

Dr Oliver Mathematics
OCR FMSQ Additional Mathematics
2023 Paper
2 hours

The total number of marks available is 100.

You must write down all the stages in your working.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given correct to three significant figures where appropriate.

1. Find the term independent of x in the expansion of (3)

$$\left(x + \frac{2}{x}\right)^6.$$

2. Jack throws 4 ordinary six-sided dice numbered 1 to 6. (3)

Find the probability that he throws at least one 3.

3. Use long division to find the quotient and the remainder when (3)

$$x^3 + 3x^2 + 5x - 3$$

is divided by $(x + 1)$.

4. Simplify the following.

(a) $\frac{1}{x-2} - \frac{2}{x+1}$. (2)

(b) **In this question you must show detailed reasoning.** (3)
 $\frac{2}{5-\sqrt{2}} + \frac{1}{5+\sqrt{2}}$.

5. You are given that

$$\sin \theta = -0.6 \text{ for } 270^\circ \leq \theta \leq 360^\circ.$$

(a) Find the value of θ . (2)

(b) Using Pythagoras' theorem, determine the **exact** value of $\tan \theta$. (4)

6. A car accelerates from rest in a straight line.

At time t seconds its velocity, $v \text{ m s}^{-1}$, is given by the equation

$$v = 20 \left(1 - 2^{-\frac{1}{2}t}\right).$$

(a) Calculate the velocity of the car when $t = 4, 6,$ and 8 seconds. (2)

(b) Hence calculate an estimate of the acceleration of the car at $t = 6$ seconds. (2)
Give your answer correct to **2** significant figures.

(c) Explain how this estimate could be improved. (1)

7. You are given that the equation

$$3^x - 4x^2 = 0$$

has three roots, $\alpha, \beta,$ and γ where $\alpha < 0$ and $\gamma > 3$.

(a) By considering the value of (2)

$$3^x - 4x^2 = 0$$

when $x = 0$ and $x = 1$, show that β lies between 0 and 1 .

(b) By considering appropriate values of x , determine the value of β correct to 1 decimal (3)
place.

8. The triangle ABC is such that $AB = 12$ cm and angle $BAC = 50^\circ$.

(a) Given that $BC = 10$ cm, determine the two possible values of angle ACB . (4)

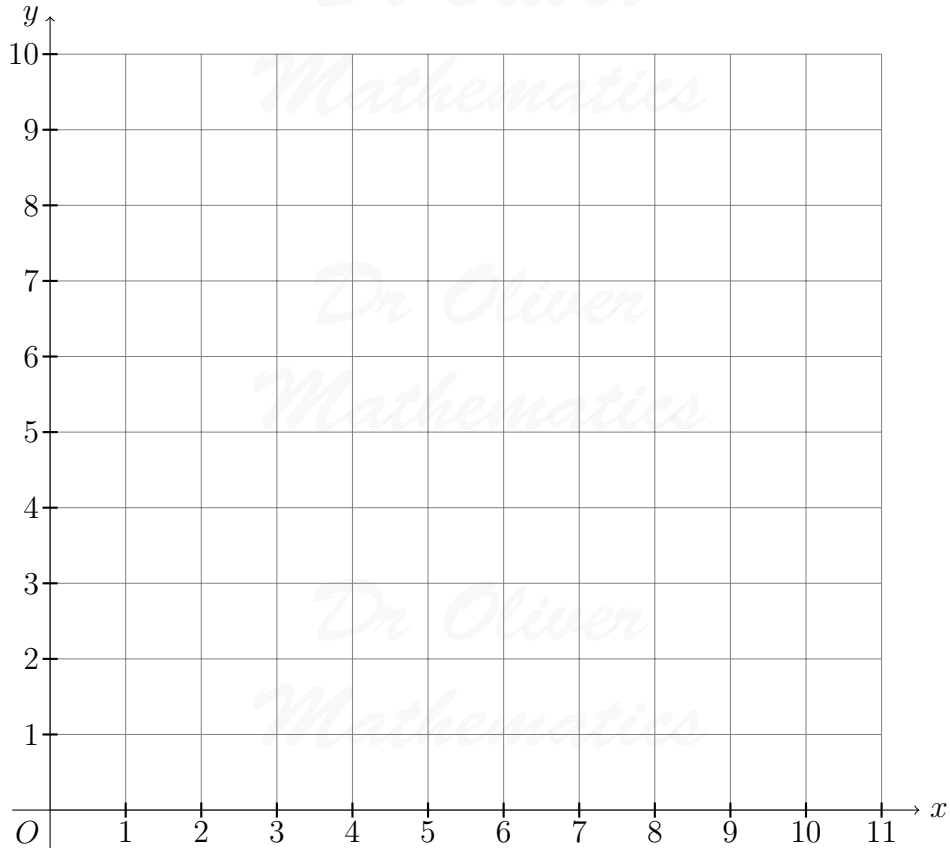
(b) State **two** conditions for BC such that if **either** of them is satisfied there will be (2)
only **one** value for the angle ACB .

9. The point A has the coordinate $(3, 7)$ and the point B has the coordinate $(7, 1)$. (5)

Find the equation of the perpendicular bisector of AB .

10. (a) On the grid below, indicate the region for which the following inequalities hold. (4)
You should shade the region that is **not** satisfied by the inequalities.

- $y \geq x + 1,$
- $x \geq 1,$ and
- $x + 2y \leq 11.$



(b) Find the maximum value of $x + y$ (2)

subject to these conditions.

11. Amir asked 80 people about their preferences for the drinks tea, coffee or hot chocolate. The results of his investigation were as follows.

- 25 liked all three drinks.
- 3 liked tea and coffee but not hot chocolate.
- 4 liked hot chocolate and coffee but not tea.
- 5 liked tea but neither of the other two drinks.
- 35 liked tea and hot chocolate.
- 48 liked hot chocolate.
- 47 liked coffee.

(a) Draw a Venn diagram to represent these data. (3)

(b) Hence determine how many people said that they did **not** like any of the drinks. (2)

12. The point $A(1, 3)$ lies on a circle with centre $(4, 5)$.
 (a) Determine the equation of the circle. (2)

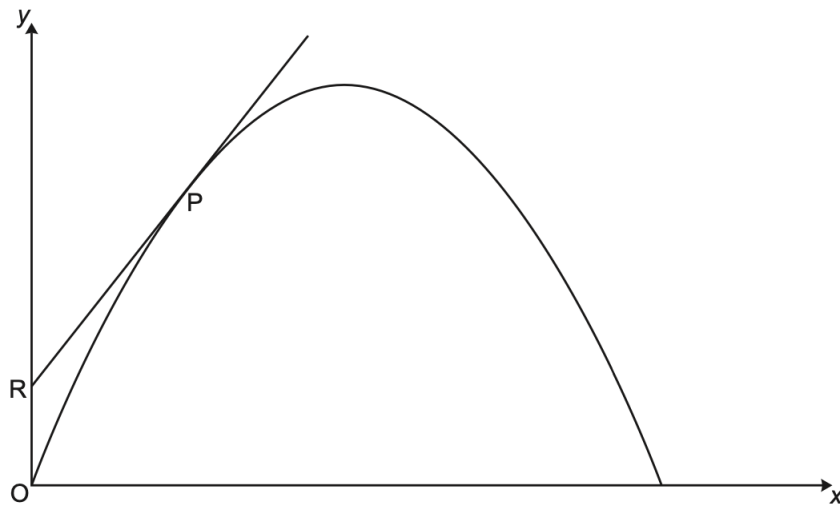
B is a point on the circle such that AB is a diameter of the circle.
 (b) Find the coordinates of B . (2)

D is the point $(2, 8)$.
 (c) Show that AD and DB are perpendicular. (3)

(d) Explain what this tells you about the point D . (1)

13. The point P with coordinates $(2, 12)$ lies on the curve

$$y = 8x - x^2.$$



- The tangent to this curve at P meets the y -axis at the point R , as shown in the diagram.
- The origin is O .

(a) Determine the coordinates of the point R . (5)

(b) Determine the exact area of the region OPR that is bounded by the curve from O to P , the tangent PR , and the y -axis. (6)

14. Sarah brings a saucepan of water to the boil.
 She leaves the water to cool, measuring its temperature every 10 minutes for 30 minutes.
 The results are shown in the table below.

Time (t minutes)	0	10	20	30
Temperature ($T^\circ\text{C}$)	100	60	40	30

Sarah believes that the temperature of the water as it cools can be modelled by the equation

$$T - 20 = A \times 2^{-\frac{t}{b}},$$

where A and b are constants.

(a) (i) Explain the significance of the number 20 in this equation. (1)

(ii) Use the fact that the initial temperature of the water is 100°C to determine the value of A . (2)

(b) By taking logs of both sides of Sarah's equation, show that plotting (3)

$$\log_{10}(T - 20)$$

against t will give a straight line.

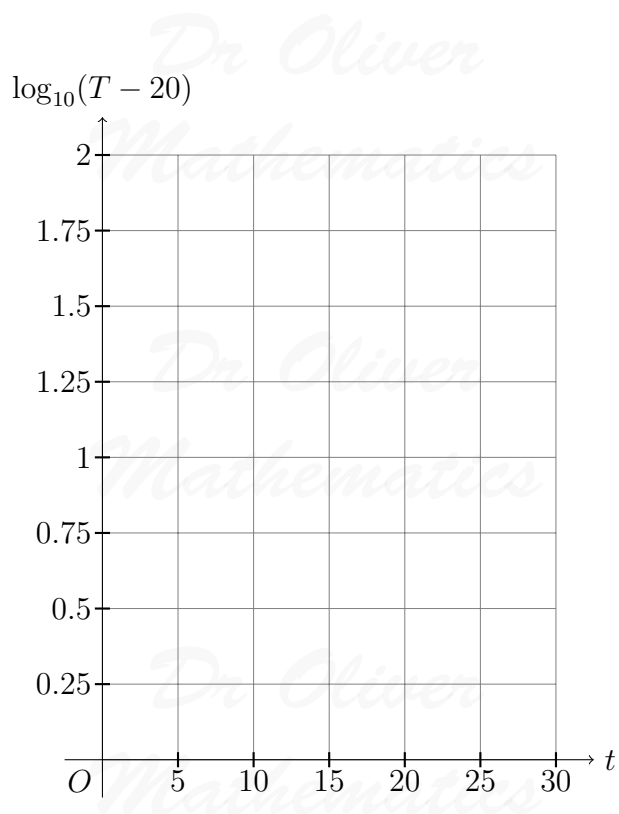
(c) Complete the table below. (2)

Time (t minutes)		0	10	20	30
Temperature ($T^\circ\text{C}$)		100	60	40	30
$T - 20$					
$\log_{10}(T - 20)$					

(d) Plot the values of (1)

$$\log_{10}(T - 20)$$

against t on the grid below.



(e) Hence estimate the value of b . (2)

15. **In this question you must show detailed reasoning.**

You are given that the curve

$$y = 2x^3 + 3x^2 - 12x + 8$$

has two stationary points.

(a) (i) Show that one of the stationary points has coordinates $(1, 1)$. (4)

(ii) Determine the nature of this stationary point. (2)

(b) Find the coordinates of the other stationary point. (2)

16. I can drive my motor boat at a maximum speed of 4 kilometres per hour in still water.

One day, I drive at maximum speed up a river from a point A to a point B , a distance of 9 km.

The constant speed of the current down the river is r kilometres per hour.

(a) Show that the time it takes me to drive up the river from A to B is (2)

$$\left(\frac{9}{4 - r} \right) \text{ hours.}$$

Dr. Oliver
Mathematics
(b) Write down, in terms of r , the time it takes me to drive down the river from B to A . (1)

(c) Given that the difference between the time to drive up the river (a) and the time to drive down the river (b) is 1.2 hours, form an equation in r and show that it simplifies to (4)

$$r^2 + 15r - 16 = 0.$$

(d) Hence find the speed of the current down the river. (3)

Dr. Oliver
Mathematics

Dr. Oliver
Mathematics

Dr. Oliver
Mathematics

Dr. Oliver
Mathematics

Dr. Oliver
Mathematics