Dr Oliver Mathematics
Mathematics
Integration Part 1
Past Examination Questions

This booklet consists of 44 questions across a variety of examination topics.
The total number of marks available is 190.

1. Find $\displaystyle\int \left(1 + 3\sqrt{x} - \frac{1}{x^2}\right)\,\mathrm{d}x$. (4)

2. The gradient of the curve C is given by (5)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (3x-1)^2.$$

   The point $P(1, 4)$ lies on C. Find an equation for the curve C in the form $y = \mathrm{f}(x)$.

3. Find $\displaystyle\int \left(6x - \frac{4}{x^2}\right)\,\mathrm{d}x$. (3)

4. (a) Show that $\dfrac{(3-\sqrt{x})^2}{\sqrt{x}}$ can be written is $9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}$. (2)

   Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(3-\sqrt{x})^2}{\sqrt{x}}$, $x > 0$, and that $y = \frac{2}{3}$ at $x = 1$,

   (b) find y in terms of x. (6)

5. Find $\displaystyle\int \left(2x^2 - \frac{6}{x^3}\right)\,\mathrm{d}x$. (3)

6. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(1, 6)$. Given that (7)

$$\mathrm{f}'(x) = 3 + \frac{5x^2 + 2}{x^{\frac{1}{2}}},\ x > 0,$$

   find $\mathrm{f}(x)$ and simplify your answer.

7. Find $\displaystyle\int \left(6x^2 + 2 + x^{-\frac{1}{2}}\right)\,\mathrm{d}x$. (3)

8. The curve C with equation $y = \mathrm{f}(x)$, $x \neq 0$, passes through the point $(3, 7\frac{1}{2})$. Given that $\mathrm{f}'(x) = 2x + \dfrac{3}{x^2}$, find $\mathrm{f}(x)$. (5)

9. (a) Show that $(4 + 3\sqrt{x})^2$ can be written as $16 + k\sqrt{x} + 9x$, where k is a constant to be found. (2)

   (b) Find $\int (4 + 3\sqrt{x})^2 \, \mathrm{d}x$. (3)

10. The curve C with equation $y = \mathrm{f}(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C. Given that (5)

$$\mathrm{f}'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

find $\mathrm{f}(x)$.

11. Find $\int \left(3x^2 + 4\sqrt{x}\right) \mathrm{d}x$, $x > 0$. (3)

12. The curve C with equation $y = \mathrm{f}(x)$ passes through the point $(5, 65)$. Given that $\mathrm{f}'(x) = 6x^2 - 10x - 12$, use integration to find $\mathrm{f}(x)$. (4)

13. Find $\int \left(3x^2 + 4x^5 - 7\right) \mathrm{d}x$.

14. The curve C with equation $y = \mathrm{f}(x)$, $x > 0$, and $\mathrm{f}'(x) = 4x - 6\sqrt{x} + \dfrac{8}{x^2}$. Given that the point $P(4, 1)$ lies on C, find $\mathrm{f}(x)$ and simplify your answer. (4)

15. Find $\int \left(2 + 5x^2\right) \mathrm{d}x$.

16. The gradient of a curve C is given by $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(x^2 + 3)^2}{x^2}$, $x \neq 0$.

   (a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 6 + 9x^{-2}$. (2)

   The point $(3, 20)$ lies on C.

   (b) Find an equation for curve C in the form $y = \mathrm{f}(x)$. (6)

17. Find $\int \left(12x^5 - 8x^3 + 3\right) \mathrm{d}x$, giving each term in its simplest form.

18. The curve has equation $y = \mathrm{f}(x)$ and passes through the point $(4, 22)$. Given that

$$\mathrm{f}'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find $\mathrm{f}(x)$, giving each term in its simplest form.

19. Given that $y = 2x^3 + \dfrac{3}{x^2}$, $x \neq 0$, find $\int y \, \mathrm{d}x$, simplifying each term. (3)

20. (7)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 5x^{-\frac{1}{2}} + x\sqrt{x}.$$

Given that $y = 35$ at $x = 4$, find y in terms of x, given each term in its simplest form.

21. Find (4)

$$\int \left(8x^3 + 6x^{\frac{1}{2}} - 5\right) \mathrm{d}x,$$

given each term in its simplest form.

22. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where (5)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x - \frac{5}{\sqrt{x}} - 2.$$

Given that the point $P(4, 5)$ lies on C, find $\mathrm{f}(x)$.

23. Find (5)

$$\int \left(12x^5 - 3x^2 + 4x^{\frac{1}{3}}\right) \mathrm{d}x,$$

given each term in its simplest form.

24. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(-1, 0)$. Given that (5)

$$\mathrm{f}'(x) = 12x^2 - 8x + 1,$$

find $\mathrm{f}(x)$.

25. Given that $y = 2x^5 + 7 + \dfrac{1}{x^3}$, $x \neq 0$, find, in its simplest form, $\displaystyle\int y \, \mathrm{d}x$. (5)

26. Given that $\dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$,

(a) write down the value of p and write down the value of q. (2)

Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$, and that $y = 90$ when $x = 4$,

(b) find y in terms of x, simplifying the coefficient of each terms. (5)

27. Given that $y = x^4 + 6x^{\frac{1}{2}}$, find, in its simplest form, $\displaystyle\int y \, \mathrm{d}x$. (3)

28. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(2, 10)$. Given that (5)

$$\mathrm{f}'(x) = 3x^2 - 3x + 5,$$

find $\mathrm{f}(1)$.

29. Find (4)

$$\int \left(6x^2 + \frac{2}{x^2} + 5\right) \mathrm{d}x,$$

giving each term in its simplest form.

30. The point $P(4, -1)$ lies on the curve C with equation $y = \mathrm{f}(x)$, $x > 0$, and (4)

$$\mathrm{f}'(x) = \tfrac{1}{2}x - \frac{6}{\sqrt{x}} + 3.$$

Find $\mathrm{f}(x)$.

31. (6)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -x^3 + \frac{4x - 5}{2x^3},\ x \neq 0.$$

Given that $y = 7$ at $x = 1$, find y in terms of x, giving each term in its simplest form.

32. Find (4)

$$\int \left(10x^4 - 4x - \frac{3}{\sqrt{x}}\right) \mathrm{d}x,$$

giving each term in its simplest form.

33.

$$\mathrm{f}'(x) = \frac{(3 - x^2)^2}{x^2},\ x \neq 0.$$

(a) Show that (3)

$$\mathrm{f}'(x) = 9x^{-2} + A + Bx^2,$$

where A and B are constants to be found.

Given that the point $(-3, 10)$ lies on the curve with equation $y = \mathrm{f}(x)$,

(b) find $\mathrm{f}(x)$. (5)

34. Find (4)

$$\int \left(3x^2 - \frac{4}{x^2}\right) \mathrm{d}x,$$

giving each term in its simplest form.

35. The curve with equation $y = \mathrm{f}(x)$ passes through the point $P(9, 0)$. Given that (6)

$$\mathrm{f}'(x) = \frac{x + 9}{\sqrt{x}},\ x > 0,$$

find $\mathrm{f}(x)$.

36. Find (3)

$$\int \left(8x^3 + 4\right)\, \mathrm{d}x,$$

giving each term in its simplest form.

37. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(4, 25)$. Given that (5)

$$\mathrm{f}'(x) = \tfrac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1,\ x > 0,$$

find $\mathrm{f}(x)$, simplifying each term.

38. Given that $y = 2x^5 + \dfrac{6}{\sqrt{x}}$, $x > 0$, find, in their simplest form, $\displaystyle\int y\, \mathrm{d}x$. (3)

39. (6)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^{-\frac{1}{2}} + x\sqrt{x},\ x > 0.$$

Given that $y = 37$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

40. Given that $y = 4x^3 - \dfrac{5}{x^2}$, $x > 0$, find, in their simplest form, $\displaystyle\int y\, \mathrm{d}x$. (3)

41. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(4, 9)$. Given that (5)

$$\mathrm{f}'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2,\ x > 0,$$

find $\mathrm{f}(x)$, simplifying each term.

42. Find (4)

$$\int \left(2x^4 - \frac{4}{\sqrt{x}} + 3\right) \mathrm{d}x,$$

giving each term in its simplest form.

43. Find (4)

$$\int \left(2x^5 - \frac{1}{4x^3} - 5\right) \mathrm{d}x,$$

giving each term in its simplest form.

44. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where (5)

$$\mathrm{f}'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}.$$

Given that the point $P(4, -8)$ lies on C, find $\mathrm{f}(x)$, giving each term in its simplest form.