Dr Oliver Mathematics Worked Examples Lower Bounds and Upper Bounds 1

From: Edexcel 2018 November Paper 3H (Calculator)

1. A high speed train travels a distance of 487 km in 3 hours.

(5)

The distance is measured correct to the nearest kilometre.

The time is measured correct to the nearest minute.

By considering bounds, work out the average speed, in km/minute, of the train to a suitable degree of accuracy.

You must show all your working and give a reason for your answer.

Solution

487 km is measured correct to the nearest kilometre:

$$486.5 \le \text{distance} < 487.5.$$

3 hours is 180 minutes and is measured correct to the nearest minute:

$$179.5 \le \text{time} < 180.5.$$

Now,

$$\frac{\text{LB distance}}{\text{UB time}} < \text{speed} < \frac{\text{UB distance}}{\text{LB time}}$$

$$\Rightarrow \frac{486.5}{180.5} < \text{speed} < \frac{487.5}{179.5}$$

$$\Rightarrow 2.695290859 < \text{speed} < 2.715877437 \text{ (FCD)}.$$

Next, we approximate it as follows:

Number	Lower Bound	Upper Bound	Agree?
Nearest Natural Number	3	3	\checkmark
1 dp	2.7	2.7	\checkmark
2 dp	2.70	2.72	X

Hence, as the lower and upper bounds agree to 1, but not 2, decimal places,

average speed =
$$2.7 \text{ km/minute } (1 \text{ dp}).$$