Dr Oliver Mathematics
GCSE Mathematics
2019 Paper 2H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. (a) Solve (2)

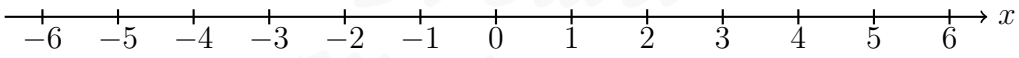
$$14n > 11n + 6.$$

Solution

$$14n > 11n + 6 \Rightarrow 3n > 6$$
$$\Rightarrow \underline{\underline{n > 2}}.$$

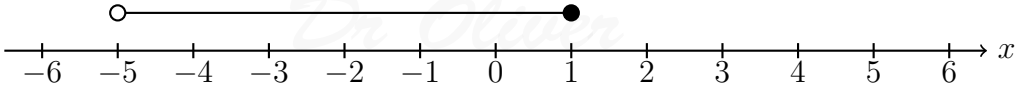
(b) On the number line below, show the set of values of x for which (3)

$$-2 < x + 3 \leqslant 4.$$



Solution

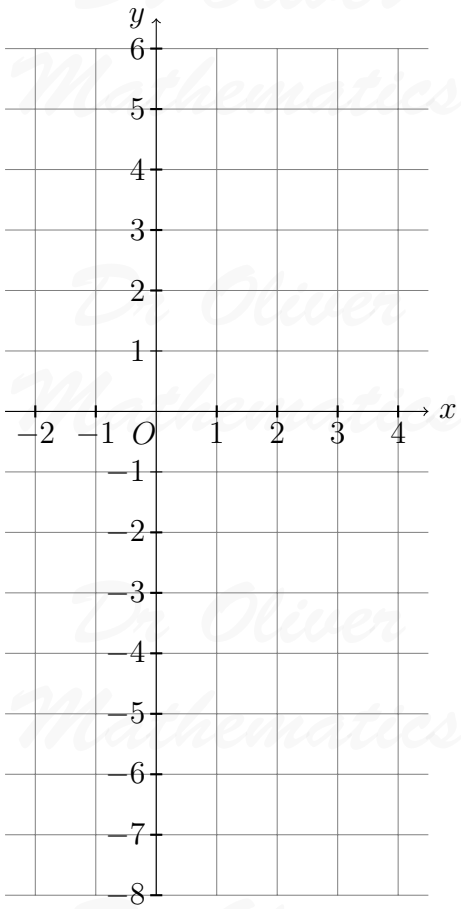
$$-2 < x + 3 \leqslant 4 \Rightarrow -5 < x \leqslant 1.$$



2. On the grid below, draw the graph of (3)

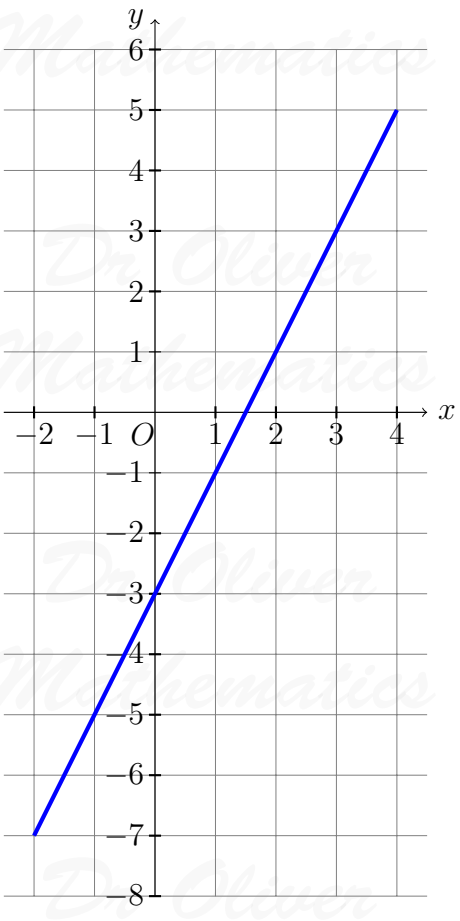
$$y = 2x - 3$$

for values of x from -2 to 4.



Solution

x	-2	-1	0	1	2	3	4
y	-7	-5	-3	-1	1	3	5



3. Hannah is planning a day trip for 195 students.

She asks a sample of 30 students where they want to go.
Each student chooses one place.

The table shows information about her results.

Place	Number of students
Theme Park	10
Theatre	5
Sports Centre	8
Seaside	7

(a) Work out how many of the 195 students you think will want to go to the Theme Park. (2)

Solution

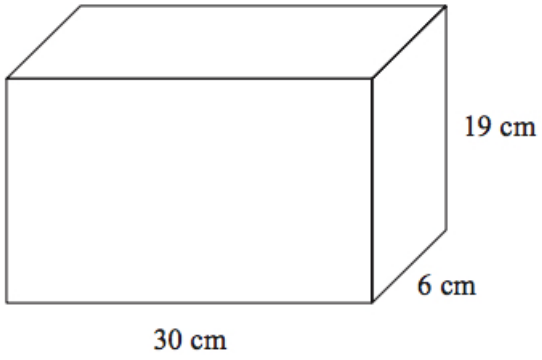
$$\frac{10}{30} \times 195 = \underline{\underline{65 \text{ students}}}.$$

(b) State any assumption you made **and** explain how this may affect your answer. (1)

Solution

E.g., the sample is representative otherwise the answer is wrong,
there is a random sample otherwise the answer is wrong,
there is no bias otherwise the answer is wrong, etc.

4. A container is in the shape of a cuboid. (4)



The container is $\frac{2}{3}$ full of water.
A cup holds 275 ml of water.

What is the greatest number of cups that can be completely filled with water from the container?

Solution

$$\begin{aligned} \text{Full container} &= 30 \times 6 \times 19 \\ &= 3\,420 \text{ cm}^3 \end{aligned}$$

and when the container is $\frac{2}{3}$ full of water

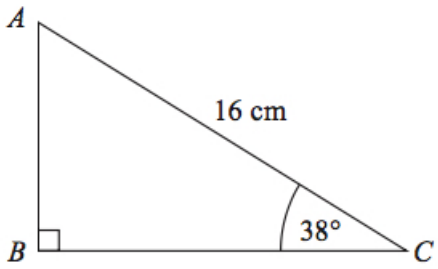
$$\frac{2}{3} \times 3\,420 = 2\,280 \text{ cm}^3.$$

Now, the number of cups that can be completely filled with water from the container is

$$\frac{2\,280}{275} = 8\tfrac{16}{55};$$

hence, he can fill 8 cups.

5. ABC is a right-angled triangle. (2)



Calculate the length of AB.
Give your answer correct to 2 decimal places.

Solution

$$\begin{aligned}\sin = \frac{\text{opp}}{\text{hyp}} &\Rightarrow \sin 38^\circ = \frac{AB}{16}\\ &\Rightarrow AB = 16\sin 38^\circ\\ &\Rightarrow AB = 9.850\,583\,605 \text{ (FCD)}\\ &\Rightarrow AB = 9.85 \text{ cm (2 dp)}.\end{aligned}$$

6. Sally used her calculator to work out the value of a number y. (2)
The answer on her calculator display began

8.3.

Complete the error interval for y:

$$\ldots\ldots\ldots \leqslant y < \ldots\ldots\ldots$$

Solution

$$\underline{\underline{8.3}} \leqslant y < \underline{\underline{8.4}}.$$

7. £360 is shared between Abby, Ben, Chloe, and Denesh. (4)

   The ratio of the amount Abby gets to the amount Ben gets is $2 : 7$.

   Chloe and Denesh each get 1.5 times the amount Abby gets.

   Work out the amount of money that Ben gets.

Solution

Now, Chloe and Denesh get

$$1.5 \times 2 = 3$$

times the amount Abby gets. Next,

$$2 + 7 + 3 + 3 = 15$$

and the amount of money that Ben gets is

$$\tfrac{7}{15} \times 360 = \underline{\underline{£168}}.$$

8. (a) Write 0.005 62 in standard form. (1)

Solution

$$0.005\,62 = \underline{\underline{5.62 \times 10^{-3}}}.$$

   (b) Write 1.452×10^3 as an ordinary number. (1)

Solution

$$1.452 \times 10^3 = \underline{\underline{1\,452}}.$$

9. The circumference of circle **B** is 90% of the circumference of circle **A**.

   (a) Find the ratio of the area of circle **A** to the area of circle **B**. (2)

Solution

$$\text{LSF}_A : \text{LSF}_B = 1 : 0.9 \Rightarrow \text{ASF}_A : \text{ASF}_B = 1^2 : 0.9^2$$
$$\Rightarrow \underline{\underline{\text{ASF}_A : \text{ASF}_B = 1 : 0.81}}.$$

Square **E** has sides of length e cm.
Square **F** has sides of length f cm.

The area of square **E** is 44% greater than the area of square **F**.

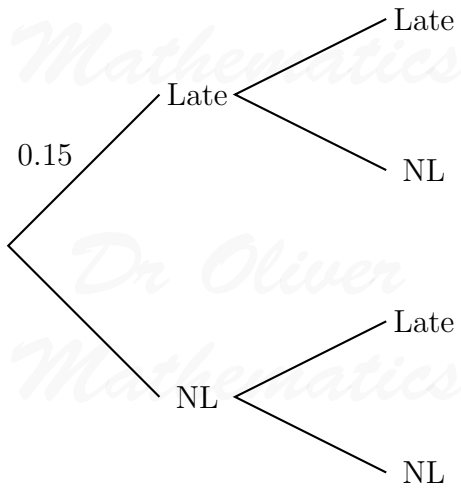
(b) Work out the ratio $e : f$. (2)

Solution

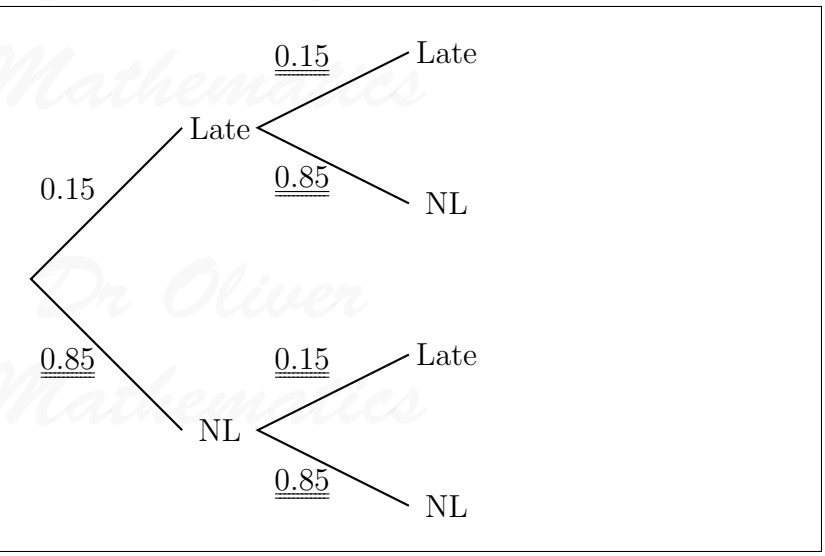
$$\text{ASF}_E : \text{ASF}_F = 1.44 : 1 \Rightarrow \text{LSF}_E : \text{LSF}_F = \sqrt{1.44} : \sqrt{1}$$
$$\Rightarrow \underline{\underline{\text{LSF}_E : \text{LSF}_F = 1.2 : 1}}.$$

10. Mary travels to work by train every day.
The probability that her train will be late on any day is 0.15.

(a) Complete the probability tree diagram for Thursday and Friday. (2)



Solution



(b) Work out the probability that her train will be late on at least one of these two days. (3)

Solution

$$\begin{aligned} \text{P(at least one of these two days)} &= 1 - \text{P(on time both days)} \\ &= 1 - (0.85 \times 0.85) \\ &= 1 - 0.722\,5 \\ &= \underline{\underline{0.277\,5}}. \end{aligned}$$

11. The grouped frequency table gives information about the times, in minutes, that 80 office workers take to get to work.

Time (t minutes)	Frequency
$0 < t \leqslant 20$	5
$20 < t \leqslant 40$	30
$40 < t \leqslant 60$	20
$60 < t \leqslant 80$	15
$80 < t \leqslant 100$	8
$100 < t \leqslant 120$	2

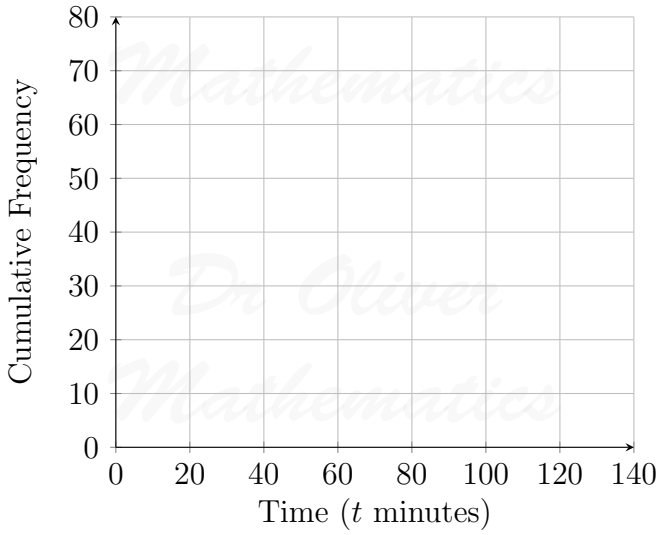
(a) Complete the cumulative frequency table. (1)

Time (t minutes)	Cumulative Frequency
$0 < t \leqslant 20$	
$0 < t \leqslant 40$	
$0 < t \leqslant 60$	
$0 < t \leqslant 80$	
$0 < t \leqslant 100$	
$0 < t \leqslant 120$	

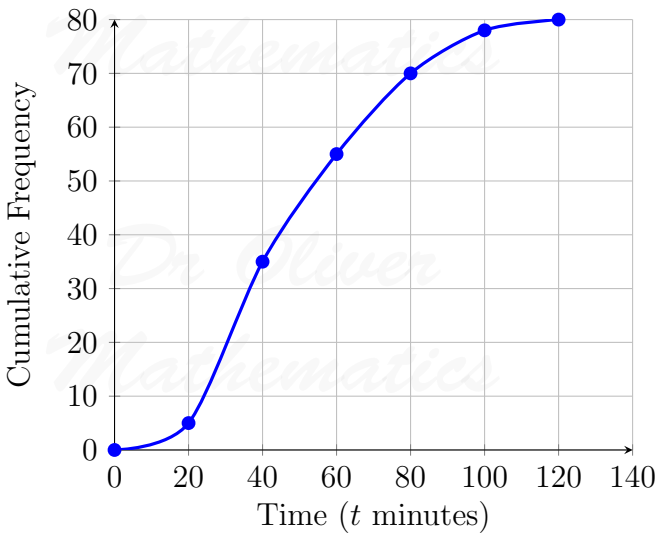
Solution

Time (t minutes)	Frequency
$0 < t \leqslant 20$	$\underline{\underline{5}}$
$0 < t \leqslant 40$	$5 + 30 = \underline{\underline{35}}$
$0 < t \leqslant 60$	$35 + 20 = \underline{\underline{55}}$
$0 < t \leqslant 80$	$55 + 15 = \underline{\underline{70}}$
$0 < t \leqslant 100$	$70 + 8 = \underline{\underline{78}}$
$0 < t \leqslant 120$	$78 + 2 = \underline{\underline{80}}$

(b) On the grid, draw the cumulative frequency graph for this information. (2)

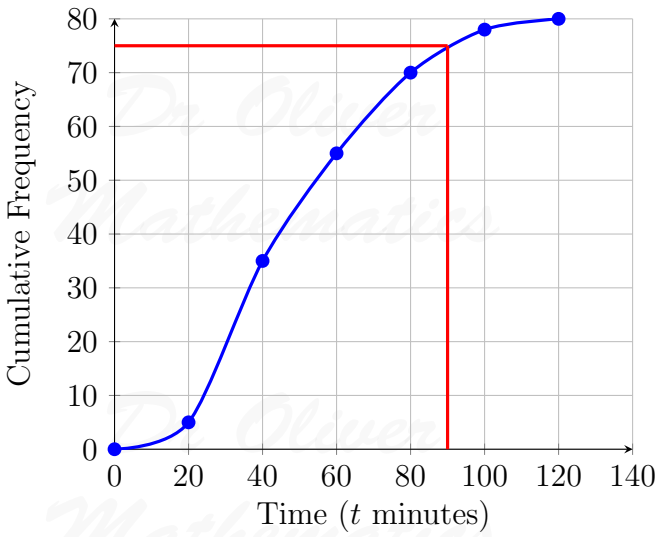


Solution



(c) Use your graph to find an estimate for the percentage of these office workers who take more than 90 minutes to get to work. (3)

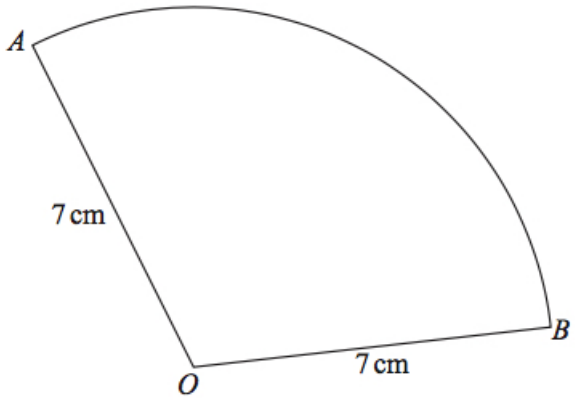
Solution



Correct read-off: approximately

$$\left(\frac{80-75}{80}\right) \times 100\% = \underline{\underline{6\tfrac{1}{4}\%}}.$$

12. OAB is a sector of a circle with centre O and radius 7 cm. (4)



The area of the sector is 40 cm^2.

Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

Solution

Let the angle be x°. Then

$$\frac{x}{360} \times \pi \times 7^2 = 40 \Rightarrow x = \frac{14\,400}{49\pi}$$

and

$$\begin{aligned} \text{perimeter} &= 7 + 7 + \left(\frac{x}{360} \times 2 \times \pi \times 7\right) \\ &= 25.428\,571\,43 \text{ (FCD)} \\ &= \underline{\underline{25.4 \text{ cm (3 sf)}}}. \end{aligned}$$

13. Show that (4)

$$6 + \left[(x+5) \div \frac{x^2 + 3x - 10}{x - 1}\right]$$

simplifies to

$$\frac{ax - b}{cx - d}$$

where a, b, c, and d are integers.

Solution

$$6 + \left[(x+5) \div \frac{x^2+3x-10}{x-1}\right] = 6 + \frac{(x+5)(x-1)}{x^2+3x-10}$$

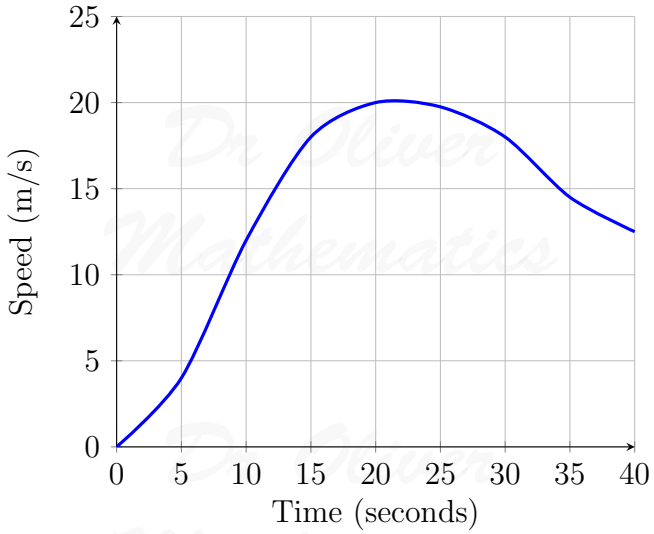
$$\left.\begin{array}{rr} \text{add to:} & +3 \\ \text{multiply to:} & -10 \end{array}\right\} -2,\ +5$$

$$\begin{aligned} &= 6 + \frac{(x+5)(x-1)}{(x+5)(x-2)} \\ &= 6 + \frac{x-1}{x-2} \\ &= \frac{6(x-2)+x-1}{x-2} \\ &= \frac{(6x-12)+x-1}{x-2} \\ &= \underline{\underline{\frac{7x-13}{x-2}}}; \end{aligned}$$

hence, $\underline{\underline{a=7}}$, $\underline{\underline{b=13}}$, $\underline{\underline{c=1}}$, and $\underline{\underline{d=2}}$.

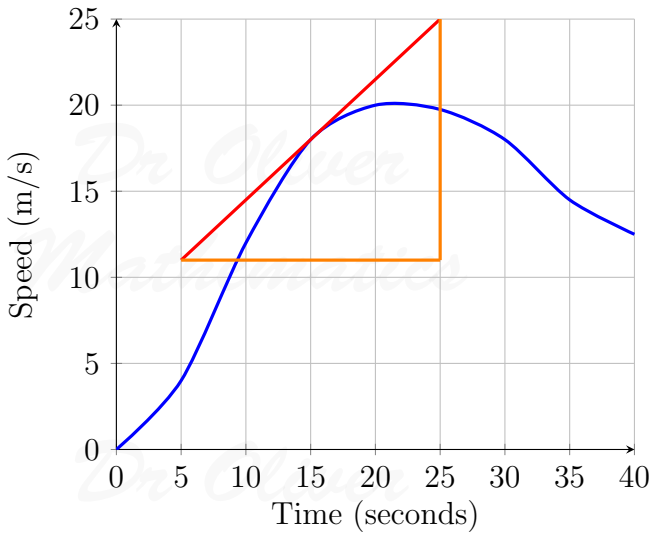
14. A car moves from rest.

The graph gives information about the speed, v metres per second, of the car t seconds after it starts to move.



(a) (i) Calculate an estimate of the gradient of the graph at $t = 15$. (3)

Solution



An attempt at finding the gradient:

$$\text{gradient} = \frac{25 - 11}{25 - 5}$$
$$= \underline{\underline{0.7}}.$$

(ii) Describe what your answer to part (i) represents. (1)

Solution

The <u>acceleration</u> of the car.

(b) Work out an estimate for the distance the car travels in the first 20 seconds of its journey. (3)
Use 4 strips of equal width.

Solution

$$\text{Distance} \approx \tfrac{1}{2} \times 5 \times [0 + 2(4 + 12 + 18) + 20]$$
$$= \underline{\underline{220 \text{ m}}}.$$

15. Make m the subject of the formula (3)

$$f = \frac{3m+4}{m-1}.$$

Solution

$$\begin{aligned} f = \frac{3m+4}{m-1} &\Rightarrow f(m-1) = 3m+4 \\ &\Rightarrow fm - f = 3m+4 \\ &\Rightarrow fm - 3m = f+4 \\ &\Rightarrow m(f-3) = f+4 \\ &\Rightarrow \underline{\underline{m = \frac{f+4}{f-3}}}. \end{aligned}$$

16. The straight line **L** has the equation $3y = 4x + 7$. (3)
The point A has coordinates $(3, -5)$.

Find an equation of the straight line that is perpendicular to **L** and passes through A.

Solution

$$3y = 4x + 7 \Rightarrow y = \tfrac{4}{3}x + \tfrac{7}{3};$$

hence, the gradient of the straight line **L** is $\frac{4}{3}$ and the straight line that is perpendicular to **L** is

$$-\frac{1}{\frac{4}{3}} = -\tfrac{3}{4}.$$

Now, an equation of the straight line is

$$\begin{aligned} y - (-5) = -\tfrac{3}{4}(x-3) &\Rightarrow 4(y+5) = -3(x-3) \\ &\Rightarrow 4y + 20 = -3x + 9 \\ &\Rightarrow \underline{\underline{3x + 4y + 11 = 0}}. \end{aligned}$$

17. There are some small cubes and some large cubes in a bag.
The cubes are red or the cubes are yellow.

The ratio of the number of small cubes to the number of large cubes is 4 : 7.

The ratio of the number of red cubes to the number of yellow cubes is 3 : 5.

(a) Explain why the least possible number of cubes in the bag is 88. (1)

Solution

3, 4, 5, and 7 are all co-prime and

$$(4 + 7) \times (3 + 5) = 11 \times 8 = 88$$

so there the least possible number of cubes in the bag is $\underline{\underline{88}}$.

All the small cubes are yellow.

(b) Work out the least possible number of large yellow cubes in the bag. (3)

Solution

$$\text{Red cubes} = \frac{3}{(3 + 5)} \times 88 = 33$$

and

$$\text{yellow cubes} = 88 - 33 = 55.$$

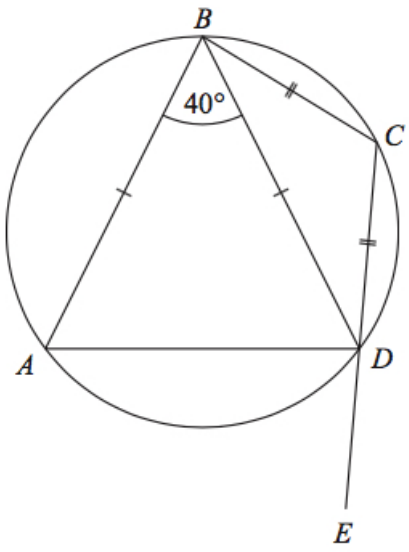
Now,

$$\text{small cubes} = \frac{4}{(4 + 7)} \times 88 = 32.$$

However, all the small cubes are yellow and the least possible number of large yellow cubes in the bag is

$$55 - 32 = \underline{\underline{23}}.$$

18. The points A, B, C, and D lie on a circle. (5)
CDE is a straight line.



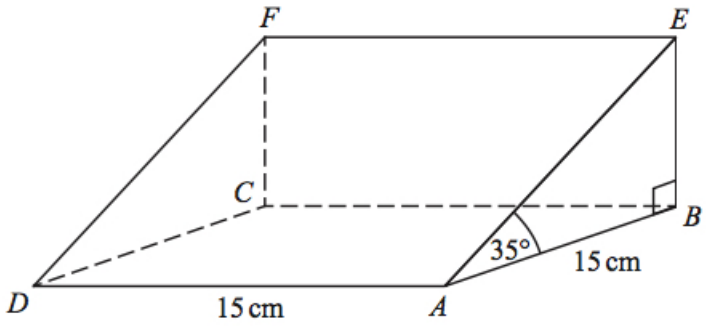
$BA = BD$.
$CB = CD$.
Angle $ABD = 40°$.

Work out the size of angle ADE.
You must give a reason for each stage of your working.

Solution

$\angle BAD = \angle BDA = \frac{1}{2}(180 - 40) = 70°$ (base angles in a triangle)
$\angle BCD = 180 - 70 = 110°$ (opposite angle in a cyclic quadrilateral)
$\angle DBC = \angle BDC = \frac{1}{2}(180 - 110) = 35°$ (base angles in a triangle)
$\angle ADE = 180 - 70 - 35 = \underline{\underline{75°}}$ (supplementary angles)

19. The diagram shows a triangular prism. (4)



The base, $ABCD$, of the prism is a square of side length 15 cm.
Angle ABE and angle CBE are right angles.
Angle $EAB = 35^\circ$.

M is the point on DA such that

$$DM : MA = 2 : 3.$$

Calculate the size of the angle between EM and the base of the prism.
Give your answer correct to 1 decimal place.

Solution

Let x° be the angle between EM and the base of the prism.

$$\begin{aligned}\tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan 35^\circ = \frac{BE}{15} \\ &\Rightarrow BE = 15 \tan 35^\circ.\end{aligned}$$

Now,

$$\begin{aligned}MA &= \frac{3}{(2+3)} \times 15 \\ &= 9 \text{ cm}\end{aligned}$$

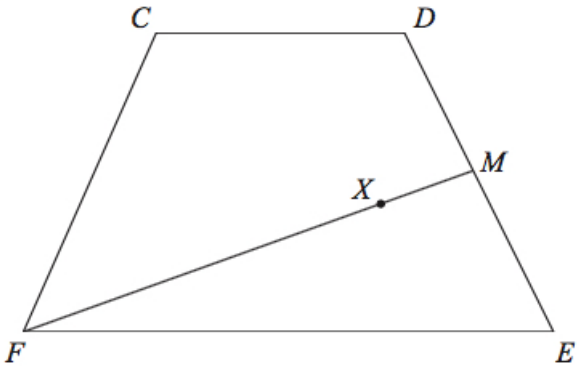
and

$$\begin{aligned}MB &= \sqrt{MA^2 + AB^2} \\ &= \sqrt{9^2 + 15^2} \\ &= 3\sqrt{34} \text{ cm}.\end{aligned}$$

Finally,

$$\begin{aligned}\tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan x^\circ = \frac{15 \tan 35^\circ}{3\sqrt{34}} \\ &\Rightarrow x^\circ = 30.981\,574\,32 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x^\circ = 31.0^\circ \text{ (1 dp)}}}.\end{aligned}$$

20. $CDEF$ is a quadrilateral.



$\overrightarrow{CD} = \mathbf{a}$.
$\overrightarrow{DE} = \mathbf{b}$.
$\overrightarrow{FC} = \mathbf{a} - \mathbf{b}$.

(a) Express $\overrightarrow{FE}$ in terms of $\mathbf{a}$ and/or $\mathbf{b}$. (2)
Give your answer in its simplest form.

Solution

$$\begin{aligned}\overrightarrow{FE} &= \overrightarrow{FC} + \overrightarrow{CD} + \overrightarrow{DE} \\ &= (\mathbf{a} - \mathbf{b}) + \mathbf{a} + \mathbf{b} \\ &= \underline{\underline{2\mathbf{a}}}.\end{aligned}$$

M is the midpoint of DE.
X is the point on FM such that

$$FX : XM = n : 1.$$

CXE is a straight line.

(b) Work out the value of n. (4)

Solution

$$\begin{aligned}\overrightarrow{FM} &= \overrightarrow{FC} + \overrightarrow{CD} + \overrightarrow{DM} \\ &= \overrightarrow{FC} + \overrightarrow{CD} + \tfrac{1}{2}\overrightarrow{DE} \\ &= (\mathbf{a} - \mathbf{b}) + \mathbf{a} + \tfrac{1}{2}\mathbf{b} \\ &= 2\mathbf{a} - \tfrac{1}{2}\mathbf{b}.\end{aligned}$$

Now,

$$\overrightarrow{FX} = \lambda(2\mathbf{a} - \tfrac{1}{2}\mathbf{b}) = 2\lambda\mathbf{a} - \tfrac{1}{2}\lambda\mathbf{b}$$

and

$$\begin{aligned}\overrightarrow{FX} &= \overrightarrow{FC} + \overrightarrow{CX} \\ &= \overrightarrow{FC} + \mu\overrightarrow{CE} \\ &= (\mathbf{a} - \mathbf{b}) + \mu(\mathbf{a} + \mathbf{b}) \\ &= (\mu + 1)\mathbf{a} + (\mu - 1)\mathbf{b}\end{aligned}$$

for some λ and μ. Next,

$$\begin{aligned}2\lambda &= \mu + 1 \quad (1) \\ -\tfrac{1}{2}\lambda &= \mu - 1 \quad (2)\end{aligned}$$

Add (1) − (2):

$$\begin{aligned}\tfrac{5}{2}\lambda = 2 &\Rightarrow \lambda = \tfrac{4}{5} \\ &\Rightarrow \mu = \tfrac{3}{5}.\end{aligned}$$

Hence,

$$\begin{aligned}FX : XM = n : 1 &\Rightarrow \tfrac{8}{5}\mathbf{a} - \tfrac{2}{5}\mathbf{b} : \tfrac{2}{5}\mathbf{a} - \tfrac{1}{10}\mathbf{b} \\ &\Rightarrow 4(\tfrac{2}{5}\mathbf{a} - \tfrac{1}{10}\mathbf{b}) : \tfrac{2}{5}\mathbf{a} - \tfrac{1}{10}\mathbf{b} \\ &\Rightarrow 4 : 1;\end{aligned}$$

hence, $\underline{\underline{n = 4}}$.