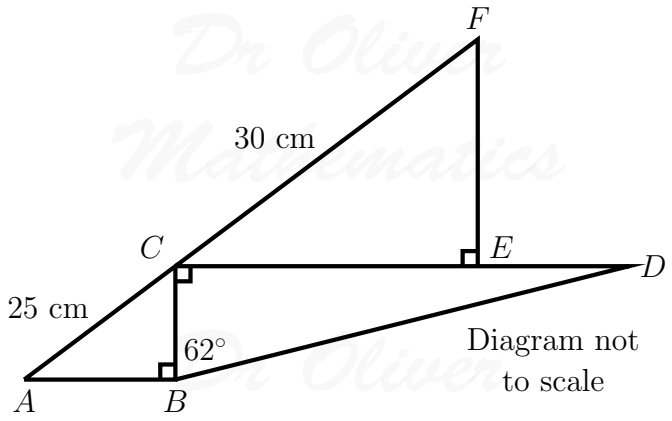
Dr Oliver Mathematics
Trigonometry: Part 1

1. We have three right-angled triangles – triangles ABC, BCD, and CDE – arranged as follows.

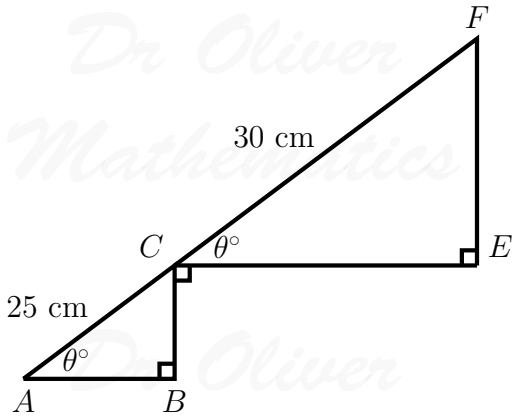


- ACF is a straight line.
- $AC = 25$ cm and $CF = 30$ cm.
- $\angle CBD = 62°$.
- $3CD = 4CE$.

Find the area of $\triangle CEF$, to 3 significant figures.

Solution

Now, AB is parallel to CED (why?) which means that $\triangle ABC$ and $\triangle CEF$ are similar (why?). Let us call $\angle BAC = \angle ECF = \theta°$.



So,

$$3CD = 4CE \Rightarrow CD = \tfrac{4}{3}CE \quad (1)$$

and

$$\begin{aligned}\cos = \frac{\text{adj}}{\text{hyp}} &\Rightarrow \cos\theta^\circ = \frac{CE}{30} \\ &\Rightarrow CE = 30\cos\theta^\circ \quad (2).\end{aligned}$$

Next,

$$\begin{aligned}\sin = \frac{\text{opp}}{\text{hyp}} &\Rightarrow \sin\theta^\circ = \frac{BC}{25} \\ &\Rightarrow BC = 25\sin\theta^\circ \quad (3).\end{aligned}$$

Now,

$$\begin{aligned}\tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan 62^\circ = \frac{CE}{BC} \\ &\Rightarrow BC = \frac{CE}{\tan 62^\circ} \\ &\Rightarrow BC = \frac{\frac{4}{3}CE}{\tan 62^\circ} \text{ (by (1))} \\ &\Rightarrow BC = \frac{4CE}{3\tan 62^\circ} \\ &\Rightarrow BC = \frac{4 \times 30\cos\theta^\circ}{3\tan 62^\circ} \text{ (by (2))} \\ &\Rightarrow BC = \frac{40\cos\theta^\circ}{\tan 62^\circ} \quad (4).\end{aligned}$$

So,

$$\begin{aligned}(3) = (4) &\Rightarrow 25\sin\theta^\circ = \frac{40\cos\theta^\circ}{\tan 62^\circ} \\ &\Rightarrow \frac{\sin\theta^\circ}{\cos\theta^\circ} = \frac{40}{25\tan 62^\circ} \\ &\Rightarrow \tan\theta^\circ = \frac{8}{5\tan 62^\circ} \\ &\Rightarrow \theta = 40.388\,979\,14 \text{ (FCD)}.\end{aligned}$$

Finally, using area $= \frac{1}{2}ab\sin C$, we get

$$\begin{aligned}\text{area of } \triangle CEF &= \tfrac{1}{2} \times CE \times CF \times \sin ECF \\ &= \tfrac{1}{2} \times 30\cos 40.388\ldots^\circ \times 30 \times \sin 40.388\ldots^\circ \\ &= 222.091\,804\,3 \text{ (FCD)} \\ &= \underline{\underline{222 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$