# Dr Oliver Mathematics
# GCSE Mathematics
# 2023 June Paper 2H: Calculator
# 1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. (a) Work out the value of (2)

$$\frac{25 - \sqrt{43.87}}{6 + 2.1^2}.$$

Write down all the figures on your calculator display.

**Solution**

$$\frac{25 - \sqrt{43.87}}{6 + 2.1^2} = \underline{\underline{1.765\,279\,23 \text{ (FCD)}}}.$$

(b) Work out the value of the reciprocal of 0.625. (1)

**Solution**

$$\frac{1}{0.625} = \underline{\underline{1\tfrac{3}{5}}}.$$

2. Write 60 as a product of its prime factors. (2)

**Solution**

Well,

$$\begin{array}{r|l} & 60 \\ \hline 2 & 30 \\ \hline 2 & 15 \\ \hline 3 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

so and

$$60 = \underline{\underline{2^2 \times 3 \times 5}}.$$

3. There are 48 counters in a bag. (1)
There are only red counters and blue counters in the bag.

$$\text{Number of red counters} : \text{number of blue counters} = 1 : 2.$$

Helen has to work out how many red counters are in the bag.

She says, "There are 24 red counters in the bag because 1 is half of 2 and 24 is half of 48."

Is Helen correct?
You must give a reason for your answer.

**Solution**

No: there are

$$\tfrac{1}{3} \times 48 = 16 \text{ red counters.}$$

4.

$$-2 \leqslant n < 5.$$

$n$ is an integer.

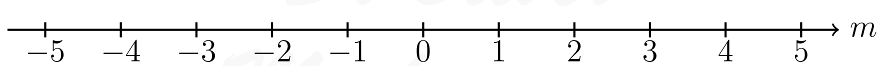
(a) Write down the greatest possible value of $n$. (1)

**Solution**

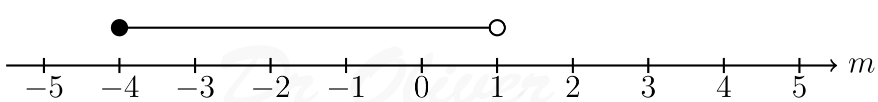
$n = 4$.

(b) On the number line below, show the inequality (2)

$$-4 \leqslant m < 1.$$



**Solution**



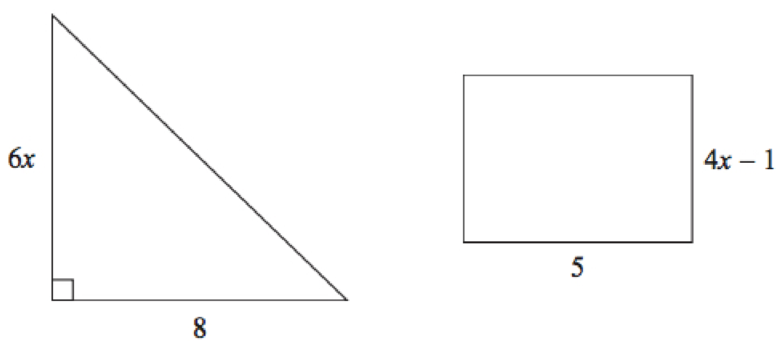
(c) Solve (3)

$$\tfrac{2}{5}g - 4 < 6.$$

**Solution**

$$\begin{aligned} \tfrac{2}{5}g - 4 < 6 &\Rightarrow \tfrac{2}{5}g < 10 \\ &\Rightarrow g < \tfrac{5}{2} \times 10 \\ &\Rightarrow \underline{\underline{g < 25}}. \end{aligned}$$

5. Here is a triangle and a rectangle. (4)




All measurements are in centimetres.

The area of the triangle is 10 cm$^2$ greater than the area of the rectangle.

Work out the value of $x$.

**Solution**

Now,

$$\begin{aligned} & \text{area of the triangle} - \text{area of the rectangle} = 10 \\ \Rightarrow\quad & (\tfrac{1}{2} \times 6x \times 8) - 5(4x - 1) = 10 \\ \Rightarrow\quad & 24x - 20x + 5 = 10 \\ \Rightarrow\quad & 4x = 5 \\ \Rightarrow\quad & \underline{\underline{x = 1\tfrac{1}{4}}}. \end{aligned}$$

6. Last year a family recycled 800 kg of household waste. (3)
57% of this waste was paper and glass.

$$\text{Weight of paper recycled : weight of glass recycled} = 12 : 7.$$

Calculate the weight of glass the family recycled.

**Solution**

The amount which was paper and glass is

$$800 \times 0.57 = 456 \text{ kg}.$$

Hence, the weight of glass the family recycled was

$$\begin{aligned}\left(\tfrac{7}{12+7}\right) \times 456 &= \tfrac{7}{19} \times 456 \\ &= \underline{\underline{168 \text{ kg}}}.\end{aligned}$$

7. A number, $d$, is rounded to 1 decimal place. (2)
The result is 12.7.

Complete the error interval for $d$.

$$\ldots\ldots\ldots \leqslant d < \ldots\ldots\ldots$$

**Solution**

$$\underline{\underline{12.65 \leqslant d < 12.75}}$$

8. Tamsin buys a house with a value of £150 000. (4)
The value of Tamsin's house increases by 4% each year.

Rachel buys a house with a value of £160 000.
The value of Rachel's house increases by 1.5% each year.

At the end of 2 years, whose house has the greater value?
You must show how you get your answer.

**Solution**

Tamsin:

$$150\,000 \times 1.04^2 = 162\,240.$$

Rachel:

$$160\,000 \times 1.015^2 = 164\,836.$$

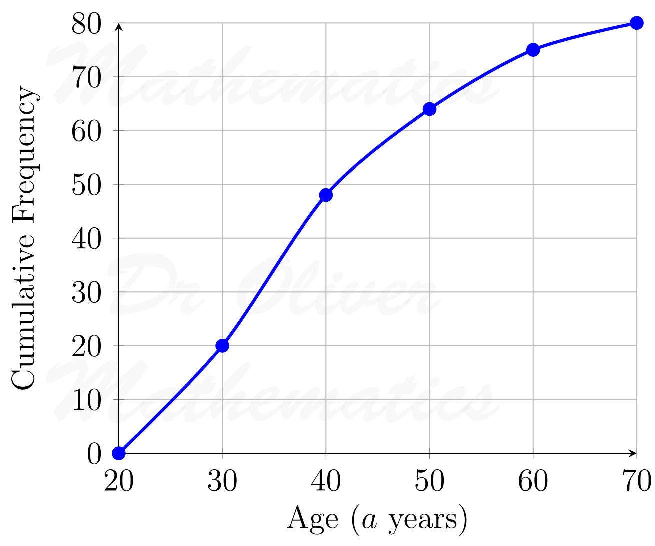
Hence, Rachel does.

9. The cumulative frequency table gives information about the ages of 80 people working for a company.

| Age ($a$ years) | Cumulative frequency |
|---|---|
| $20 < a \leqslant 30$ | 20 |
| $20 < a \leqslant 40$ | 48 |
| $20 < a \leqslant 50$ | 64 |
| $20 < a \leqslant 60$ | 75 |
| $20 < a \leqslant 70$ | 80 |

(a) Draw a cumulative frequency graph for this information. (2)

**Solution**

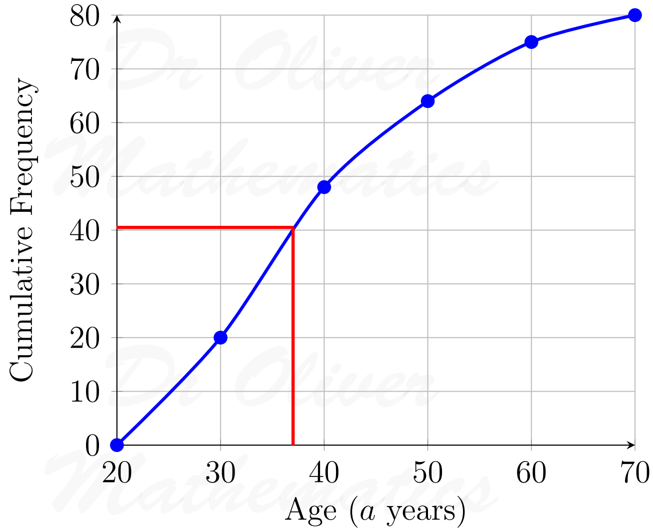



(b) Use your graph to find an estimate for the median age. (1)

**Solution**

The median is at

$$\frac{80+1}{2} = 40\tfrac{1}{2}\text{th place.}$$




Correct read-off: approximately $\underline{\underline{37}}$.

10. A biased dice is thrown 60 times.

The table shows information about the number that the dice lands on each time.

| Number on dice | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Frequency | 12 | 7 | 8 | 9 | 9 | 15 |

Gethin throws the dice twice.

(a) Work out an estimate for the probability that the dice will land on 6 both times. (3)

**Solution**

$$\tfrac{15}{60} \times \tfrac{15}{60} = \underline{\underline{\tfrac{1}{16}}}.$$

Sally is going to throw the same dice $n$ times and record the number it lands on each time.

She will use her results to work out a more reliable estimate for the probability in part (a).

(b) What can you say about the value of $n$? (1)

**Solution**

E.g., $n$ should be greater than 60.

11. Use algebra to solve the simultaneous equations (4)

$$2x + 6y = 5$$
$$3x - 4y = -12.$$

**Solution**

$$2x + 6y = 5 \quad (1)$$
$$3x - 4y = -12 \quad (2)$$

Do $2 \times (1)$ and $3 \times (2)$:

$$4x + 12y = 10 \quad (3)$$
$$9x - 12y = -36 \quad (4)$$

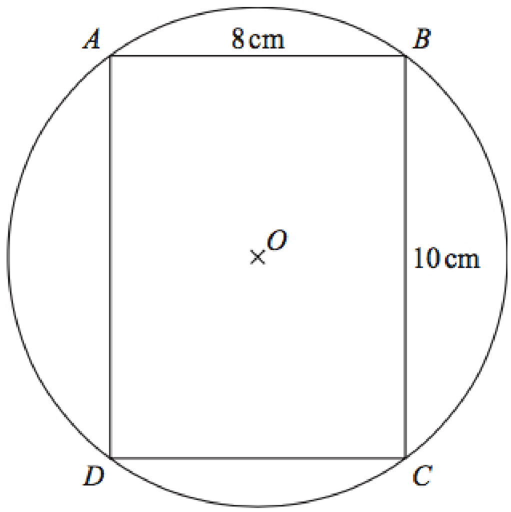
Add $(3) + (4)$:

$$13x = -26 \Rightarrow \underline{\underline{x = -2}}.$$

Now,

$$\begin{aligned} x = -2 &\Rightarrow 2(-2) + 6y = 5 \\ &\Rightarrow -4 + 6y = 5 \\ &\Rightarrow 6y = 9 \\ &\Rightarrow \underline{\underline{y = 1\tfrac{1}{2}}}. \end{aligned}$$

12. The points $A$, $B$, $C$, and $D$ lie on a circle, centre $O$. (4)
$ABCD$ is a rectangle.




- $AB = 8$ cm.
- $BC = 10$ cm.

Work out the circumference of the circle.
Give your answer correct to 3 significant figures.

**Solution**

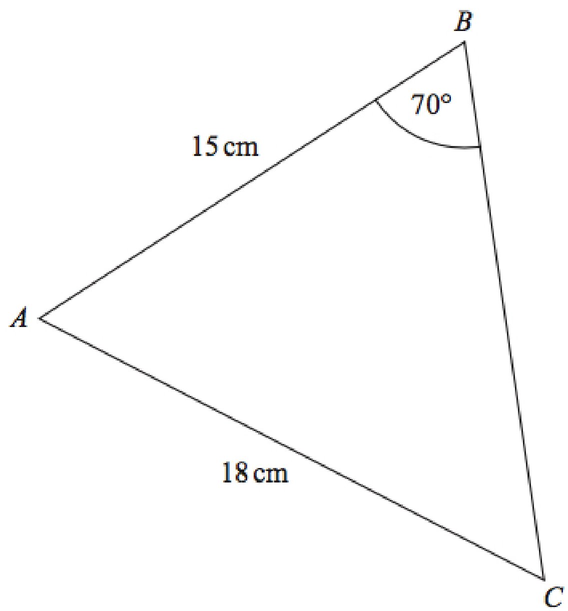
Pythagoras' theorem:

$$\begin{aligned} AC^2 = AB^2 + BC^2 &\Rightarrow AC^2 = 8^2 + 10^2 \\ &\Rightarrow AC^2 = 64 + 100 \\ &\Rightarrow AC^2 = 164 \\ &\Rightarrow AC = 2\sqrt{41} \\ &\Rightarrow OA = \sqrt{41} \end{aligned}$$

and

$$\begin{aligned} \text{circumference} &= 2 \times \pi \times \sqrt{41} \\ &= 40.232\,016\,13 \text{ (FCD)} \\ &= \underline{\underline{40.2 \text{ cm (3 sf)}}}. \end{aligned}$$

13. $ABC$ is a triangle. (4)




Calculate the size of angle $BAC$.
Give your answer correct to 1 decimal place.

**Solution**

Sine rule:

$$\frac{\sin BCA}{AB} = \frac{\sin ABC}{AC} \Rightarrow \frac{\sin BCA}{15} = \frac{\sin 70^\circ}{18}$$
$$\Rightarrow \sin BCA = \tfrac{5}{6} \sin 70^\circ$$
$$\Rightarrow \angle BCA = 51.543\,189\,37 \text{ (FCD)}$$
$$\Rightarrow \angle BAC = 180 - 70 - 51.543\ldots$$
$$\Rightarrow \angle BAC = 58.456\,810\,63 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle BAC = 58.5^\circ \text{ (1 dp)}}}.$$

14. Show that (3)

$$\frac{x^2 - x - 6}{2x^2 - 5x - 3}$$

can be written in the form

$$\frac{ax + b}{cx + d},$$

where $a$, $b$, $c$, and $d$ are integers.

**Solution**

$$\left.\begin{array}{ll}\text{add to:} & -1 \\ \text{multiply to:} & -6\end{array}\right\} -3,\ +2$$

So

$$x^2 - x - 6 = (x - 3)(x + 2).$$

Now,

$$\left.\begin{array}{lr}\text{add to:} & -5 \\ \text{multiply to:} & (+2) \times (-3) = -6\end{array}\right\} -5,\ -1$$

e.g.,

$$\begin{aligned} 2x^2 - 5x - 3 &= 2x^2 - 6x + x - 3 \\ &= 2x(x - 3) + 1(x - 3) \\ &= (2x + 1)(x - 3). \end{aligned}$$

Next,

$$\begin{aligned} \frac{x^2 - x - 6}{2x^2 - 5x - 3} &= \frac{(x - 3)(x + 2)}{(2x + 1)(x - 3)} \\ &= \underline{\underline{\frac{x + 2}{2x + 1}}}; \end{aligned}$$

hence, $\underline{\underline{a = 1}}$, $\underline{\underline{b = 2}}$, $\underline{\underline{c = 2}}$, and $\underline{\underline{d = 1}}$.

15. Here are the first four terms of a quadratic sequence: (3)

$$3 \quad 9 \quad 17 \quad 27.$$

Find an expression, in terms of $n$, for the $n$th term of this sequence.

**Solution**

Let the

$$n\text{th term} = an^2 + bn + c.$$

Then

$$\begin{array}{ccccccc} 3 & & 9 & & 17 & & 27 \\ & 6 & & 8 & & 10 & \\ & & 2 & & 2 & & \end{array}$$

and

$$\begin{array}{ccccccc} a+b+c & & 4a+2b+c & & 9a+3b+c & & 16a+4b+c \\ & 3a+b & & 5a+b & & 7a+b & \\ & & 2a & & 2a & & \end{array}$$

We compare terms:

$$2a = 2 \Rightarrow a = 1,$$

$$\begin{aligned} 3a + b = 6 &\Rightarrow 3 \times 1 + b = 6 \\ &\Rightarrow b = 3, \end{aligned}$$

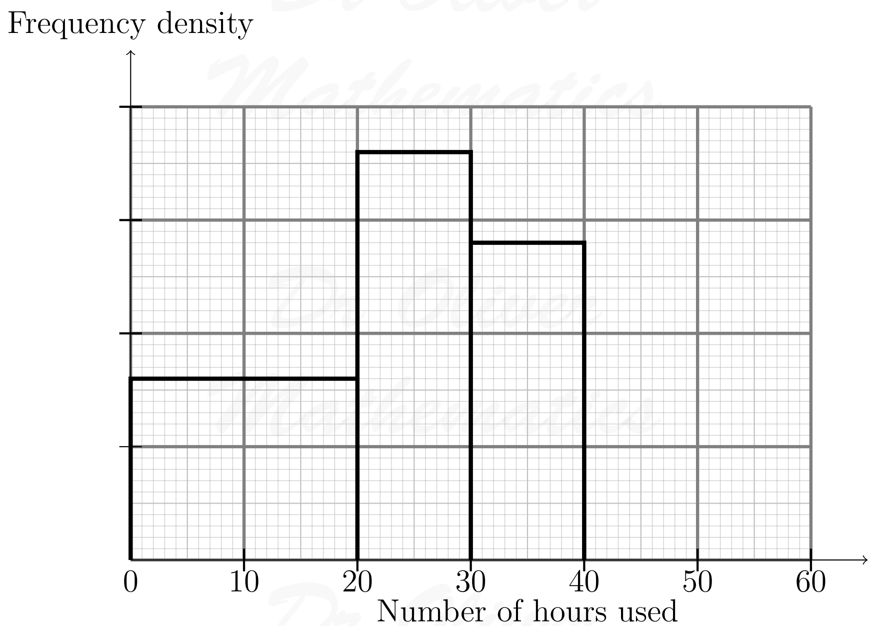
and

$$\begin{aligned} a + b + c = 3 &\Rightarrow 1 + 3 + c = 3 \\ &\Rightarrow c = -1; \end{aligned}$$

hence,

$$n\text{th term} = \underline{\underline{n^2 + 3n - 1}}.$$

16. The histogram gives information about the number of hours some students used their phones last week.
The histogram is incomplete.




28 students used their phones for between 30 and 40 hours.
24 students used their phones for between 40 and 60 hours.

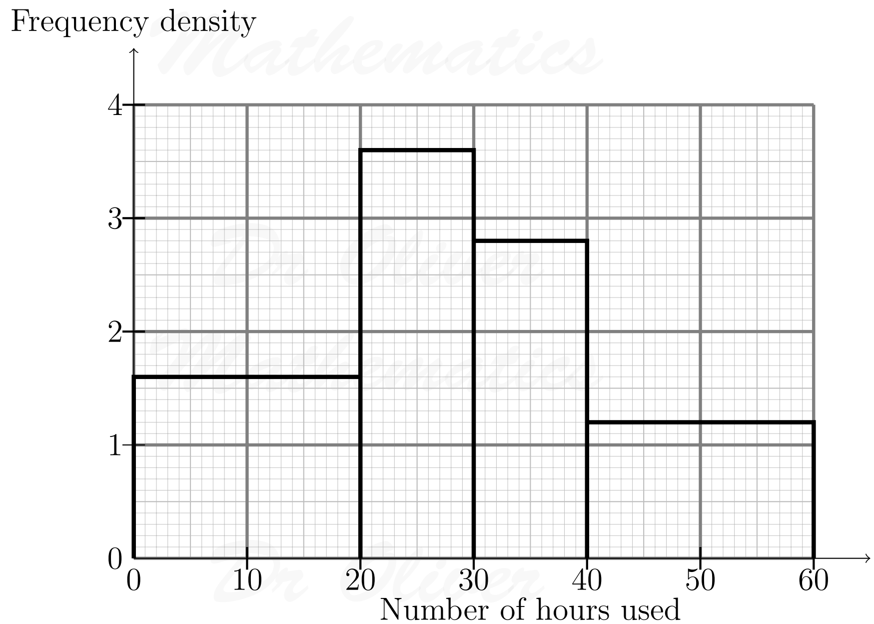
(a) Use this information to complete the histogram. (2)

**Solution**

Well,

| Interval | Frequency | Width | Frequency density |
|---|---|---|---|
| $0-20$ | $1.6 \times 20 = 32$ | 20 | 1.6 |
| $20-30$ | $3.6 \times 10 = 36$ | 10 | 3.6 |
| $30-40$ | $2.8 \times 10 = 28$ | 10 | 2.8 |
| $40-60$ | 24 | 20 | $\frac{24}{20} = 1.2$ |

and we complete the histogram:




No student used their phone for more than 60 hours.

(b) Work out the total number of students. (2)

**Solution**

$$32 + 36 + 28 + 24 = \underline{\underline{120 \text{ students}}}.$$

17. (a) Show that the equation (1)

$$x^4 - x^2 - 5 = 0$$

can be written in the form

$$x = \sqrt[4]{x^2 + 5}.$$

**Solution**

$$\begin{aligned} x^4 - x^2 - 5 = 0 &\Rightarrow x^4 = x^2 + 5 \\ &\Rightarrow \underline{\underline{x = \sqrt[4]{x^2 + 5}}}, \end{aligned}$$

as required.

(b) Starting with $x_0 = 1.5$, use the iteration formula (3)

$$x_{n+1} = \sqrt[4]{x_n^2 + 5}$$

three times to find an estimate for a solution of

$$x^4 - x^2 - 5 = 0.$$

**Solution**

Now,

$$\begin{aligned} x_1 &= \sqrt[4]{x_0^2 + 5} \\ &= \sqrt[4]{1.5^2 + 5} \\ &= \underline{\underline{1.640\,909\,017 \text{ (FCD)}}} \\ x_2 &= \sqrt[4]{1.640\ldots^2 + 5} \\ &= \underline{\underline{1.665\,398\,02 \text{ (FCD)}}} \\ x_3 &= \sqrt[4]{1.665\ldots^2 + 5} \\ &= \underline{\underline{1.669\,763\,088 \text{ (FCD)}}}. \end{aligned}$$

18. $2a : 5c = 6 : 25$. (3)
$4b : 7c = 20 : 21$.

Show that

$$(a + b) : (b + c) = 17 : 20.$$

**Solution**

Well,

$$\begin{aligned} 2a : 5c = 6 : 25 &\Rightarrow 25(2a) = 6(5c) \\ &\Rightarrow 50a = 30c \\ &\Rightarrow a = \tfrac{3}{5}c \end{aligned}$$

and

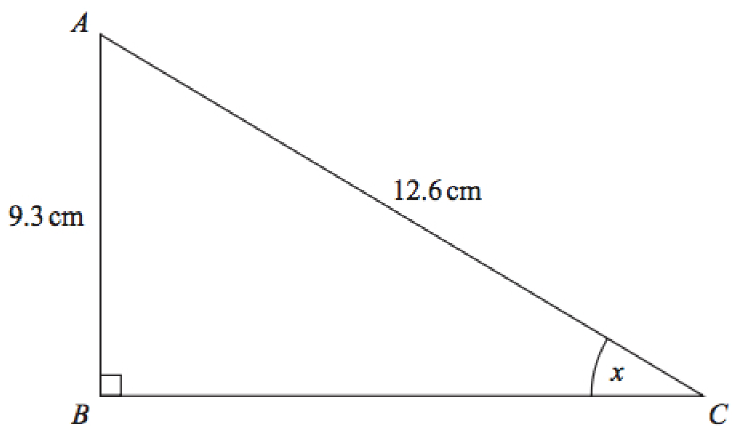
$$\begin{aligned} 4b : 7c = 20 : 21 &\Rightarrow 21(4b) = 20(7c) \\ &\Rightarrow 84b = 140c \\ &\Rightarrow b = \tfrac{5}{3}c. \end{aligned}$$

Now,

$$
\begin{aligned}
(a+b):(b+c) &= \left(\tfrac{3}{5}c + \tfrac{5}{3}c\right) : \left(\tfrac{5}{3}c + c\right) \\
&= \tfrac{34}{15}c : \tfrac{8}{3}c \\
&= \tfrac{34}{15} : \tfrac{8}{3} \\
&= \left(15 \times \tfrac{34}{15}\right) : \left(15 \times \tfrac{8}{3}\right) \\
&= 34 : 40 \\
&= \underline{\underline{17 : 20}},
\end{aligned}
$$

as required.

19. $ABC$ is a right-angled triangle. (3)




$AB = 9.3$ cm correct to the nearest mm.
$AC = 12.6$ cm correct to the nearest mm.

Calculate the lower bound for the size of the angle marked $x$.
You must show all your working.

**Solution**

Now,

$$9.25 \leqslant AB < 9.35$$

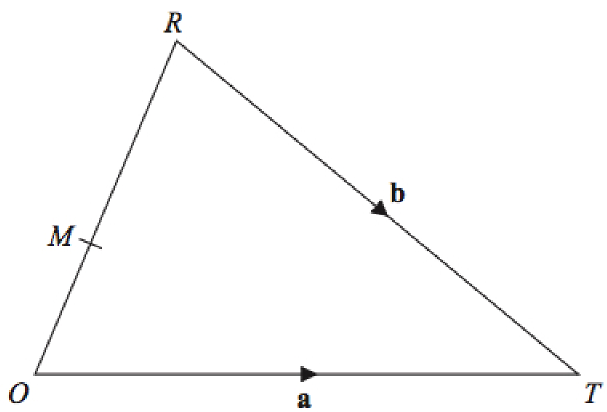
and

$$12.55 \leqslant AC < 12.65.$$

Next,

$$\text{LB for } \sin x = \frac{\text{LB for } AB}{\text{UB for } AC} \Rightarrow \text{LB for } \sin x = \frac{9.25}{12.65}$$
$$\Rightarrow \text{LB for } \angle x = 46.989\,213\,56 \text{ (FCD)}$$
$$\Rightarrow \text{LB for } \angle x = \underline{\underline{47.0^\circ \text{ (3 sf)}}}.$$

20. $ORT$ is a triangle. (4)




- $\overrightarrow{OT} = \mathbf{a}$.
- $\overrightarrow{RT} = \mathbf{b}$.
- $M$ is the point on $OR$ such that $OM : MR = 2 : 3$.

Express $\overrightarrow{MT}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

**Solution**

Well,

$$\begin{aligned} \overrightarrow{OR} &= \overrightarrow{OA} + \overrightarrow{TR} \\ &= \overrightarrow{OA} - \overrightarrow{RT} \\ &= \mathbf{a} - \mathbf{b} \end{aligned}$$

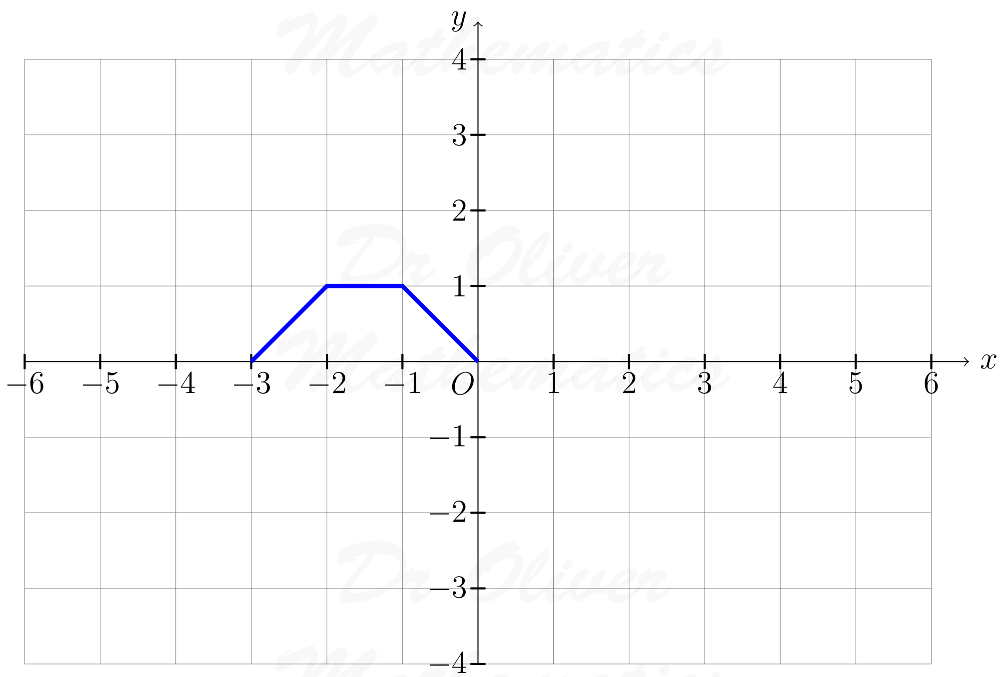
and

$$\begin{aligned}\overrightarrow{MR} &= \tfrac{3}{5}\overrightarrow{OR} \\ &= \tfrac{3}{5}(\mathbf{a}-\mathbf{b}) \\ &= \tfrac{3}{5}\mathbf{a}-\tfrac{3}{5}\mathbf{b}.\end{aligned}$$

Finally,

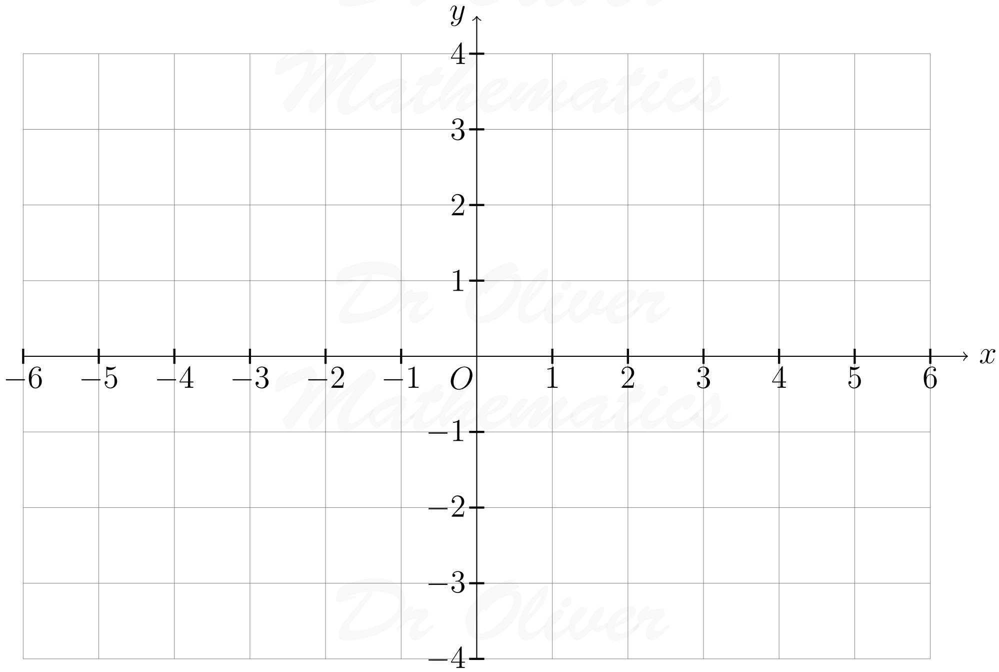
$$\begin{aligned}\overrightarrow{MT} &= \overrightarrow{MR}+\overrightarrow{TR} \\ &= \tfrac{3}{5}\mathbf{a}-\tfrac{3}{5}\mathbf{b}+\mathbf{b} \\ &= \underline{\underline{\tfrac{3}{5}\mathbf{a}+\tfrac{2}{5}\mathbf{b}}}.\end{aligned}$$

21. Here is the graph of $y = \mathrm{f}(x)$.

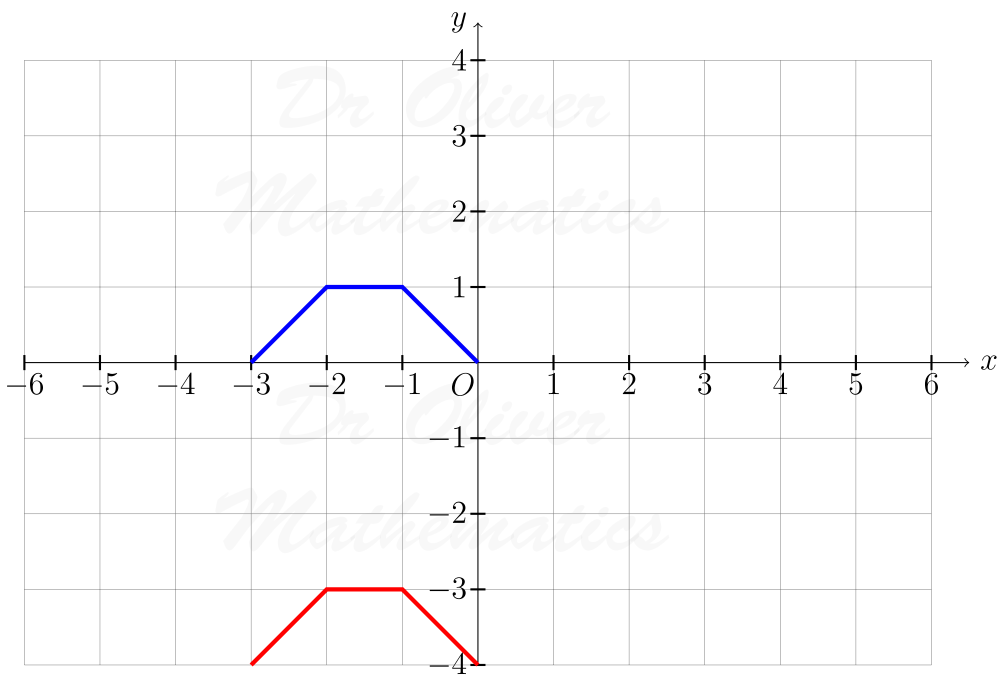



(a) On the grid below, draw the graph of (1)

$$y = \mathrm{f}(x) - 4.$$

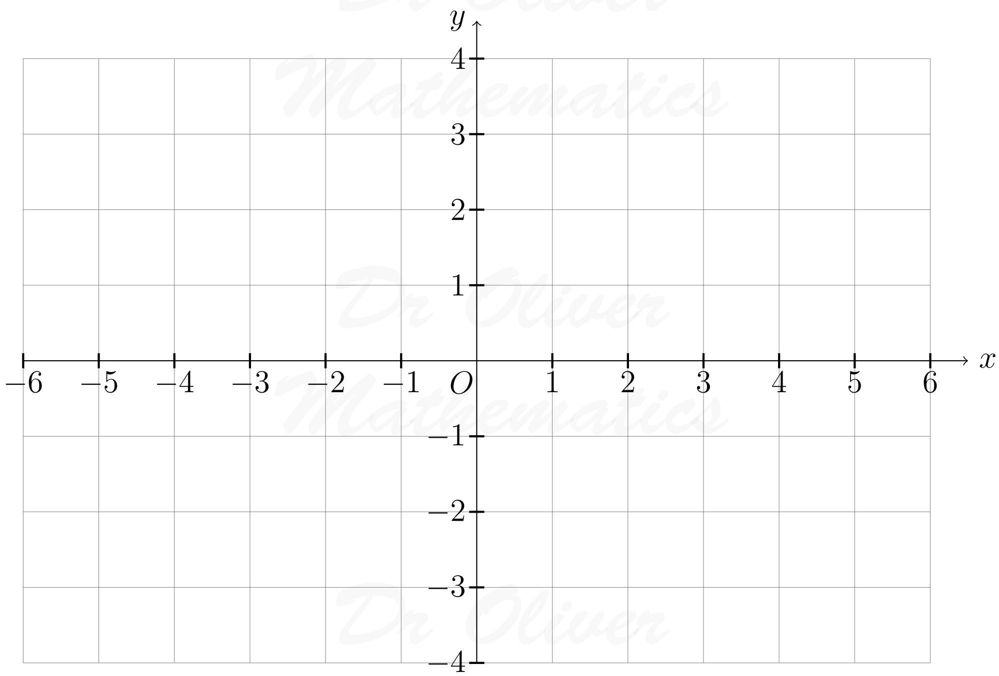


**Solution**

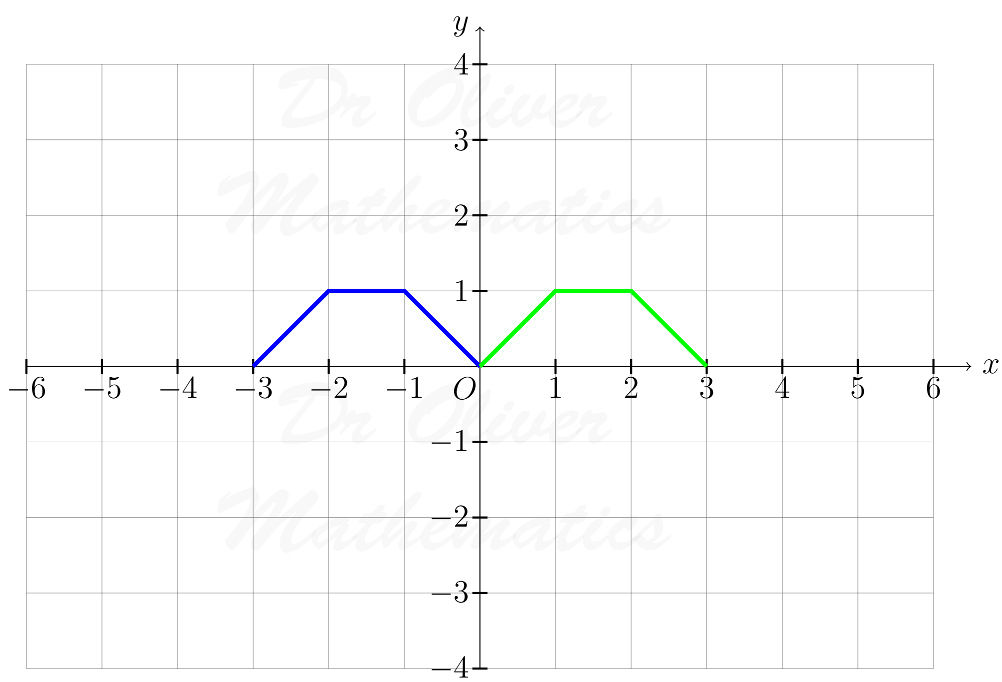


(b) On the grid below, draw the graph of (1)

$$y = \mathrm{f}(-x).$$




**Solution**




22. There are only blue pens and red pens in a box. (2)

The number of blue pens is four times the number of red pens.

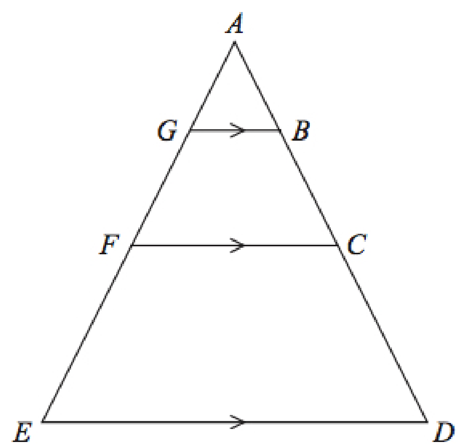
Rita takes at random one pen from the box.
She records the colour of the pen and then replaces it in the box.
Rita does this n times, where $n \geqslant 2$.

Write down an expression, in terms of $n$, for the probability that Rita gets a blue pen at least once and a red pen at least once.

**Solution**

$$\begin{aligned} & \text{P(a blue pen at least once and a red pen least once)} \\ = \ & 1 - \text{P(all blue pens)} - \text{P(all red pens)} \\ = \ & \underline{\underline{1 - (\tfrac{4}{5})^n - (\tfrac{1}{5})^n}}. \end{aligned}$$

23. Here are three similar triangles: $ABG$, $ACF$, and $ADE$. (3)




- $ABCD$ and $AGFE$ are straight lines.
- $AB : BC : CD = 1 : 2 : 3$.

Show that

$$\text{area of } ABG : \text{area of } BCFG : \text{area of } CDEF = 1 : 8 : 27.$$

**Solution**

Well,

$$\begin{aligned} AB : AC : AD &= 1 : (1+2) : (1+2+3) \\ &= 1 : 3 : 6 \end{aligned}$$

and

$$\begin{aligned} AB^2 : AC^2 : AD^2 &= 1^2 : 3^2 : 6^2 \\ &= 1 : 9 : 36. \end{aligned}$$

In particular,

$$\begin{aligned} \text{area of } BCFG &= \text{area of } AFC - \text{area of } AGB \\ &= 9 - 1 \\ &= 8 \end{aligned}$$

and

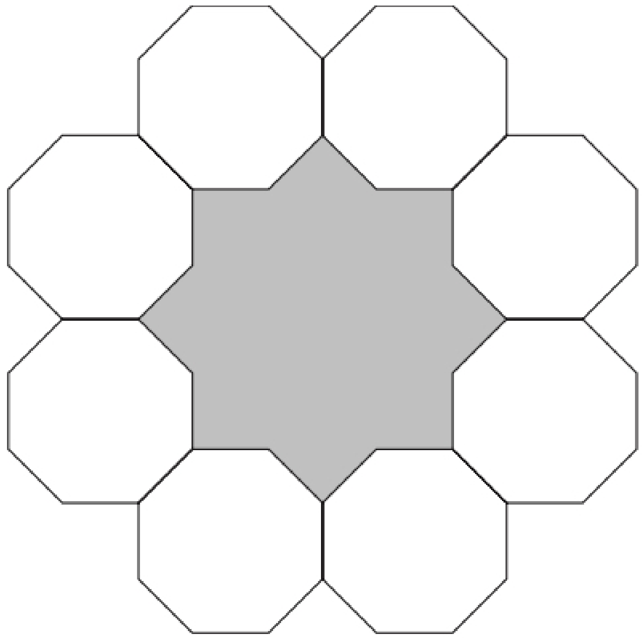
$$\begin{aligned} \text{area of } CDEF &= \text{area of } AED - \text{area of } AFC \\ &= 36 - 9 \\ &= 27; \end{aligned}$$

so,

$$\text{area of } ABG : \text{area of } BCFG : \text{area of } CDEF = \underline{\underline{1 : 8 : 27}},$$

as required.

24. The diagram shows 8 identical regular octagons joined to enclose a shaded shape. (5)



Each octagon has sides of length $a$.

Find, in terms of $a$, an expression for the area of the shaded shape.
Give your answer in the form

$$p(2+\sqrt{2})a^2,$$

where $p$ is an integer.
You must show all your working.

**Solution**

Well, there are 4 little triangles and a big square. Now,

$$\begin{aligned}\text{area of the little triangle} &= \tfrac{1}{2}\times a\times a\\ &= \tfrac{1}{2}a^2.\end{aligned}$$

Next,

$$\begin{aligned}\text{hyp}^2 = \text{opp}^2 + \text{adj}^2 &\Rightarrow \text{hyp}^2 = a^2 + a^2\\ &\Rightarrow \text{hyp}^2 = 2a^2\\ &\Rightarrow \text{hyp} = \sqrt{2}a\end{aligned}$$

and

$$\begin{aligned}\text{length of the big square} &= a + \sqrt{2}a + a \\ &= 2a + \sqrt{2}a \\ &= a(2 + \sqrt{2}).\end{aligned}$$

| $\times$ | $2$ | $+\sqrt{2}$ |
|---|---|---|
| $2$ | $4$ | $+2\sqrt{2}$ |
| $+\sqrt{2}$ | $+2\sqrt{2}$ | $+2$ |

Finally,

$$\begin{aligned}\text{area} &= (4 \times \text{area of the little triangle}) + \text{area of the big square} \\ &= (4 \times \tfrac{1}{2}a^2) + [a(2 + \sqrt{2})]^2 \\ &= 2a^2 + a^2(6 + 4\sqrt{2}) \\ &= 2a^2 + 6a^2 + 4a^2\sqrt{2} \\ &= 8a^2 + 4a^2\sqrt{2} \\ &= \underline{\underline{4(2a + \sqrt{2})a^2}},\end{aligned}$$

as required.