Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2004 June Paper 2: Calculator
2 hours

The total number of marks available is 80.
You must write down all the stages in your working.

1. The position vectors of the points A and B, relative to an origin O, are $\mathbf{i} - 7\mathbf{j}$ and $4\mathbf{i} + k\mathbf{j}$ respectively, where k is a scalar. (4)

   The unit vector in the direction of $\overrightarrow{AB}$ is $0.6\mathbf{i} + 0.8\mathbf{j}$.

   Find the value of k.

2. Given that x is measured in radians and $x > 10$, find the smallest value of x such that (4)

$$10 \cos \left(\frac{x+1}{2} \right) = 3.$$

3. Given that

   - $\mathscr{E} = \{\text{students in a college}\}$,
   - $A = \{\text{students who are over 180 cm tall}\}$,
   - $B = \{\text{students who are vegetarian}\}$, and
   - $C = \{\text{students who are cyclists}\}$,

   express in words each of the following

   (a) $A \cap B = \varnothing$, (1)

   (b) $A \subset C'$. (1)

   Express in set notation the statement

   (c) all students who are both vegetarians and cyclists are not over 180 cm tall. (2)

4. Prove the identity (4)

$$(1 + \sec \theta)(\operatorname{cosec} \theta - \cot \theta) \equiv \tan \theta.$$

5. The roots of the quadratic equation (5)

$$x^2 - \sqrt{20}\, x + 2 = 0$$

are c and d.

Without using a calculator, show that

$$\frac{1}{c} + \frac{1}{d} = \sqrt{5}.$$

6. (a) Find the values of x for which (3)

$$2x^2 > 3x + 14.$$

(b) Find the values of k for which the line (3)

$$y + kx = 8$$

is a tangent to the curve

$$x^2 + 4y = 20.$$

7. Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto \mathrm{e}^x,$$
$$\mathrm{g} : x \mapsto 2x - 3.$$

(a) Solve the equation (2)

$$\mathrm{f\,g}(x) = 7.$$

Function h is defined as g f.

(b) Express h in terms of x and state its range. (2)

(c) Express h^{-1} in terms of x. (2)

8. Solve

(a) $\log_3(2x + 1) = 2 + \log_3(3x - 11)$, (4)

(b) $\log_4 y + \log_2 y = 9$. (4)

9. (a) Express (2)

$$6 + 4x - x^2$$

in the form

$$a - (x + b)^2,$$

where a and b are integers.

(b) Find the coordinates of the turning point of the curve (3)

$$y = 6 + 4x - x^2$$

and determine the nature of this turning point.

The function f is defined by

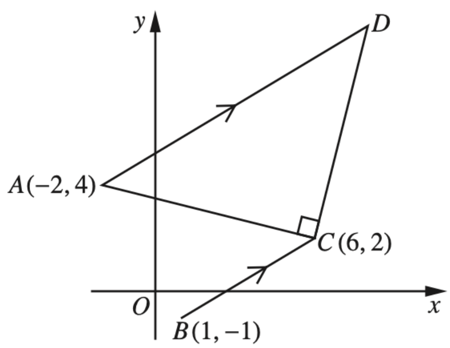
$$\mathrm{f} : x \mapsto 6 + 4x - x^2,$$

for the domain $0 \leqslant x \leqslant 5$.

(c) Find the range of f. (2)

(d) State, giving a reason, whether or not f has an inverse. (1)

10. **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, the points A, B, and C have coordinates $(-2, 4)$, $(1, -1)$, and $(6, 2)$ respectively.

The line AD is parallel to BC and angle $ACD = 90°$.

(a) Find the equations of AD and CD. (6)

(b) Find the coordinates of D. (2)

(c) Show that triangle ACD is isosceles. (2)

11. It is given that

$$y = (x + 1)(2x - 3)^{\frac{3}{2}}.$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be written in the form (4)

$$kx\sqrt{2x - 3},$$

and state the value of k.

Hence

(b) find, in terms of p, an approximate value of y when $x = 6 + p$, where p is small, (3)

(c) evaluate (3)

$$\int_2^6 x\sqrt{2x-3}\,\mathrm{d}x.$$

EITHER

12. A particle moves in a straight line so that, t s after leaving a fixed point O, its velocity, v ms^{-1}, is given by

$$v = 10(1 - \mathrm{e}^{-\frac{1}{2}t}).$$

(a) Find the acceleration of the particle when $v = 8$. (4)

(b) Calculate, to the nearest metre, the displacement of the particle from O when $t = 6$. (4)

(c) State the value which v approaches as t becomes very large. (1)

(d) Sketch the velocity-time graph for the motion of the particle. (2)

OR

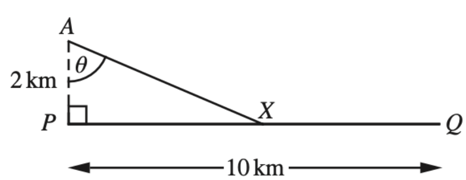
13. (a) By considering $\sec\theta$ as $(\cos\theta)^{-1}$, show that (2)

$$\frac{\mathrm{d}}{\mathrm{d}\theta}(\sec\theta) = \frac{\sin\theta}{\cos^2\theta}.$$

The diagram shows a straight road joining two points, P and Q, 10 km apart.

A man is at point A, where AP is perpendicular to PQ and AP is 2 km.

The man wishes to reach Q as quickly as possible and travels across country in a straight line to meet the road at point X, where angle $PAX = \theta$ radians.



The man travels across country along AX at 3 ms^{-1} but on reaching the road he travels at 5 ms^{-1} along XQ.

Given that he takes T hours to travel from A to Q,

(b) show that (4)

$$T = \tfrac{2}{3}\sec\theta + 2 - \tfrac{2}{5}\tan\theta.$$

(c) Given that θ can vary, show that T has a stationary value when $PX = 1.5$ km. (5)