 \\ &= 32 - 27 \\ &= \underline{\underline{5}}. \end{aligned}$$

15. The equation of line L_1 is $y = 2x - 5$. (3)
The equation of line L_2 is $6y + kx - 12 = 0$.

L_1 is perpendicular to L_2.

Find the value of k.
You must show all your working.

Solution

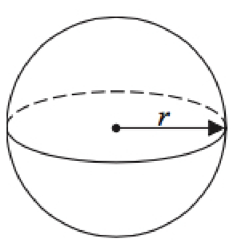
Well,

$$\begin{aligned} 6y + kx - 12 = 0 &\Rightarrow 6y = -kx + 12 \\ &\Rightarrow y = -\tfrac{1}{6}kx + 2. \end{aligned}$$

Now, L_1 is perpendicular to L_2 which means

$$\begin{aligned} 2 \times (-\tfrac{1}{6}k) = -1 &\Rightarrow -\tfrac{1}{3}k = -1 \\ &\Rightarrow \underline{\underline{k = 3}}. \end{aligned}$$

16. Here is a sphere. (4)



Surface area of sphere = $4\pi r^2$

$\frac{3}{8}$ of the surface area of this sphere is 75π cm^2.

Find the diameter of the sphere.
Give your answer in the form $a\sqrt{b}$, where a is an integer and b is a prime number.

Solution

Well,

$$\begin{aligned}\tfrac{3}{8}\text{ of the sphere} = 75\pi &\leftrightarrow \tfrac{1}{8}\text{ of the sphere} = 25\pi \\ &\leftrightarrow \text{sphere} = 200\pi.\end{aligned}$$

Now,

$$\begin{aligned}4\pi r^2 = 200\pi &\Rightarrow r^2 = 50 \\ &\Rightarrow r = \sqrt{50} \\ &\Rightarrow r = \sqrt{25 \times 2} \\ &\Rightarrow r = \sqrt{25} \times \sqrt{2} \\ &\Rightarrow r = 5\sqrt{2} \\ &\Rightarrow \underline{\underline{d = 10\sqrt{2}}};\end{aligned}$$

so, $\underline{\underline{a = 10}}$ and $\underline{\underline{b = 2}}$.

17. Make x the subject of the formula (4)

$$y = \frac{4(2x-7)}{5x+3}.$$

Solution

$$\begin{aligned}y = \frac{4(2x-7)}{5x+3} &\Rightarrow y = \frac{8x-28}{5x+3} \\ &\Rightarrow y(5x+3) = 8x - 28 \\ &\Rightarrow 5xy + 3y = 8x - 28 \\ &\Rightarrow 5xy - 8x = -3y - 28 \\ &\Rightarrow x(5y-8) = -3y - 28 \\ &\Rightarrow \underline{\underline{x = \frac{-3y-28}{5y-8}}}.\end{aligned}$$

18. 7 kg of carrots and 5 kg of tomatoes cost a total of 480 p. (4)

Cost of 1 kg of carrots : cost of 1 kg of tomatoes = 5 : 9.

Work out the cost of 1 kg of carrots and the cost of 1 kg of tomatoes.

Solution

Let c and t be the cost of one kilogram of carrots and tomatoes respectively. Now,

$$\begin{aligned} c : t = 5 : 9 &\Rightarrow \frac{c}{t} = \tfrac{5}{9} \\ &\Rightarrow c = \tfrac{5}{9}t. \end{aligned}$$

Now,

$$\begin{aligned} 7c + 5t = 480 &\Rightarrow 7(\tfrac{5}{9}t) + 5t = 480 \\ &\Rightarrow \tfrac{35}{9}t + 5t = 480 \\ &\Rightarrow \tfrac{35+45}{9}t = 480 \\ &\Rightarrow \tfrac{80}{9}t = 480 \\ &\Rightarrow \tfrac{1}{9}t = 6 \\ &\Rightarrow \underline{\underline{t = 54}} \\ &\Rightarrow c = \tfrac{5}{9}(54) \\ &\Rightarrow c = 5 \times 6 \\ &\Rightarrow \underline{\underline{c = 30}}. \end{aligned}$$

19. The menu in a restaurant has starters, main courses, and desserts. (2)

- There are 5 starters.
- There are 12 main courses.
- There are x desserts.

There are 420 different ways to choose one starter, one main course, and one dessert.

Work out the value of x.

Solution

$$\begin{aligned} 5 \times 12 \times x = 420 &\Rightarrow 60x = 420 \\ &\Rightarrow \underline{\underline{x = 7}}. \end{aligned}$$

20. For $x \geqslant 0$, the functions f and g are such that

$$\mathrm{f}(x) = 3x + 4 \text{ and } \mathrm{g}(x) = \frac{\sqrt{x} + 2}{5}.$$

(a) Find $\mathrm{g}^{-1}(x)$. (2)

Solution

$$\begin{aligned} y = \frac{\sqrt{x} + 2}{5} &\Rightarrow 5y = \sqrt{x} + 2 \\ &\Rightarrow 5y - 2 = \sqrt{x} \\ &\Rightarrow (5y - 2)^2 = x \end{aligned}$$

and so

$$\mathrm{g}^{-1}(x) = \underline{\underline{(5x - 2)^2}}.$$

(b) Solve (3)

$$\mathrm{g\,f}(x) = 3.$$

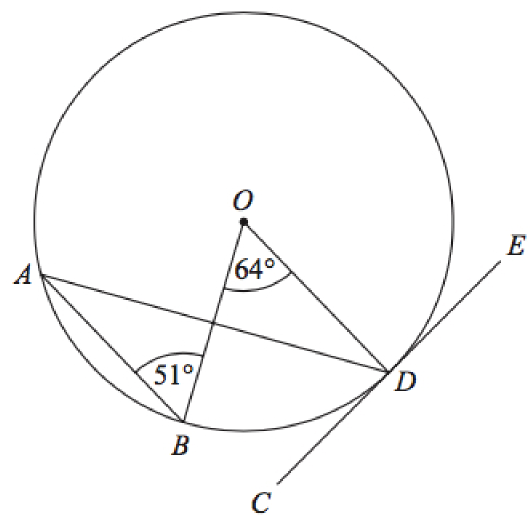
Solution

$$\begin{aligned} \mathrm{g\,f}(x) &= \mathrm{g}(\mathrm{f}(x)) \\ &= \mathrm{g}(3x + 4) \\ &= \frac{\sqrt{3x + 4} + 2}{5} \end{aligned}$$

and so

$$\begin{aligned}
\mathrm{g\,f}(x) = 3 &\Rightarrow \frac{\sqrt{3x+4}+2}{5} = 3 \\
&\Rightarrow \sqrt{3x+4}+2 = 15 \\
&\Rightarrow \sqrt{3x+4} = 13 \\
&\Rightarrow 3x+4 = 169 \\
&\Rightarrow 3x = 165 \\
&\Rightarrow \underline{\underline{x = 55}}.
\end{aligned}$$

21. A, B, and D are points on a circle with centre O. (4)
CDE is the tangent to the circle at D.



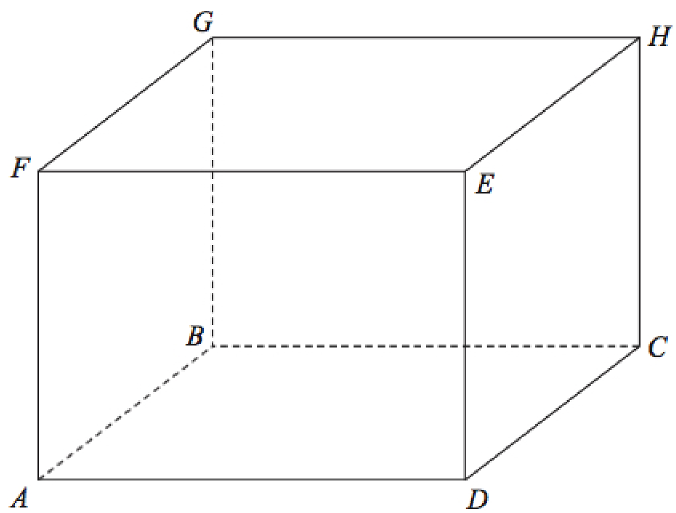
Work out the size of angle ADC.
Write down any circle theorems you use.

Solution

Let F be where the lines AD and OB intersect.
$\angle BAD = \frac{1}{2} \times 64 = 32°$ (the angle at the centre of a circle is twice the angle at the circumference)
$\angle AFB = 180 - (32 + 51) = 180 - 83 = 97°$ (completing the triangle)
$\angle OFD = 97°$ (vertically opposite angles)

$\angle ODA = 180 - (97 + 64) = 180 - 161 = 19°$ (completing the triangle)
$\angle ADC = 90 - 19 = \underline{\underline{71°}}$ (the tangent of a circle is perpendicular to the radius of the circle)

22. $ABCDEFGH$ is a cuboid. (2)



$AF = 6.8$ cm.
$FC = 13.6$ cm.

Work out the size of the angle between FC and the plane $ABCD$.

Solution

Well,

$$\begin{aligned} \sin = \frac{\text{opp}}{\text{hyp}} &\Rightarrow \sin FCA = \frac{AF}{FC} \\ &\Rightarrow \sin FCA = \tfrac{6.8}{13.6} \\ &\Rightarrow \sin FCA = \tfrac{1}{2} \\ &\Rightarrow \underline{\underline{\angle FCA = 30°}}. \end{aligned}$$

23. Write (4)

$$\frac{3\sqrt{3}}{4 - \sqrt{3}} - \frac{2}{\sqrt{3}}$$

in the form

$$\frac{a\sqrt{3}+b}{c},$$

where a, b, and c are integers.

Solution

Now,

$\times$	4	$-\sqrt{3}$
4	16	$-4\sqrt{3}$
$+\sqrt{3}$	$+4\sqrt{3}$	-3

and so

$$\begin{aligned}
\frac{3\sqrt{3}}{4-\sqrt{3}} - \frac{2}{\sqrt{3}} &= \left(\frac{3\sqrt{3}}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}}\right) - \left(\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) \\
&= \frac{3\sqrt{3}(4+\sqrt{3})}{13} - \frac{2\sqrt{3}}{3} \\
&= \frac{12\sqrt{3}+9}{13} - \frac{2\sqrt{3}}{3} \\
&= \tfrac{1}{39}\left[3(12\sqrt{3}+9) - 13(2\sqrt{3})\right] \\
&= \tfrac{1}{39}\left[36\sqrt{3}+27-26\sqrt{3}\right] \\
&= \underline{\underline{\frac{10\sqrt{3}+27}{39}}};
\end{aligned}$$

hence, $\underline{\underline{a = 10}}$, $\underline{\underline{b = 27}}$, and $\underline{\underline{c = 39}}$.

24. Find the set of possible values of x for which (5)

$$4x^2 - 25 < 0 \textbf{ and } 12 - 5x - 3x^2 > 0.$$

You must show all your working.

Solution

Difference of two squares:

$$\begin{aligned} 4x^2 - 25 < 0 &\Rightarrow (2x)^2 - 5^2 < 0 \\ &\Rightarrow (2x-5)(2x+5) < 0. \end{aligned}$$

Now,

$$12 - 5x - 3x^2 > 0 \Rightarrow 3x^2 + 5x - 12 < 0$$

$$\left.\begin{array}{ll} \text{add to:} & +5 \\ \text{multiply to:} & (+3) \times (-12) = -36 \end{array}\right\} +9,\ -4$$

e.g.,

$$\begin{aligned} &\Rightarrow 3x^2 + 9x - 4x - 12 < 0 \\ &\Rightarrow 3x(x+3) - 4(x+3) < 0 \\ &\Rightarrow (3x-4)(x+3) < 0. \end{aligned}$$

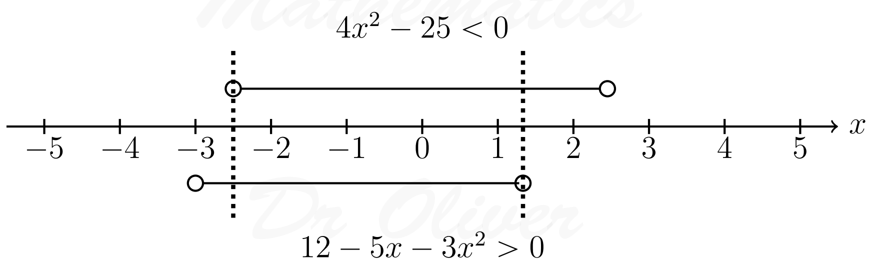
We need two ‘tables of signs’:

	$x < -\frac{5}{2}$	$x = -\frac{5}{2}$	$-\frac{5}{2} < x < \frac{5}{2}$	$x = \frac{5}{2}$	$x > \frac{5}{2}$
$(2x+5)$	$-$	0	$+$	$+$	$+$
$(2x-5)$	$-$	$-$	$-$	0	$+$
$(2x-5)(2x+5)$	$+$	0	$-$	0	$+$

and

	$x < -3$	$x = -3$	$-3 < x < \frac{4}{3}$	$x = \frac{4}{3}$	$x > \frac{4}{3}$
$(x+3)$	$-$	0	$+$	$+$	$+$
$(3x+4)$	$-$	$-$	$-$	0	$+$
$(3x-4)(x+3)$	$+$	0	$-$	0	$+$

We draw a number line:



Hence, the inequalities are true if

$$\underline{\underline{-\tfrac{5}{2} < x < \tfrac{4}{3}}}.$$