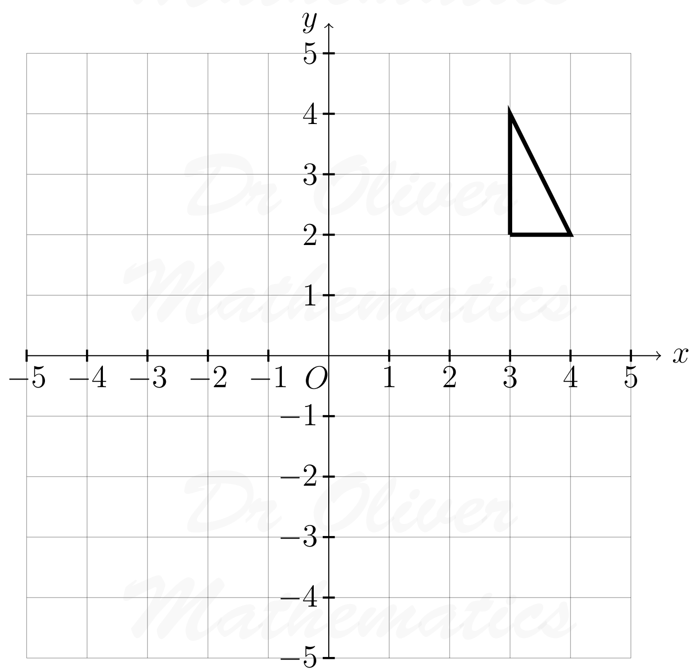
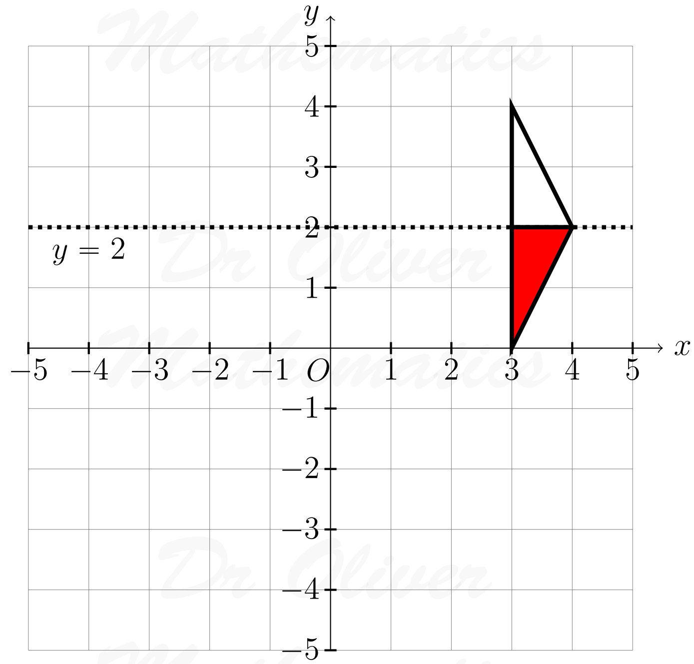
Dr Oliver Mathematics
AQA GCSE Mathematics
2012 June Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.

1. Reflect the triangle in the line $y = 2$. (2)



Solution



2. (a) Expand (1)

$$3(x - 6).$$

Solution

$$3(x - 6) = \underline{\underline{3x - 18}}.$$

(b) Factorise (1)

$$5y - 10.$$

Solution

$$5y - 10 = \underline{\underline{5(y - 2)}}.$$

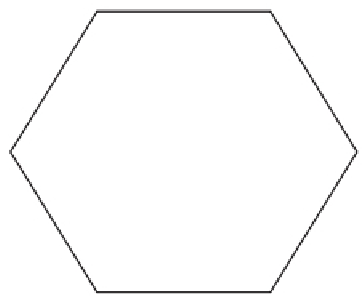
(c) Expand and simplify (3)

$$3(4w + 1) - 5(3w - 2).$$

Solution

$$\begin{aligned} 3(4w+1) - 5(3w-2) &= 12w + 3 - 15w + 10 \\ &= \underline{\underline{-3w + 13}}. \end{aligned}$$

3. Show that the interior angle of a regular hexagon is $120°$. (2)



Solution

Cut it up into identical triangles: there are $360°$ in a whole circle which means

$$\frac{360}{6} = 60°$$

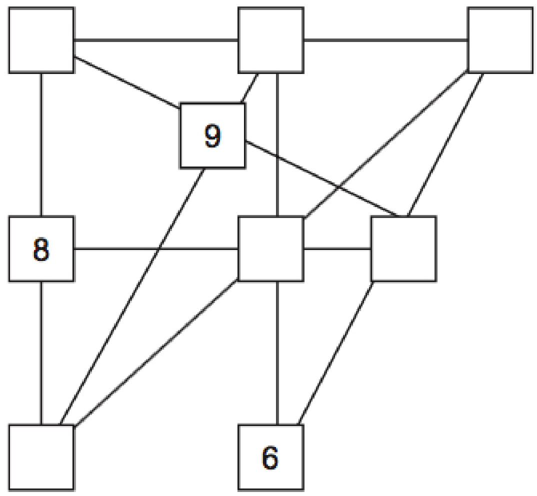
in each of the six triangles. Now, each piece is an isosceles triangle and

$$\tfrac{1}{2}(180 - 60) = 60°$$

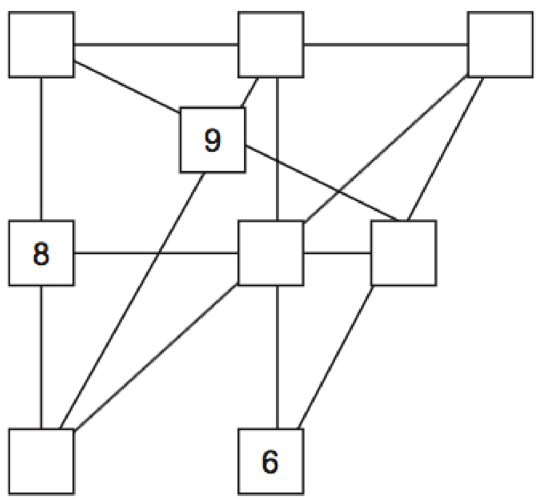
in each of the two corners. Simply add them up and you get that the interior angle of a regular hexagon is

$$60 + 60 = \underline{\underline{120°}}.$$

4. In the diagram, the three boxes in each straight line have a total of 14. (3)
Complete the diagram using the numbers 1, 2, 3, 4, 5, and 7.
You can use this diagram to practise.



Put your final answer on this diagram.



Solution

Clearly, if we imagine that 1 goes in the top-left, we get a 5 on the bottom-left and that is not allowed (can you see why?)

If we imagine that 2 goes in the top-left, we get a 4 on the bottom-left, and 1 on the top-middle and that is not allowed (can you see why?)

If we imagine that 3 goes in the top-left, we get a 3 on the bottom-left – oops!

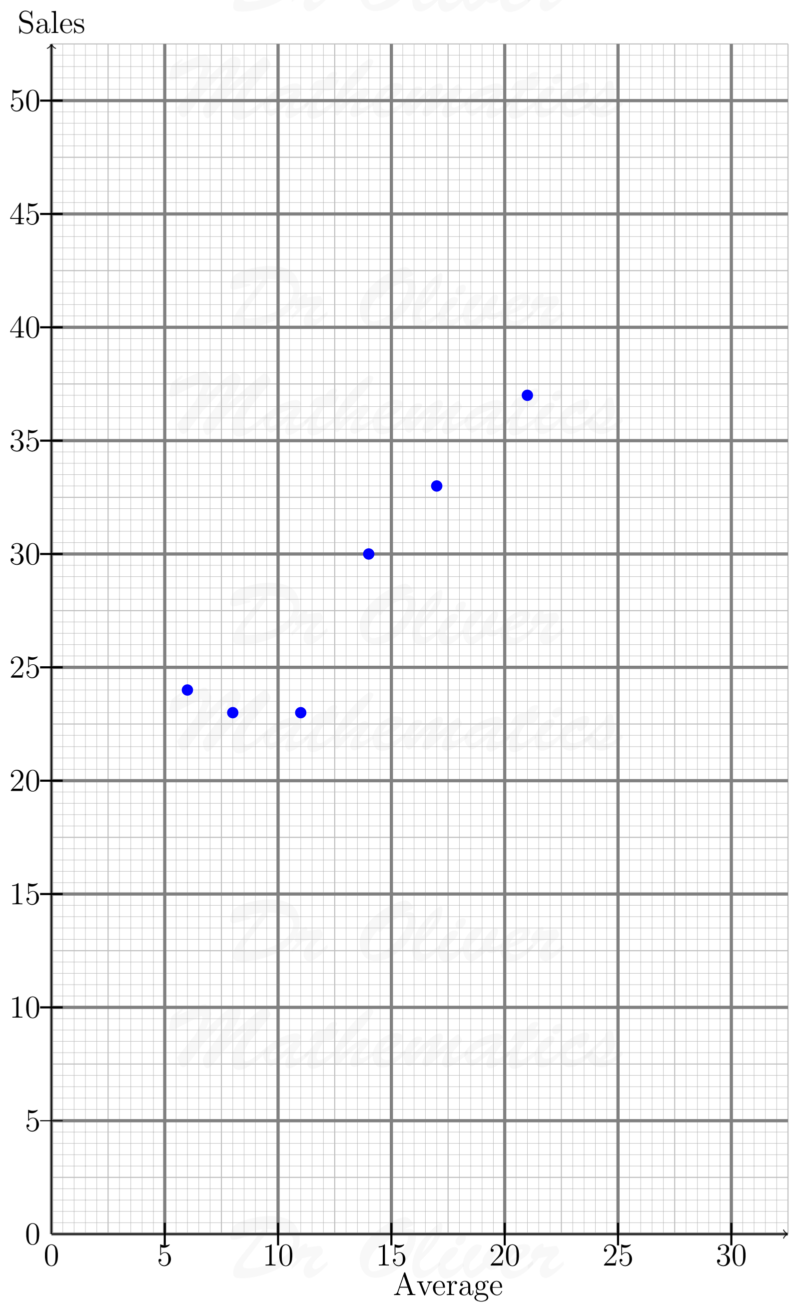
So that leaves 4:

First line	$\underline{\underline{4}}$	$\underline{\underline{3}}$	$\underline{\underline{7}}$
Second line		$\underline{\underline{5}}$	$\underline{\underline{1}}$
Third line	$\underline{\underline{2}}$		

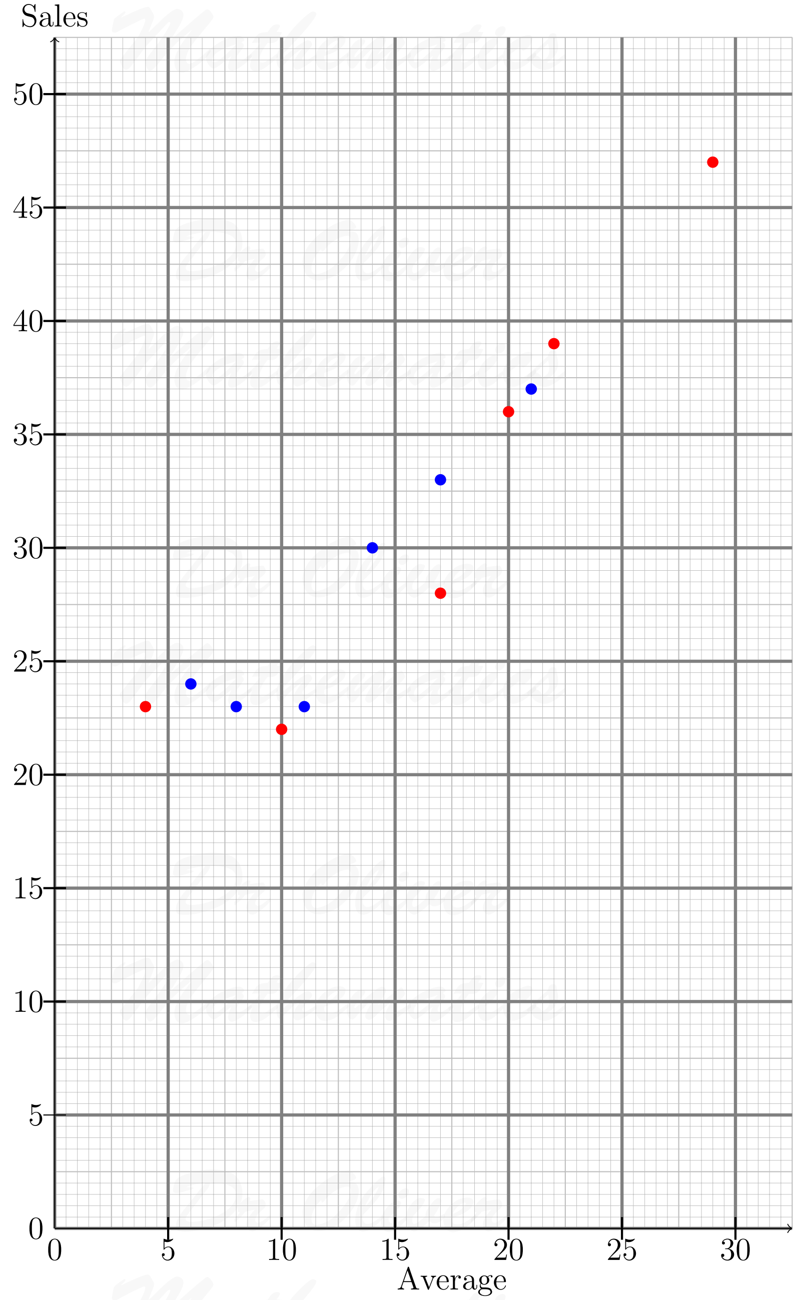
5. A company sells ice cream.
The average midday temperature (in °C) and the sales (tonnes) for each month in 2011 are shown.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Average	8	6	11	14	17	21	22	29	20	14	10	4
Sales	23	24	23	30	33	37	39	47	36	28	22	23

(a) Complete the scatter diagram by plotting the values for July to December. (2)
The values for January to June have been done for you.



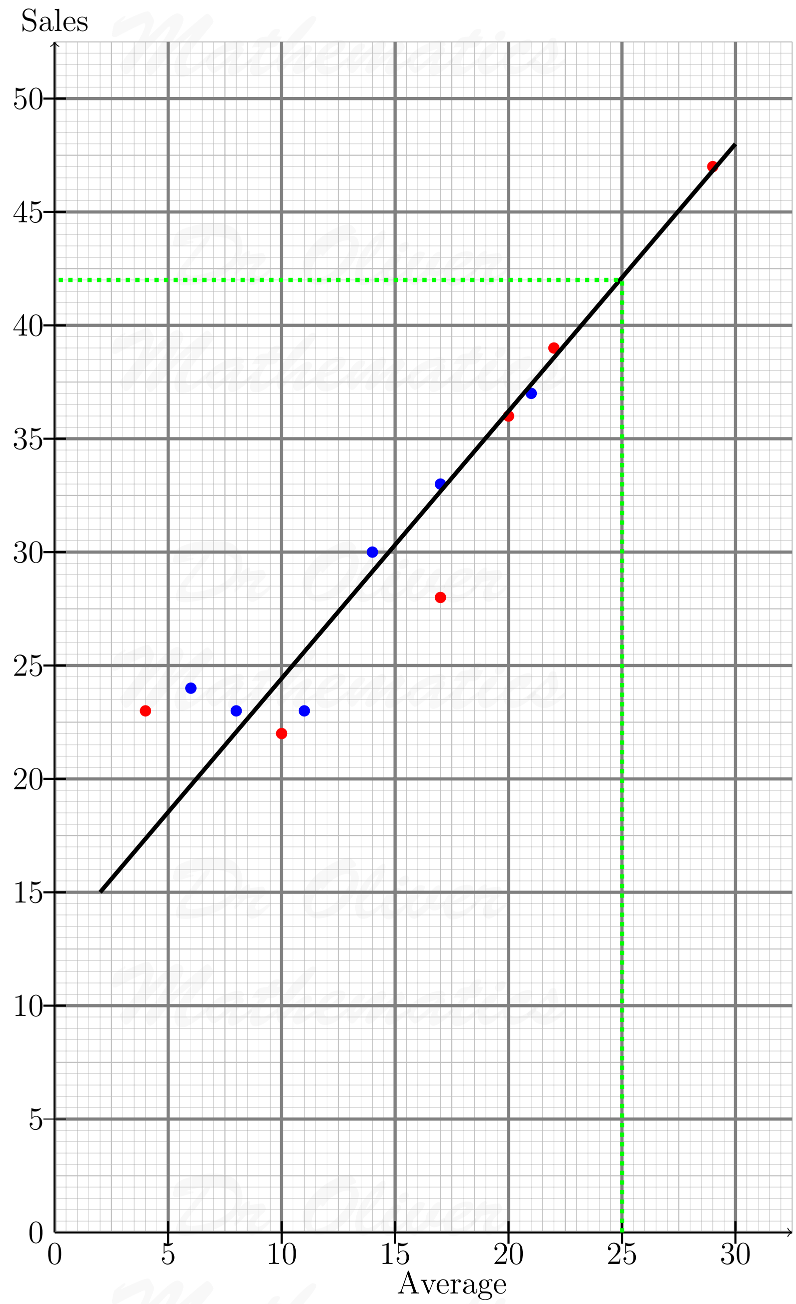
Solution



(b) In July 2012, the average midday temperature is predicted to be 25°C. Use the graph to estimate the sales of ice cream in July 2012. (2)
Show clearly how you obtain your answer.

Solution

Draw a line of best fit and read off the answer:



42 tonnes.

(c) In December 2012, the average midday temperature is predicted to be 5°C higher than in December 2011. (1)
Should the company increase its production of ice cream for December 2012?
Tick a box.

Yes ☐ No ☐

Give a reason for your answer.

Solution

No; as at 9°C, the sales are the same.

6. This circle is drawn accurately. (4)

Work out the area of the circle.
Give your answer in terms of π.
State the units of your answer.

Solution

The radius is 3 cm which means that area is

$$3^2 \times \pi = \underline{\underline{9\pi \text{ cm}^2}}.$$

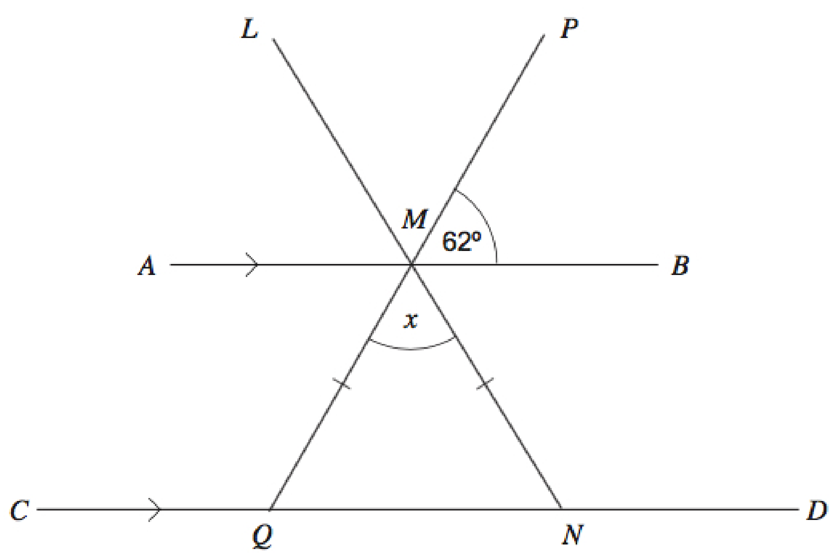
7. Solve (3)

$$6x - 5 = 2x + 13.$$

Solution

$$6x - 5 = 2x + 13 \Rightarrow 4x = 18$$
$$\Rightarrow \underline{\underline{x = 4\tfrac{1}{2}}}.$$

8. AB is parallel to CD. (3)
LMN and PMQ are straight lines.
$MQ = MN$.

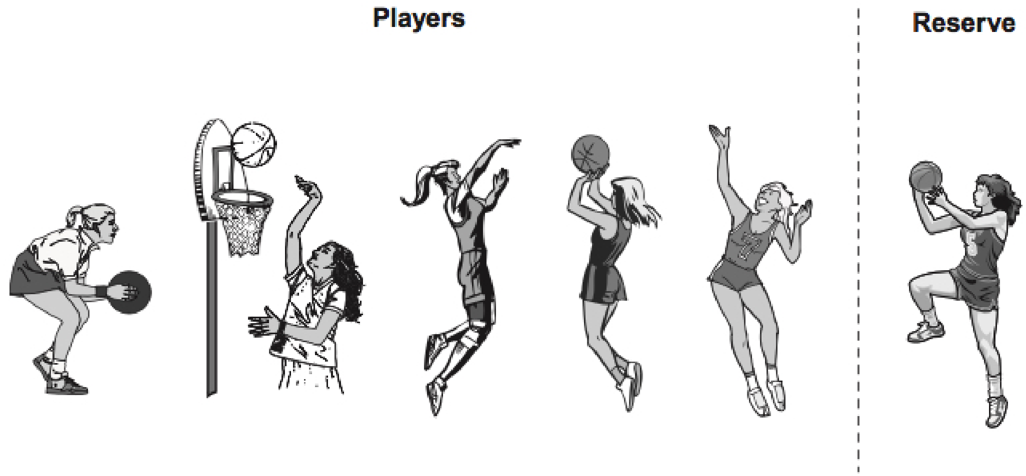


Work out the value of x.

Solution

Well,
$$\angle MQN = \angle MNQ = \tfrac{1}{2}(180 - x) \text{ (base angles)}$$
and
$$\angle MNQ = \angle NMB \text{ (alternate angles)}.$$
Given that PMQ is a straight line,
$$x + \tfrac{1}{2}(180 - x) + 62 = 180 \Rightarrow x + 90 - \tfrac{1}{2}x = 118$$
$$\Rightarrow \tfrac{1}{2}x = 28$$
$$\Rightarrow \underline{\underline{x = 56^\circ}}.$$

9. A basketball team has five players and one reserve. (3)



The mean weight of the **five** players is 58 kg.
The reserve weighs 64 kg.

Work out the mean weight of all **six** team members.

Solution

The actual mass of the five players is

$$5 \times 58 = 290 \text{ kg}.$$

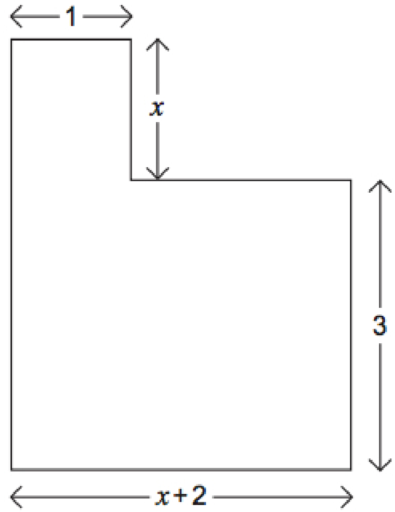
Add this to the mass of the reserve:

$$290 + 64 = 354 \text{ kg}.$$

Now do the average of the six players:

$$\frac{354}{6} = \underline{\underline{59 \text{ kg}}}.$$

10. The L-shape below has an area of 12 cm^2. (4)
All corners are right angles.
All lengths are in centimetres.

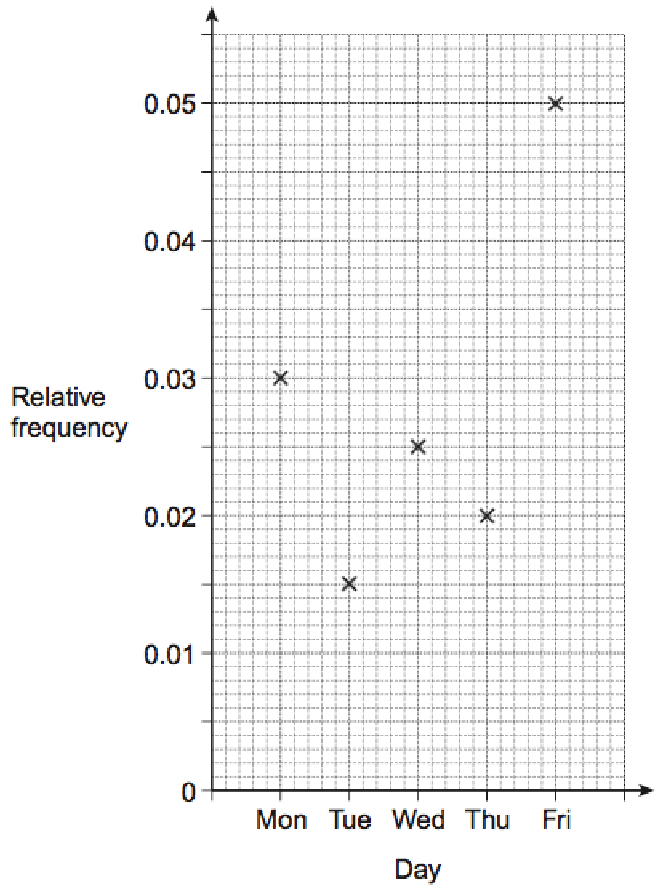


Work out the value of x.

Solution

$$\begin{aligned}\text{Area} = 12 &\Rightarrow (1 \times x) + [3 \times (x+2)] = 12 \\ &\Rightarrow x + 3(x+2) = 12 \\ &\Rightarrow x + 3x + 6 = 12 \\ &\Rightarrow 4x = 6 \\ &\Rightarrow \underline{\underline{x = 1\tfrac{1}{2}}}.\end{aligned}$$

11. The relative frequencies of the number of absences in a school on 5 days are shown. (3)



There are 1600 students in the school.
How many more absences were there on Friday than on Monday?

Solution

There were

$$\begin{aligned} 1600 \times (0.05 - 0.03) &= 1600 \times 0.02 \\ &= 16 \times 2 \\ &= \underline{\underline{32}} \end{aligned}$$

more on Friday than on Monday.

12. Solve the simultaneous equations (4)

$$\begin{aligned} 2x + 4y &= 1 \\ 3x - 5y &= 7. \end{aligned}$$

Do **not** use trial and improvement.
You **must** show your working.

Solution

$$
\begin{aligned}
2x + 4y &= 1 \quad (1) \\
3x - 5y &= 7 \quad (2)
\end{aligned}
$$

Do $3 \times (1)$ and $2 \times (2)$:

$$
\begin{aligned}
6x + 12y &= 3 \quad (3) \\
6x - 10y &= 14 \quad (4)
\end{aligned}
$$

Do $(3) - (4)$:

$$
\begin{aligned}
22y = -11 &\Rightarrow \underline{\underline{y = -\tfrac{1}{2}}} \\
&\Rightarrow 2x + 4(-\tfrac{1}{2}) = 1 \\
&\Rightarrow 2x - 2 = 1 \\
&\Rightarrow 2x = 3 \\
&\Rightarrow \underline{\underline{x = 1\tfrac{1}{2}}}.
\end{aligned}
$$

13. (a) Work out (2)

$$(3 \times 10^5) \times (6 \times 10^9).$$

Give your answer in standard form.

Solution

$$
\begin{aligned}
(3 \times 10^5) \times (6 \times 10^9) &= (3 \times 6) \times (10^5 \times 10^9) \\
&= 18 \times 10^{14} \\
&= \underline{\underline{1.8 \times 10^{15}}}.
\end{aligned}
$$

(b) Work out (2)

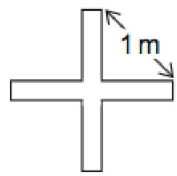
$$(3 \times 10^5) \div (6 \times 10^9).$$

Give your answer in standard form.

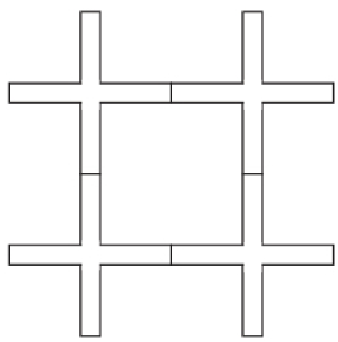
Solution

$$\begin{aligned}(3 \times 10^5) \div (6 \times 10^9) &= (3 \div 6) \times (10^5 \div 10^9)\\ &= 0.5 \times 10^{-4}\\ &= \underline{\underline{5 \times 10^{-5}}}.\end{aligned}$$

14. A cross has a distance of 1 metre between the ends of each arm. (3)



Four of these crosses are put together as shown.



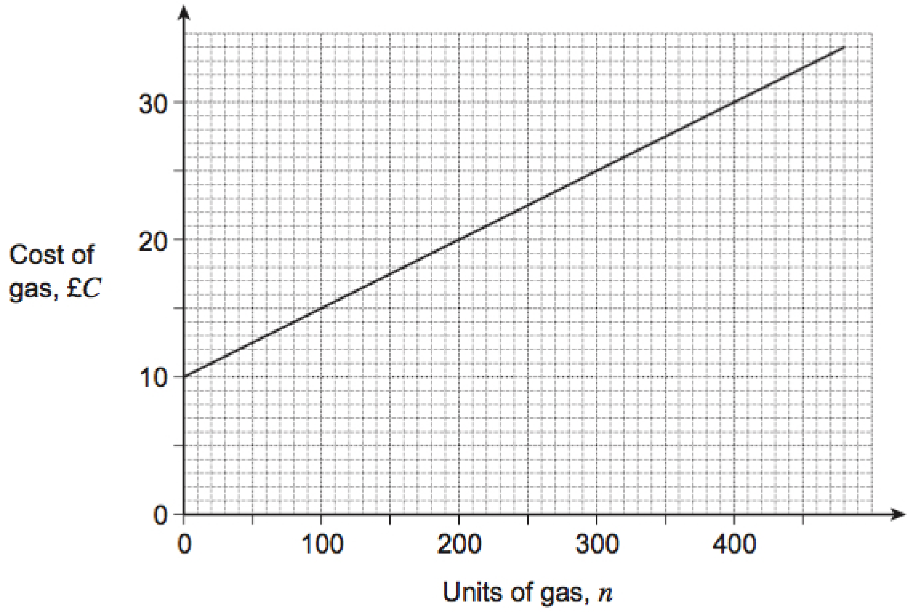
What is the area of the square formed in the middle?
Show clearly how you obtain your answer.

Solution

Let x be the opposite side of the triangle and apply Pythagoras:

$$\begin{aligned}(2x)^2 + (2x)^2 = 2^2 &\Rightarrow 4x^2 + 4x^2 = 4\\ &\Rightarrow 8x^2 = 4\\ &\Rightarrow 4x^2 = 2\\ &\Rightarrow (2x)^2 = 2\\ &\Rightarrow \text{area} = \underline{\underline{2 \text{ cm}^2}}.\end{aligned}$$

15. The graph shows the cost of gas from GasCo. (3)
£C is the cost of the gas and n is the number of units of gas used.



Use the graph to obtain a formula for C in terms of n.

Solution

We will use $(0, 10)$ and $(400, 30)$:

$$\begin{aligned} \text{gradient} &= \frac{30 - 10}{400 - 0} \\ &= \frac{20}{400} \\ &= \tfrac{1}{20} \end{aligned}$$

and, clearly, the C-intercept is 10. Hence, the formula for C in terms of n is

$$\underline{\underline{C = \tfrac{1}{20}n + 10.}}$$

16. y is inversely proportional to the square of x.
When $x = 3$, $y = 8$.

(a) Work out an equation connecting y and x. (3)

Solution

Well,

$$y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2},$$

for some constant k. Now,

$$8 = \frac{k}{3^2} \Rightarrow 8 = \frac{k}{9} \Rightarrow k = 72.$$

Hence,

$$\underline{\underline{y = \frac{72}{x^2}}}.$$

(b) Work out the value of y when $x = 12$. (2)
Give your answer as a fraction in its simplest form.

Solution

$$\begin{aligned} y = \frac{72}{12^2} &\Rightarrow y = \frac{72}{144} \\ &\Rightarrow \underline{\underline{y = \tfrac{1}{2}}}. \end{aligned}$$

17. (a) Factorise (2)

$$2x^2 - x - 3.$$

Solution

$$\left.\begin{array}{ll} \text{add to:} & -1 \\ \text{multiply to:} & (+2) \times (-3) = -6 \end{array}\right\} -3,\ +2$$

Hence, e.g.,

$$\begin{aligned} 2x^2 - x - 3 &= 2x^2 - 3x + 2x - 3 \\ &= x(2x - 3x) + 1(2x - 3) \\ &= \underline{\underline{(x + 1)(2x - 3)}}. \end{aligned}$$

(b) Hence, simplify (2)

$$\frac{2x^2 - x - 3}{4x^2 - 9}.$$

Solution

Difference of two squares:

$$4x^2 - 9 = (2x)^2 - 3^2 = (2x+3)(2x-3).$$

Hence,

$$\begin{aligned}\frac{2x^2 - x - 3}{4x^2 - 9} &= \frac{(x+1)(2x-3)}{(2x+3)(2x-3)} \\ &= \underline{\underline{\frac{x+1}{2x+3}}}.\end{aligned}$$

18. (a) Write (1)

$$\sqrt{72}$$

in the form

$$a\sqrt{2},$$

where a is an integer.

Solution

$$\begin{aligned}\sqrt{72} &= \sqrt{36 \times 2} \\ &= \sqrt{36} \times \sqrt{2} \\ &= \underline{\underline{6\sqrt{2}}};\end{aligned}$$

hence, $\underline{\underline{a = 6}}$.

(b) Work out (3)

$$\left(\sqrt{6} + \sqrt{12}\right)^2.$$

Give your answer in the form

$$c + d\sqrt{2},$$

where c and d are integers.

Solution

$\times$	$\sqrt{6}$	$\sqrt{12}$
$\sqrt{6}$	6	$\sqrt{72}$
$\sqrt{12}$	$\sqrt{72}$	12

Hence

$$\left(\sqrt{6}+\sqrt{12}\right)^2 = 6 + 2\sqrt{72} + 12$$
$$= 18 + 2 \times 6\sqrt{2}$$
$$= \underline{\underline{18 + 12\sqrt{2}}};$$

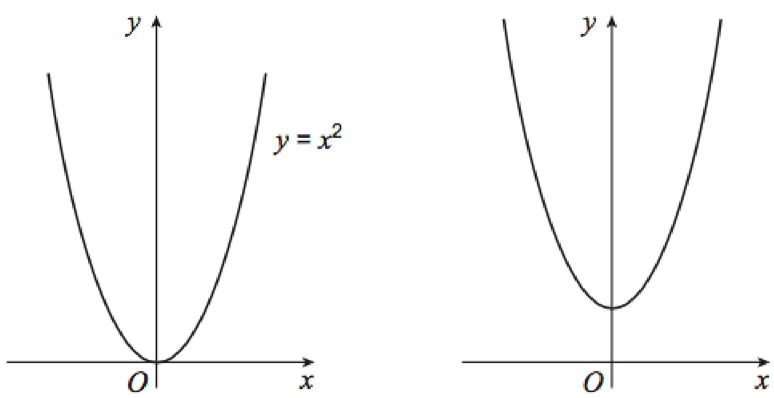
thus, $\underline{\underline{c = 18}}$ and $\underline{\underline{d = 12}}$.

19. (a) The graph of (1)

$$y = x^2$$

is transformed by the vector

$$\begin{pmatrix} 0 \\ 2 \end{pmatrix}.$$



Not drawn accurately

Write down the equation of the transformed graph.

Solution

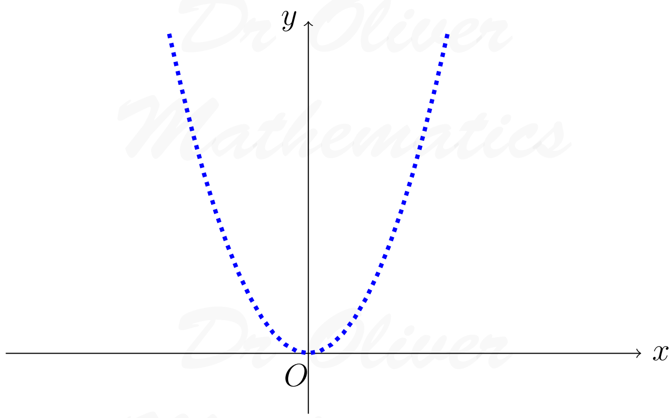
$$\underline{\underline{y = x^2 + 2}}.$$

(b) The diagram shows the graph of (1)

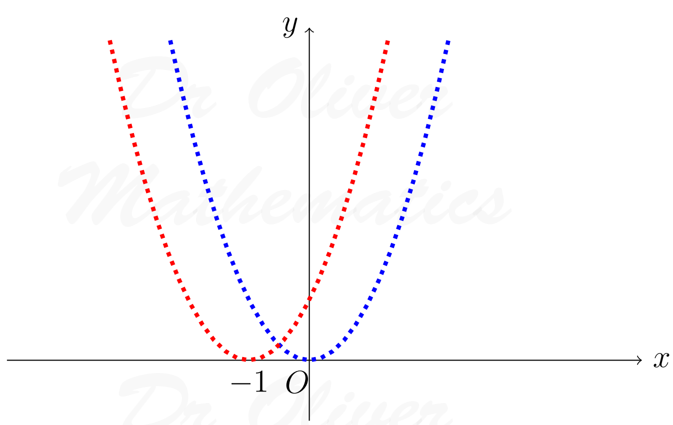
$$y = x^2.$$

On the same diagram, sketch the graph of

$$y = (x+1)^2.$$

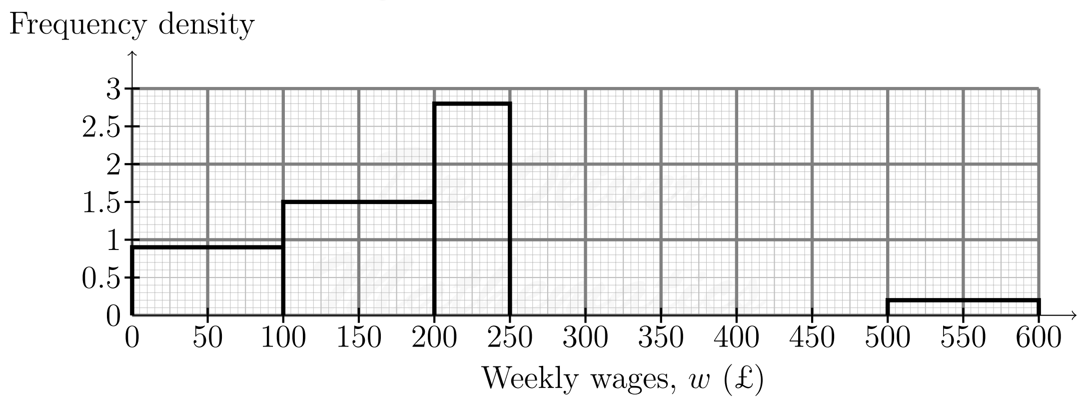


Solution



20. A company has 800 workers. (4)
The table and histogram show the distribution of weekly wages.

Weekly wages, w (£)	Frequency
$0 < w \leqslant 100$	
$100 < w \leqslant 200$	150
$200 < w \leqslant 250$	140
$250 < w \leqslant 300$	120
$300 < w \leqslant 500$	
$500 < w \leqslant 600$	20
	Total = 800



Complete **both** the table and the histogram.

Solution

Weekly wages, w (£)	Frequency	Width	Frequency Density
$0 < w \leqslant 100$		100	0.9
$100 < w \leqslant 200$	150	100	$\frac{150}{100} = 1.5$
$200 < w \leqslant 250$	140	50	$\frac{140}{50} = 2.8$
$250 < w \leqslant 300$	120	50	$\frac{120}{50} = 2.4$
$300 < w \leqslant 500$		200	
$500 < w \leqslant 600$	20	100	$\frac{20}{100} = 0.2$
	Total = 800		

$\underline{0 < w \leqslant 100}$:

$$\text{Frequency Density} = \frac{\text{Frequency}}{\text{Width}} \Rightarrow 0.9 = \frac{\text{Frequency}}{100}$$
$$\Rightarrow \text{Frequency} = 0.9 \times 100$$
$$\Rightarrow \text{Frequency} = \underline{\underline{90}}.$$

$\underline{300 < w \leqslant 500}$:

$$800 - (90 + 150 + 140 + 120 + 20) = 800 - 520 = \underline{\underline{280}}$$

and

$$\begin{aligned} \text{Frequency Density} &= \frac{\text{Frequency}}{\text{Width}} \\ &= \frac{280}{200} \\ &= \underline{\underline{1.4}}. \end{aligned}$$

So, the table is

Weekly wages, w (£)	Frequency	Width	Frequency Density
$0 < w \leqslant 100$	$\underline{\underline{90}}$	100	$\underline{\underline{0.9}}$
$100 < w \leqslant 200$	150	100	$\frac{150}{100} = 1.5$
$200 < w \leqslant 250$	140	50	$\frac{140}{50} = 2.8$
$250 < w \leqslant 300$	120	50	$\frac{120}{50} = 2.4$
$300 < w \leqslant 500$	$\underline{\underline{280}}$	200	$\underline{\underline{1.4}}$
$500 < w \leqslant 600$	20	100	$\frac{20}{100} = 0.2$
	Total = 800		

and the histogram is