Dr Oliver Mathematics
Vectors: Part 1

1. The diagram shows triangle OCF.

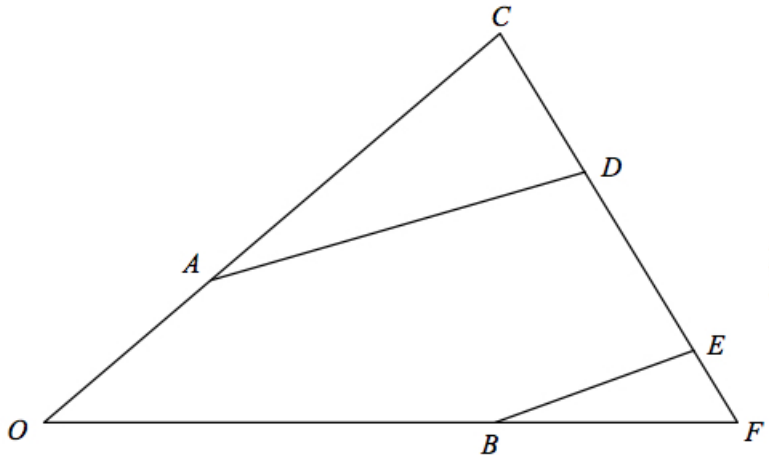

Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = 3\mathbf{a}$, $\overrightarrow{OB} = 2\mathbf{b}$, and $\overrightarrow{OF} = 3\mathbf{b}$.

(a) Express $\overrightarrow{CF}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (1)

Solution

$$\begin{aligned}\overrightarrow{CF} &= \overrightarrow{CO} + \overrightarrow{OF} \\ &= -\overrightarrow{OC} + \overrightarrow{OF} \\ &= \underline{\underline{-3\mathbf{a} + 3\mathbf{b}}}.\end{aligned}$$

Given that $\overrightarrow{CD} = \frac{1}{3}\overrightarrow{CF}$ and that $\overrightarrow{CE} = \frac{5}{6}\overrightarrow{CF}$,

(b) use vectors to prove that AD and BE are parallel. (5)

Solution

$$\begin{aligned}\overrightarrow{AD} &= \overrightarrow{AC} + \overrightarrow{CD} \\ &= \tfrac{2}{3}\overrightarrow{OC} + \tfrac{1}{3}\overrightarrow{CF} \\ &= \tfrac{2}{3}(3\mathbf{a}) + \tfrac{1}{3}(-3\mathbf{a} + 3\mathbf{b}) \\ &= 2\mathbf{a} + (-\mathbf{a} + \mathbf{b}) \\ &= \mathbf{a} + \mathbf{b}\end{aligned}$$

and

$$\begin{aligned}
\overrightarrow{BE} &= \overrightarrow{BF} + \overrightarrow{FE} \\
&= \tfrac{1}{3}\overrightarrow{OF} + \tfrac{1}{6}\overrightarrow{FC} \\
&= \tfrac{1}{3}\overrightarrow{OF} - \tfrac{1}{6}\overrightarrow{CF} \\
&= \tfrac{1}{3}(3\mathbf{b}) - \tfrac{1}{6}(-3\mathbf{a} + 3\mathbf{b}) \\
&= \mathbf{b} + \tfrac{1}{2}\mathbf{a} - \tfrac{1}{2}\mathbf{b} \\
&= \tfrac{1}{2}\mathbf{a} + \tfrac{1}{2}\mathbf{b} \\
&= \tfrac{1}{2}(\mathbf{a} + \mathbf{b}) \\
&= \tfrac{1}{2}\overrightarrow{AD};
\end{aligned}$$

hence, AD and BE are parallel.