Dr Oliver Mathematics
Mathematics Standard Grade: Credit Level
2011 Paper 1: Non-Calculator
55 minutes

The total number of marks available is 41.
You must write down all the stages in your working.

1. Evaluate (2)

$$2.4 + 5.46 \div 60.$$

Solution

$$\begin{aligned} 2.4 + (5.46 \div 60) &= 2.4 + (0.546 \div 6) \\ &= 2.4 + 0.091 \\ &= \underline{\underline{2.491}}. \end{aligned}$$

2. Factorise fully (2)

$$2m^2 - 18.$$

Solution

$$2m^2 - 18 = 2(m^2 - 9)$$

$$\left.\begin{array}{ll} \text{add to:} & 0 \\ \text{multiply to:} & -9 \end{array}\right\} -3,\ +3$$

$$\begin{aligned} &= 2[m^2 - 3m + 3m - 9] \\ &= 2[m(m-3) + 3(m-3)] \\ &= \underline{\underline{2(m+3)(m-3)}}. \end{aligned}$$

3. Given that (2)

$$f(x) = 5 - x^2,$$

evaluate $f(-3)$.

Solution

$$\begin{aligned} f(-3) &= 5 - (-3)^2 \\ &= 5 - 9 \\ &= \underline{\underline{-4}}. \end{aligned}$$

4. Solve the equation (3)

$$3x + 1 = \frac{x-5}{2}.$$

Solution

$$\begin{aligned} 3x + 1 = \frac{x-5}{2} &\Rightarrow 3x + 1 = \tfrac{1}{2}x - 2\tfrac{1}{2} \\ &\Rightarrow \tfrac{5}{2}x = -3\tfrac{1}{2} \\ &\Rightarrow \tfrac{5}{2}x = -\tfrac{7}{2} \\ &\Rightarrow \underline{\underline{x = -\tfrac{7}{5} \text{ or } -1\tfrac{2}{5}}}. \end{aligned}$$

5. Jamie is going to bake cakes for a party. (3)
He needs $\frac{2}{5}$ of a block of butter for 1 cake.
He has 7 blocks of butter.
How many cakes can Jamie bake?

Solution

$$\begin{aligned} 7 \div \tfrac{2}{5} &= 7 \times \tfrac{5}{2} \\ &= \tfrac{35}{2} \\ &= 17\tfrac{1}{2}; \end{aligned}$$

so, he can make $\underline{\underline{\text{17 cakes}}}$.

6. A driving examiner looks at her diary for the next 30 days.
She writes down the number of driving tests booked for each day as shown below.

Number of tests booked	0	1	2	3	4	5	6
Frequency	1	1	3	2	9	10	4

(a) Find the median for this data. (2)

Solution
There are
$$1+1+3+2+9+10+4=30$$
pieces of data in this set and
$$\frac{30+1}{2}=15\tfrac{1}{2}\text{th}$$
piece of data.

Number of tests booked	$\leqslant 0$	$\leqslant 1$	$\leqslant 2$	$\leqslant 3$	$\leqslant 4$	$\leqslant 5$	$\leqslant 6$
Cumulative Frequency	1	2	5	7	16	26	30

Hence, it is $\underline{\underline{4}}$.

(b) Find the probability that **more than** 4 tests are booked for one day. (1)

Solution
$$\begin{aligned}\text{P(more than 4)} &= \frac{10+4}{30}\\ &= \underline{\underline{\tfrac{14}{30}}}\\ &= \underline{\underline{\tfrac{7}{15}}}.\end{aligned}$$

7. Brian, Molly, and their four children visit Waterworld.
The total cost of their tickets is £56.
Let a pounds be the cost of an adult's ticket and c pounds the cost of a child's ticket.

(a) Write down an equation in terms of a and c to illustrate this information. (1)

Solution

$$\underline{\underline{2a + 4c = 56}} \quad (1)$$

Sarah and her three children visit Waterworld.
The total cost of their tickets is £36.

(b) Write down another equation in terms of a and c to illustrate this information. (1)

Solution

$$\underline{\underline{a + 3c = 36}} \quad (2)$$

(c) (i) Calculate the cost of a child's ticket. (2)

Solution

$$\begin{aligned} (1): \quad & 2a + 4c = 56 \\ 2 \times (2): \quad & 2a + 6c = 72 \quad (3) \end{aligned}$$

$(3) - (1)$:

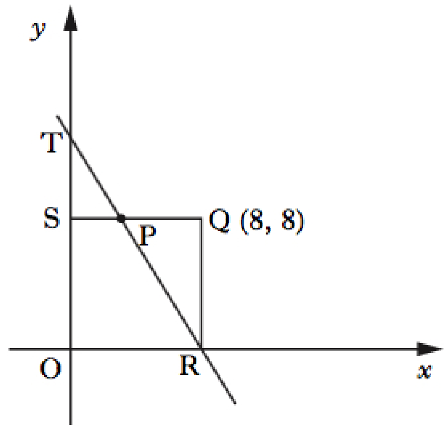
$$2c = 16 \Rightarrow \underline{\underline{c = 8}}.$$

(ii) Calculate the cost of an adult's ticket. (1)

Solution

$$a = 36 - 3 \times 8 = \underline{\underline{12}}.$$

8. A square, $OSQR$, is shown below.



Q is the point $(8, 8)$.
The straight line TR cuts the y-axis at $T(0, 12)$ and the x-axis at R.

(a) Find the equation of the line TR. (3)

Solution
Well, $R(8, 0)$ and the gradient of TR is

$$\frac{12-0}{0-8} = -\tfrac{3}{2}.$$

Finally, the equation of the line TR is

$$y - 12 = -\tfrac{3}{2}(x - 0) \Rightarrow y - 12 = -\tfrac{3}{2}x$$
$$\Rightarrow \underline{\underline{y = -\tfrac{3}{2}x + 12}}.$$

The line TR also cuts SQ at P.

(b) Find the coordinates of P. (4)

Solution

$$8 = -\tfrac{3}{2}x + 12 \Rightarrow -\tfrac{3}{2}x = -4$$
$$\Rightarrow 3x = 8$$
$$\Rightarrow x = 2\tfrac{2}{3};$$

hence, $\underline{\underline{P(2\tfrac{2}{3}, 8)}}$.

9. (a) Simplify (1)

$$2a \times a^{-4}.$$

Solution

$$2a \times a^{-4} = \underline{\underline{2a^{-3}}}.$$

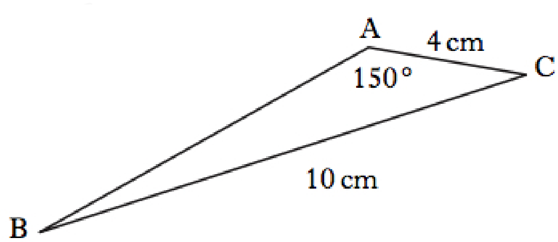
(b) Solve for x, (3)

$$\sqrt{x} + \sqrt{18} = 4\sqrt{2}.$$

Solution

$$\begin{aligned}\sqrt{x} + \sqrt{18} = 4\sqrt{2} &\Rightarrow \sqrt{x} = 4\sqrt{2} - \sqrt{9 \times 2} \\ &\Rightarrow \sqrt{x} = 4\sqrt{2} - (\sqrt{9} \times \sqrt{2}) \\ &\Rightarrow \sqrt{x} = 4\sqrt{2} - 3\sqrt{2} \\ &\Rightarrow \sqrt{x} = \sqrt{2} \\ &\Rightarrow \underline{\underline{x = 2}}.\end{aligned}$$

10. In triangle ABC, $AC = 4$ centimetres, $BC = 10$ centimetres, and angle $BAC = 150°$. (4)



Given that $\sin 30° = \frac{1}{2}$, show that

$$\sin ABC = \tfrac{1}{5}.$$

Solution

$$\begin{aligned}\frac{\sin ABC}{AC} = \frac{\sin BAC}{BC} \Rightarrow \frac{\sin ABC}{4} &= \frac{\sin 150^\circ}{10} \\ \Rightarrow \sin ABC &= \frac{4\sin(180-150)^\circ}{10} \\ \Rightarrow \sin ABC &= \frac{2\sin 30^\circ}{5} \\ \Rightarrow \underline{\underline{\sin ABC = \tfrac{1}{5}}},\end{aligned}$$

as required.

11. F varies directly as s and inversely as the square of d.

(a) Write down a relationship connecting F, s, and d. (1)

Solution

$$F \propto \frac{s}{d^2} \Rightarrow \underline{\underline{F = \frac{ks}{d^2}}}$$

for some constant, k.

(b) What is the effect on F when s is halved and d is doubled? (3)

Solution

$$\begin{aligned}F &= \frac{k(\frac{1}{2}s)}{(2d)^2} \\ &= \frac{\frac{1}{2}ks)}{4d^2} \\ &= \tfrac{1}{8} \cdot \frac{ks}{d^2};\end{aligned}$$

hence, F is $\underline{\underline{\tfrac{1}{8}}}$ of the original value.

12. The sums, S_2, S_3, and S_4 of the first 2, 3, and 4 natural numbers are given by

$$\begin{aligned}S_2 &= 1 + 2 = \tfrac{1}{2}(2 \times 3) = 3 \\ S_3 &= 1 + 2 + 3 = \tfrac{1}{2}(3 \times 4) = 6 \\ S_4 &= 1 + 2 + 3 + 4 = \tfrac{1}{2}(4 \times 5) = 10.\end{aligned}$$

(a) Find S_{10}, the sum of the first 10 natural numbers. (1)

> **Solution**
>
> $$S_{10} = \tfrac{1}{2}(10 \times 11) = \underline{\underline{55}}.$$

(b) Write down the formula for the sum, S_n, of the first n natural numbers. (1)

> **Solution**
>
> $$S_n = \underline{\underline{\tfrac{1}{2}n(n+1)}}.$$