Dr Oliver Mathematics
Worked Examples
Find the Area of the Yellow Region 1

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1. In Figure 1, a circle, centred at O and of radius 17 cm, parallel chords AB and CD are drawn.

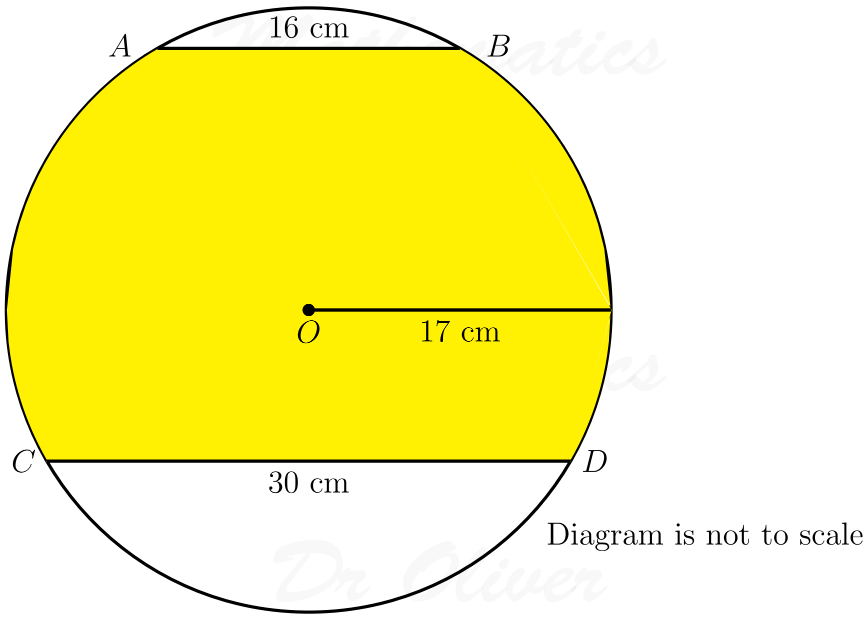

Figure 1: a circle, centred at O and of radius 17 cm

These parallel chords are, respectively, $AB = 16$ cm and $CD = 30$ cm.

Find the area which is coloured in yellow.

Solution

Let E be the midpoint on AB and let F be the midpoint on CD so that EOF is a straight line.

Let $OE = x$ cm and $OF = y$ cm, as shown in Figure 2:

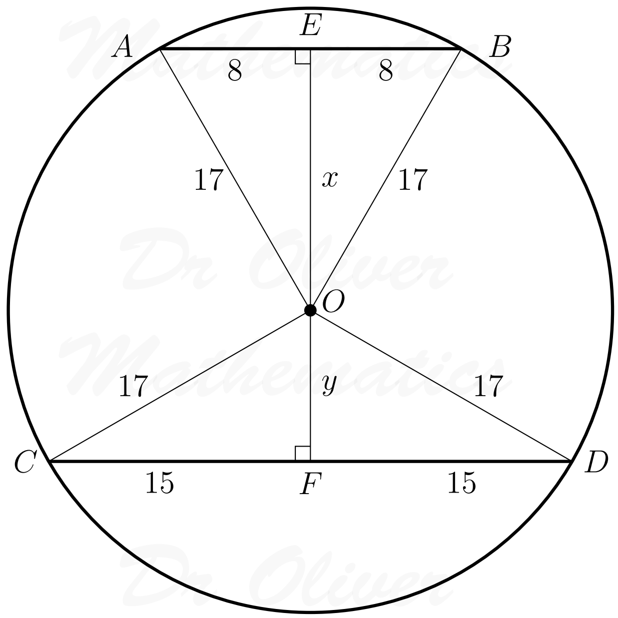

Figure 2: x, y, and the four radii

We will calculate the values of x and y using Pythagoras' theorem:

$$\begin{aligned} 8^2 + x^2 = 17^2 &\Rightarrow 64 + x^2 = 289 \\ &\Rightarrow x^2 = 225 \\ &\Rightarrow x = 15 \end{aligned}$$

and

$$\begin{aligned} 15^2 + y^2 = 17^2 &\Rightarrow 225 + y^2 = 289 \\ &\Rightarrow y^2 = 64 \\ &\Rightarrow y = 8. \end{aligned}$$

The two triangles are both $(8, 15, 17)$ Pythagorean triples.

Now,

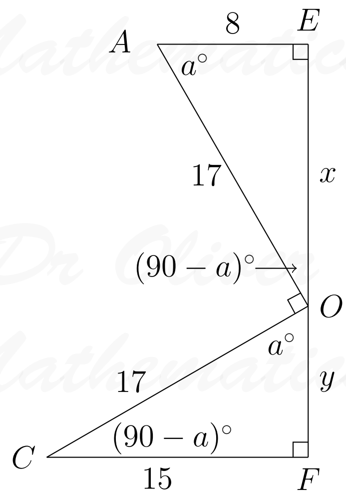

Figure 3: EOF is a straight line

$$\begin{aligned}\angle AOE + \angle AOC + \angle COF = 180 &\Rightarrow (90 - a) + \angle AOC + a = 180\\ &\Rightarrow \angle AOC = 90^\circ;\end{aligned}$$

exactly the same goes for $\angle BOD$:

$$\angle BOD = 90^\circ.$$

(In fact,

$$\angle AOD = \tan^{-1}\tfrac{8}{15} = 28.072\,486\,94^\circ \text{ (FCD)}$$

and

$$\angle COE = \tan^{-1}\tfrac{15}{8} = 61.927\,513\,06^\circ \text{ (FCD)};$$

the reader will not be surprised to find the sum the angles equals 90°.)

Now,

$$\begin{aligned}\text{area}_{\triangle OAB} &= \tfrac{1}{2} \times 16 \times 15\\ &= 120 \text{ cm}^2\end{aligned}$$

and

$$\begin{aligned}\text{area}_{\triangle OCD} &= \tfrac{1}{2} \times 30 \times 8\\ &= 120 \text{ cm}^2.\end{aligned}$$

Next,

$$\begin{aligned}\text{area}_{\text{sector } OAC} &= \tfrac{1}{4} \times (\pi \times 17^2) \\ &= \tfrac{289}{4}\pi\end{aligned}$$

and

$$\begin{aligned}\text{area}_{\text{sector } OBD} &= \tfrac{1}{4} \times (\pi \times 17^2) \\ &= \tfrac{289}{4}\pi.\end{aligned}$$

Finally,

$$\begin{aligned}\text{area} &= 120 + 120 + \tfrac{289}{4}\pi + \tfrac{289}{4}\pi \\ &= \underline{\underline{\left(240 + \tfrac{289}{2}\pi\right)\ \text{cm}^2}}.\end{aligned}$$