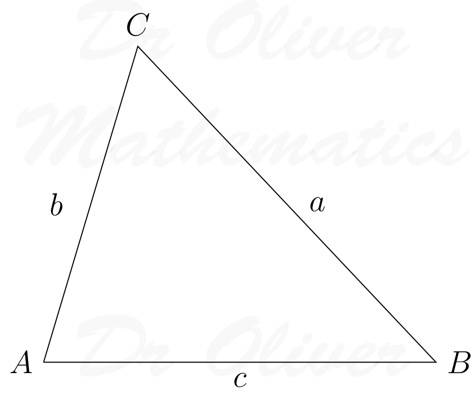
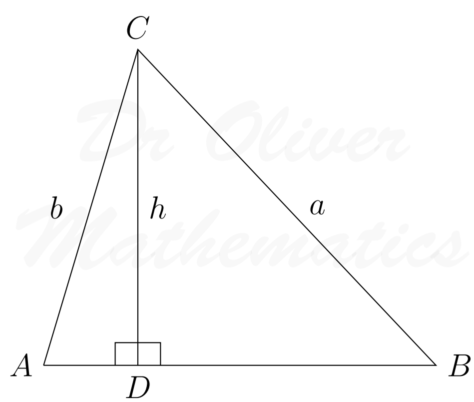
# Dr Oliver Mathematics
# Area of a Triangle

In this note, we will investigate the area of a triangle.

Suppose we have the following triangle.




Split the triangle in two and call the altitude $h$:



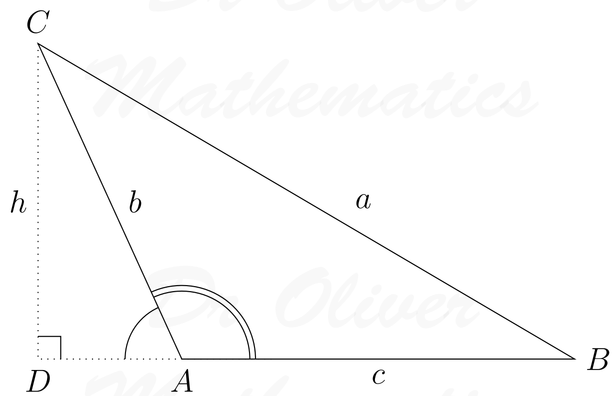

Now,

$$\frac{h}{b} = \sin A^\circ \Rightarrow h = b \sin A^\circ$$

and

$$\begin{aligned} \text{area of a triangle} &= \tfrac{1}{2}ch \\ &= \tfrac{1}{2}bc \sin A^\circ. \end{aligned}$$

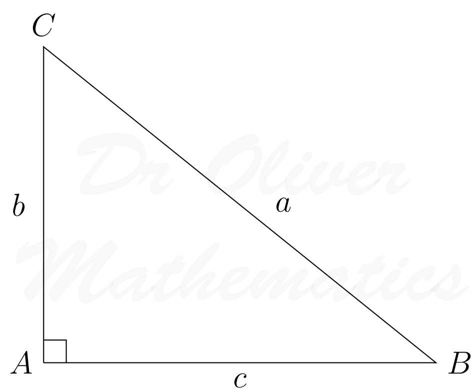
What about one obtuse angle?




$$\begin{aligned}\frac{h}{b} = \sin DAC^\circ &\Rightarrow h = b \sin DAC^\circ \\ &\Rightarrow h = b \sin BAC^\circ\end{aligned}$$

since supplementary angles have the same sine and the proof follows.

What about right-angled triangles?




Now,

$$\text{area of a triangle} = \tfrac{1}{2}bc = \tfrac{1}{2}bc \sin A^\circ$$

since $\sin A^\circ = 1$.

Hence, the area of a triangle equals

$$\tfrac{1}{2} \times \text{product of the two sides} \times \text{sine of the included angle}$$

and we are left with this

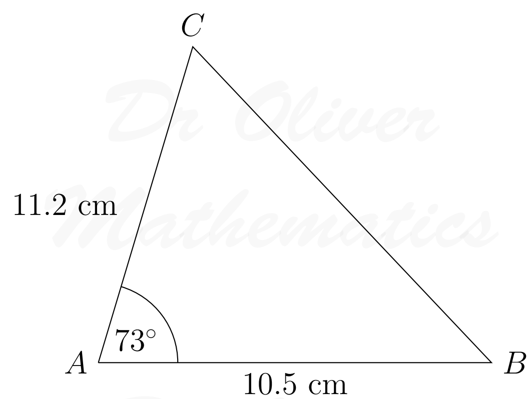
$$\boxed{\begin{aligned}\text{area of a triangle} &= \tfrac{1}{2}bc \sin A^\circ \\ &= \tfrac{1}{2}ac \sin B^\circ \\ &= \tfrac{1}{2}ab \sin C^\circ.\end{aligned}}$$

Let the area of a triangle be $A$ and we can rearrange this into

$$\boxed{\begin{aligned}\sin A^\circ &= \frac{2A}{bc}\\ \sin B^\circ &= \frac{2A}{ac}\\ \sin C^\circ &= \frac{2A}{ab}.\end{aligned}}$$

Okay: a few examples. We will give our answers to 3 significant figures. Oh, the diagrams are *not* accurately drawn...

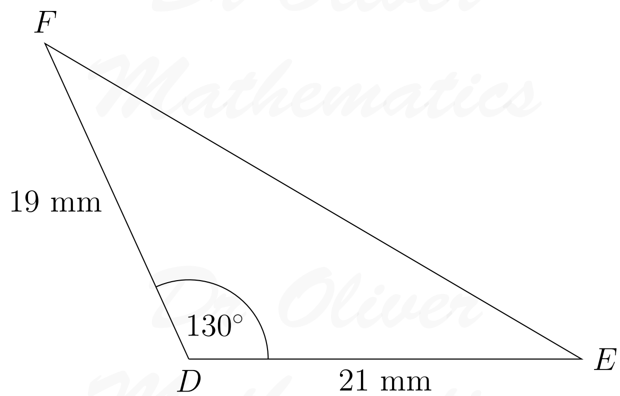
1. In $\triangle ABC$, find the area.




**Solution**

$$\begin{aligned}\text{Area} &= \tfrac{1}{2} \times 10.5 \times 11.2 \times \sin 73^\circ\\ &= 56.230\,719\,65 \text{ (FCD)}\\ &= \underline{\underline{56.2 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

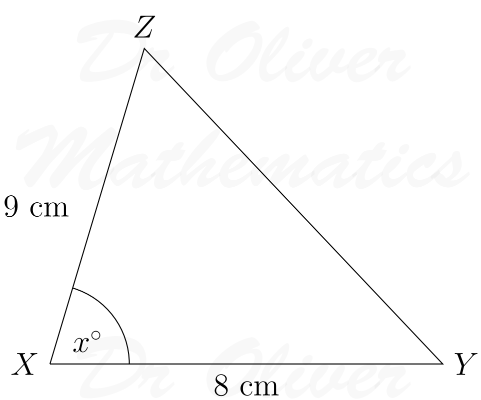
2. In $\triangle DEF$, find $EF$.




**Solution**

$$\begin{aligned}\text{Area} &= \tfrac{1}{2} \times 21 \times 19 \times \sin 130^\circ \\ &= 152.825\,866\,4 \text{ (FCD)} \\ &= \underline{\underline{153 \text{ mm}^2 \text{ (3 sf)}}}.\end{aligned}$$

3. In $\triangle XYZ$, the area equals 30 cm$^2$. Find $x^\circ$.




**Solution**

$$\begin{aligned}30 = \tfrac{1}{2} \times 8 \times 9 \times \sin x^\circ &\Rightarrow \sin x^\circ = \tfrac{5}{6} \\ &\Rightarrow x^\circ = 56.442\,690\,24 \text{ or } 123.557\,309\,8 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x^\circ = 56.2^\circ \text{ or } 124^\circ \text{ (3 sf)}}}.\end{aligned}$$