Dr Oliver Mathematics

A Polar Curve

April 3, 2018

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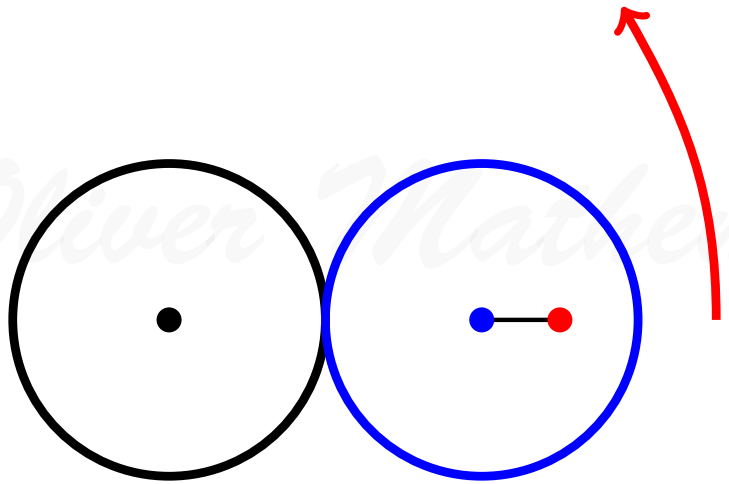
Generating a curve

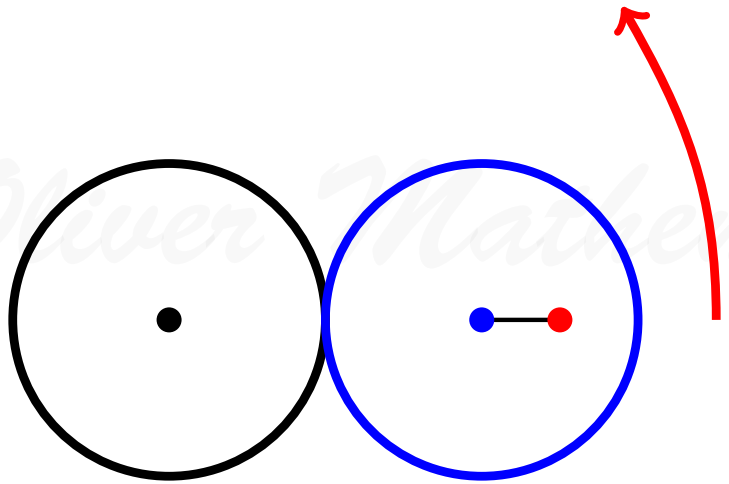
We will roll a blue circle around the outside of a black circle that has the same radius. As it does so the red dot, which is halfway between the centre and the circumference, will rotate around the centre of the blue circle and it will trace out a path.



For every time the blue circle goes around the outside of the black circle how many times will the red do go around the centre of the blue circle?

Generating a curve

We will roll a blue circle around the outside of a black circle that has the same radius. As it does so the red dot, which is halfway between the centre and the circumference, will rotate around the centre of the blue circle and it will trace out a path.



For every time the blue circle goes around the outside of the black circle how many times will the red do go around the centre of the blue circle? It will make two revolutions.

Generating a curve

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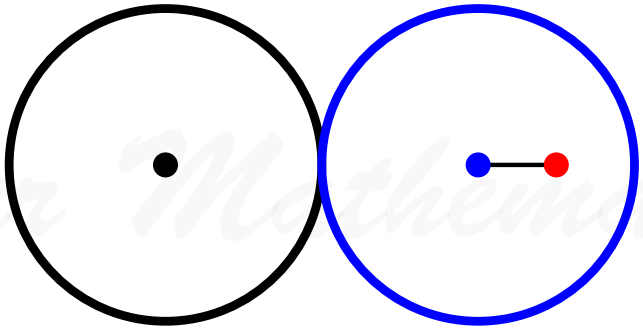

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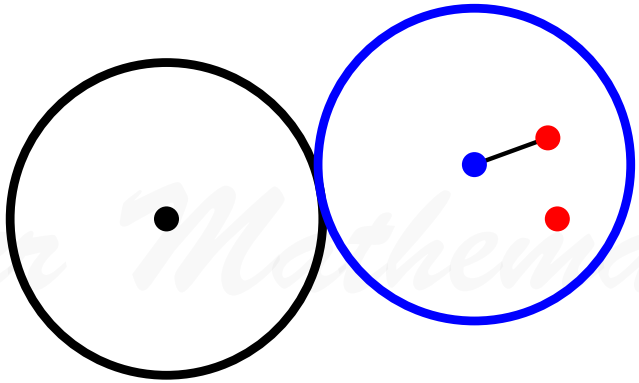
Generating a curve



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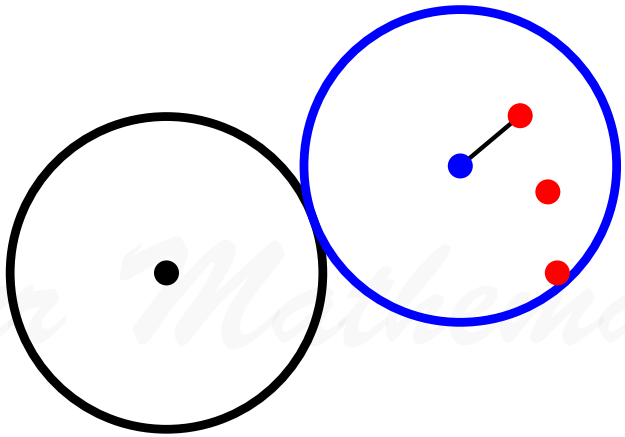

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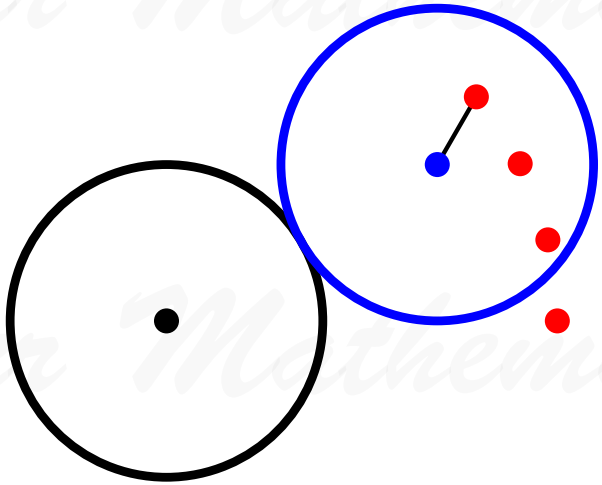

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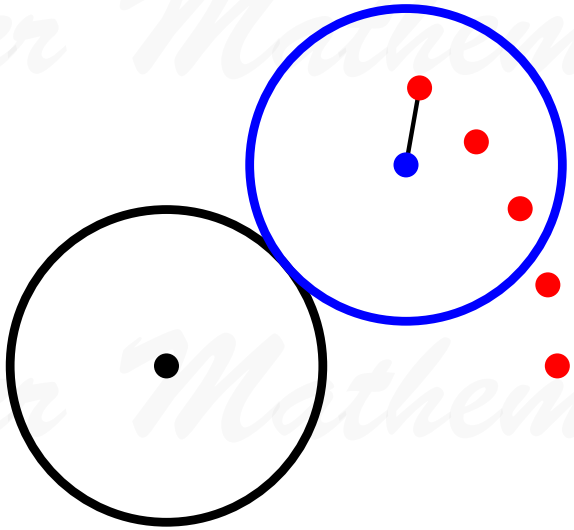

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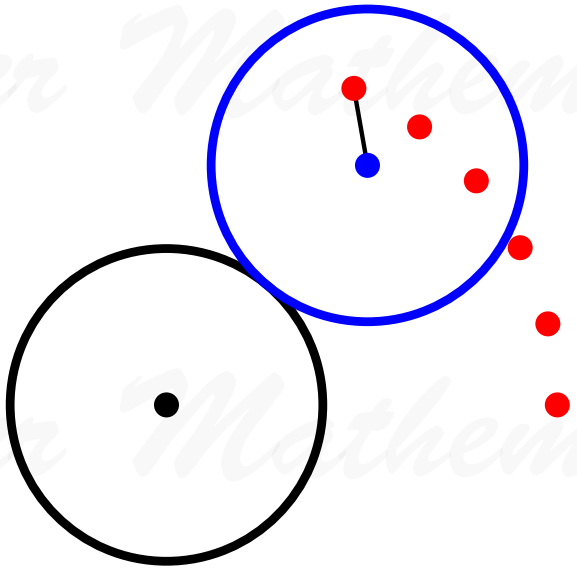

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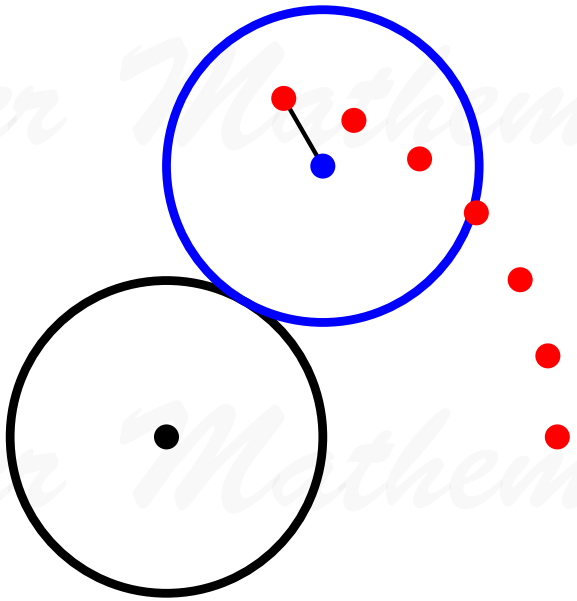

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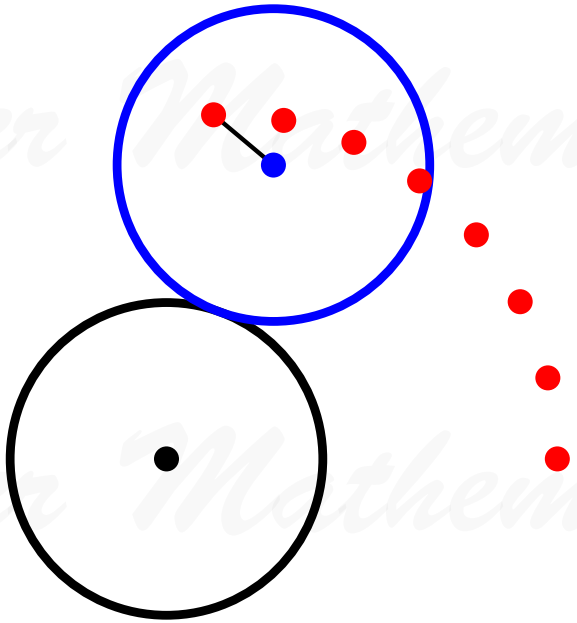

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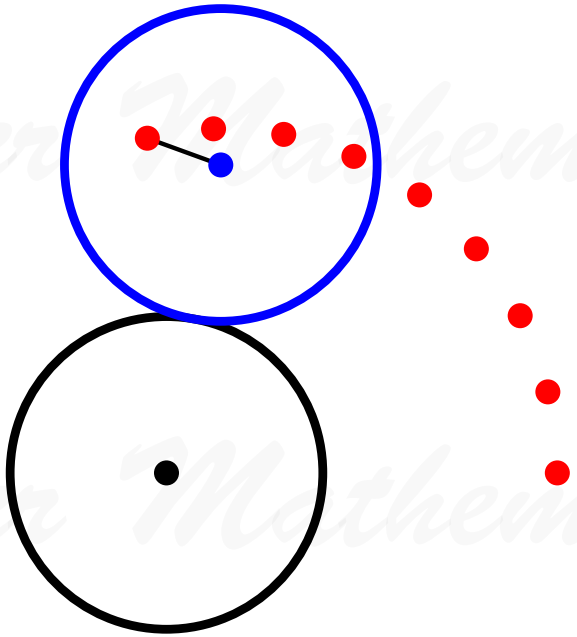

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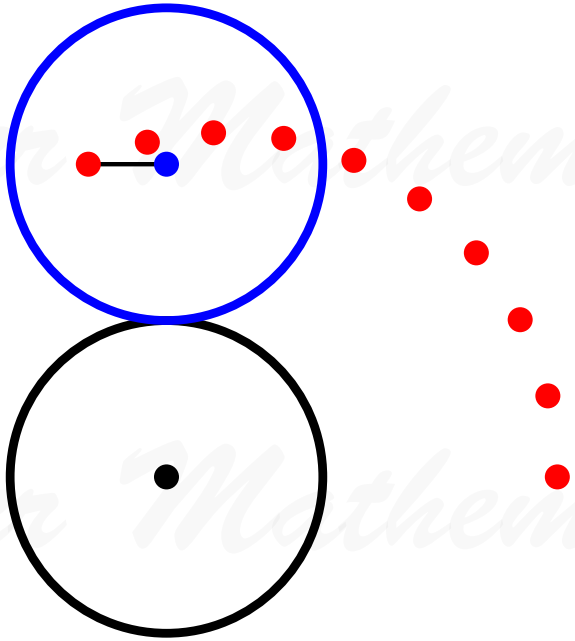

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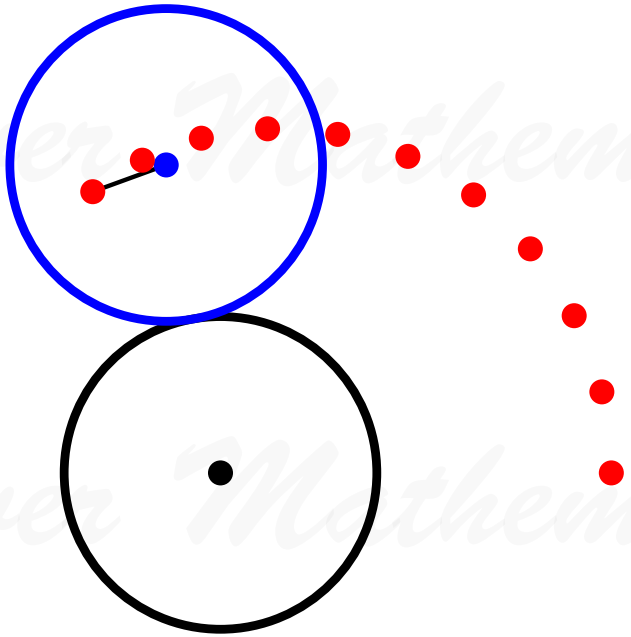

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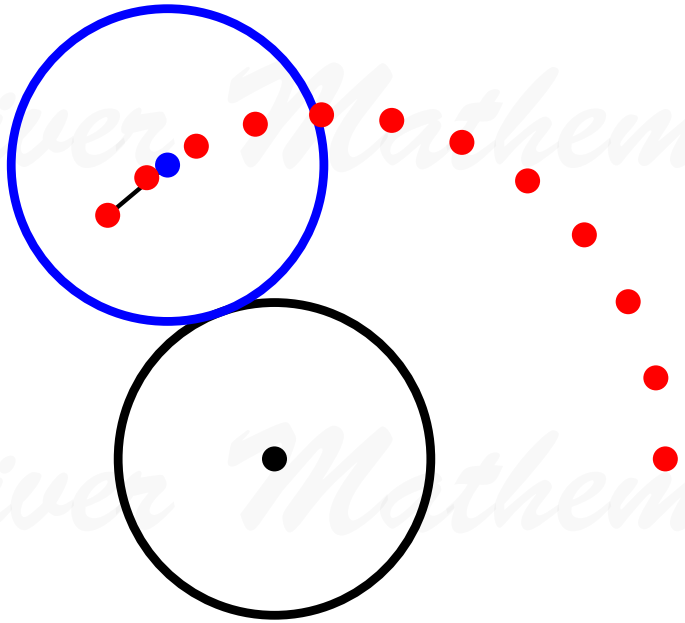

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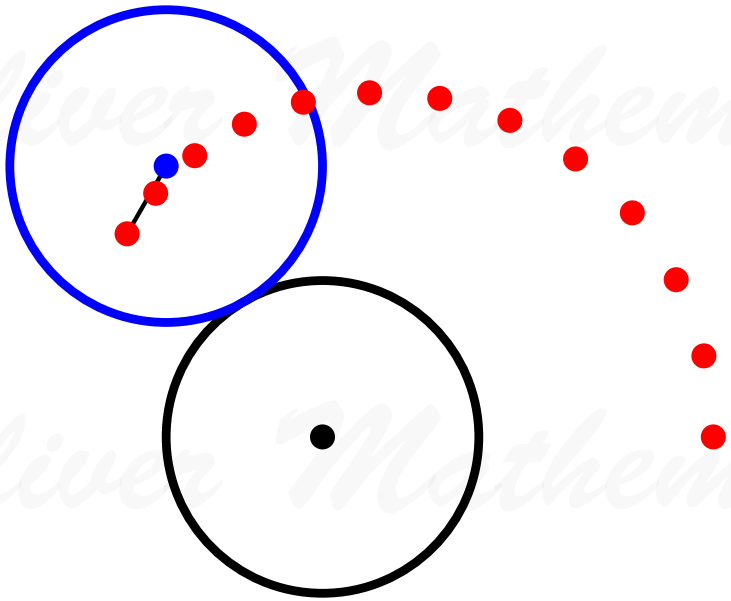

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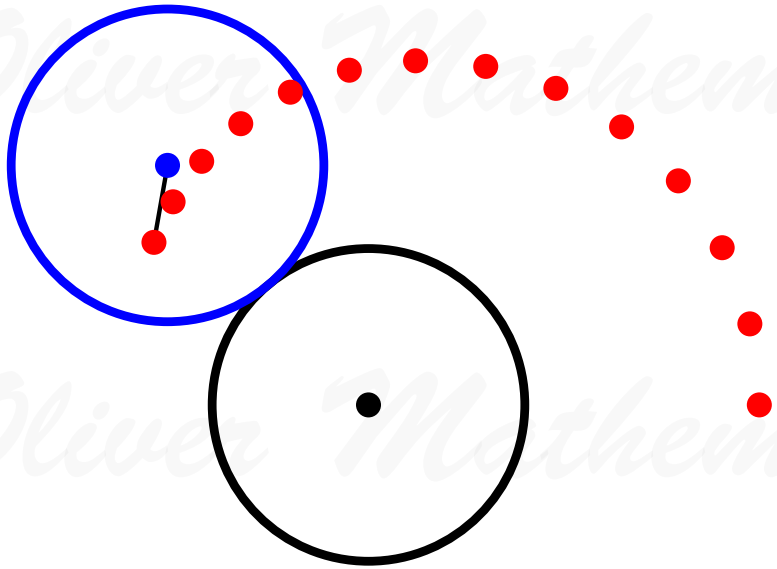

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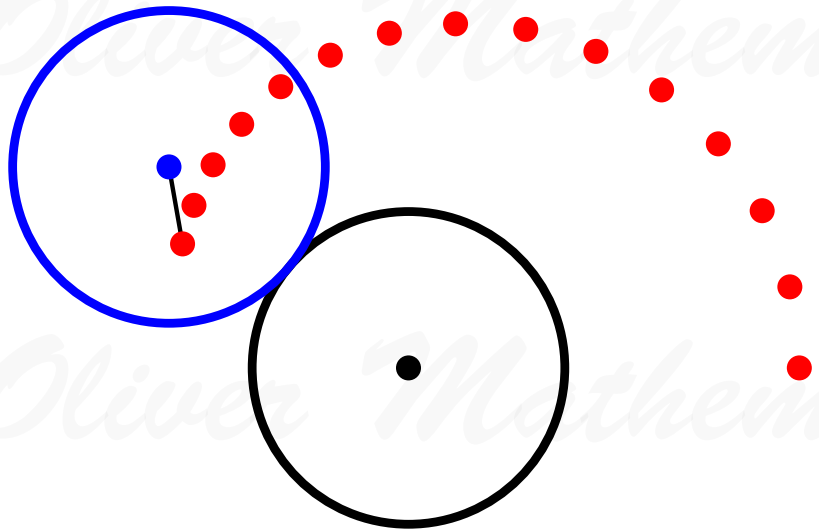

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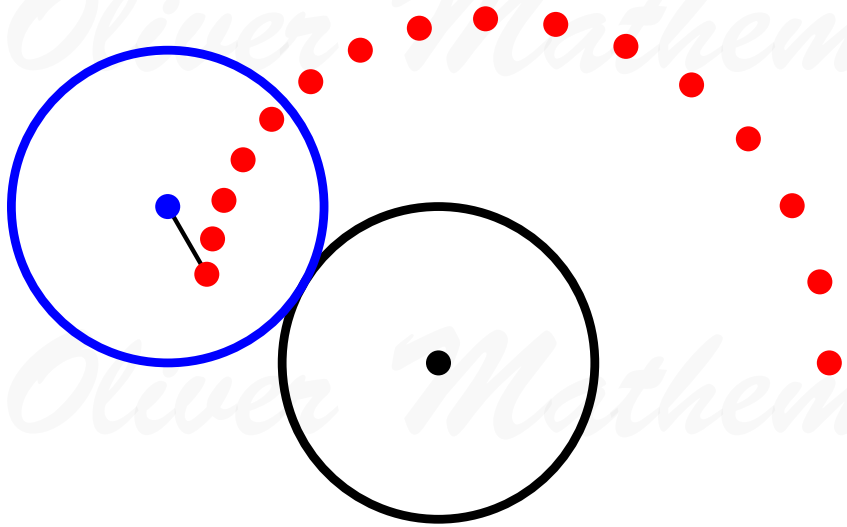

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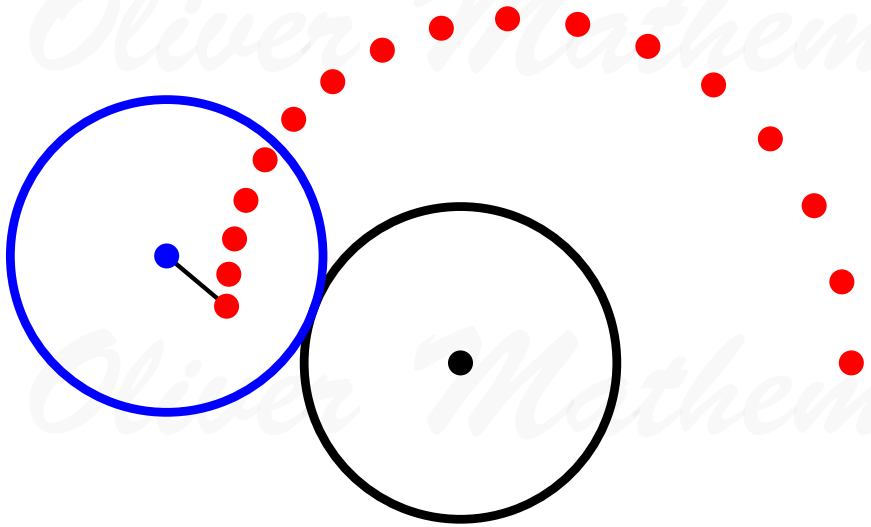

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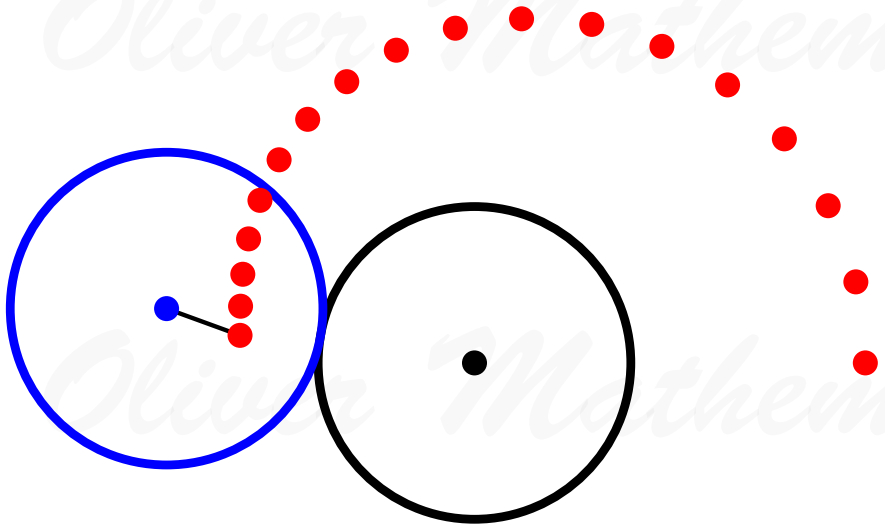

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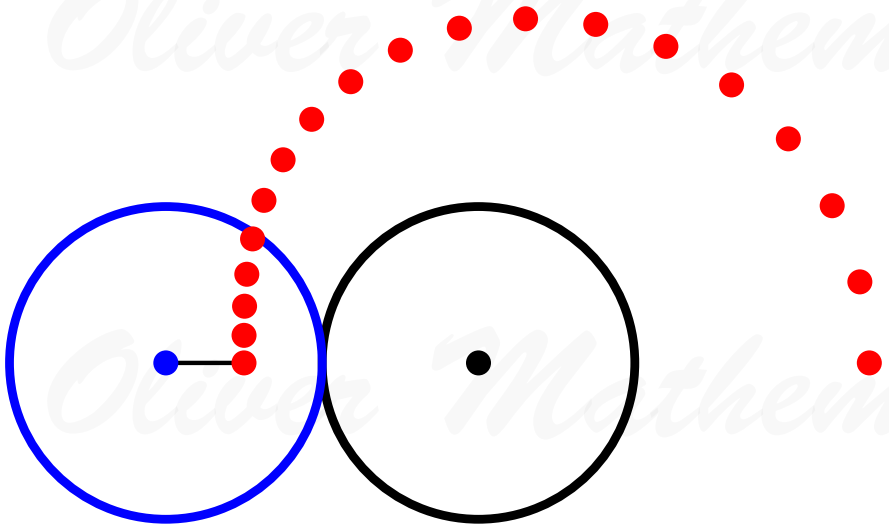

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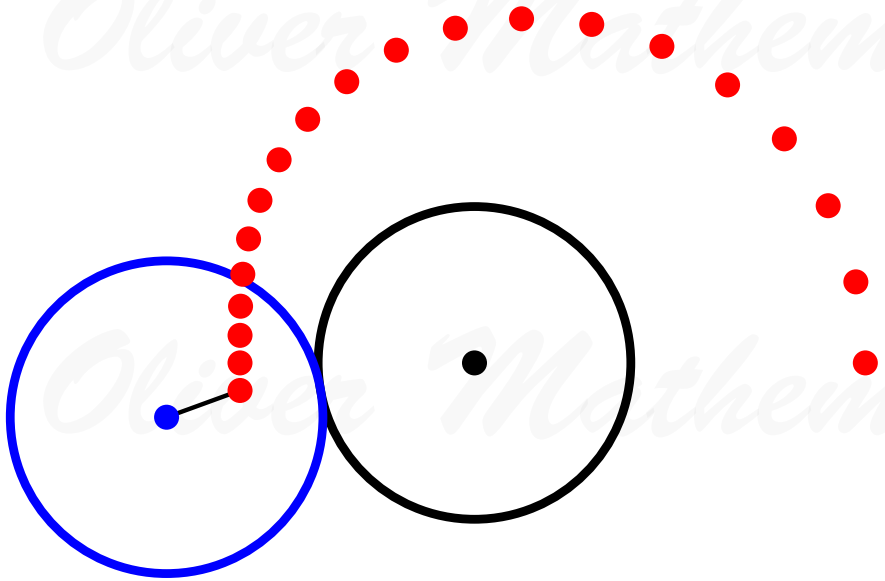

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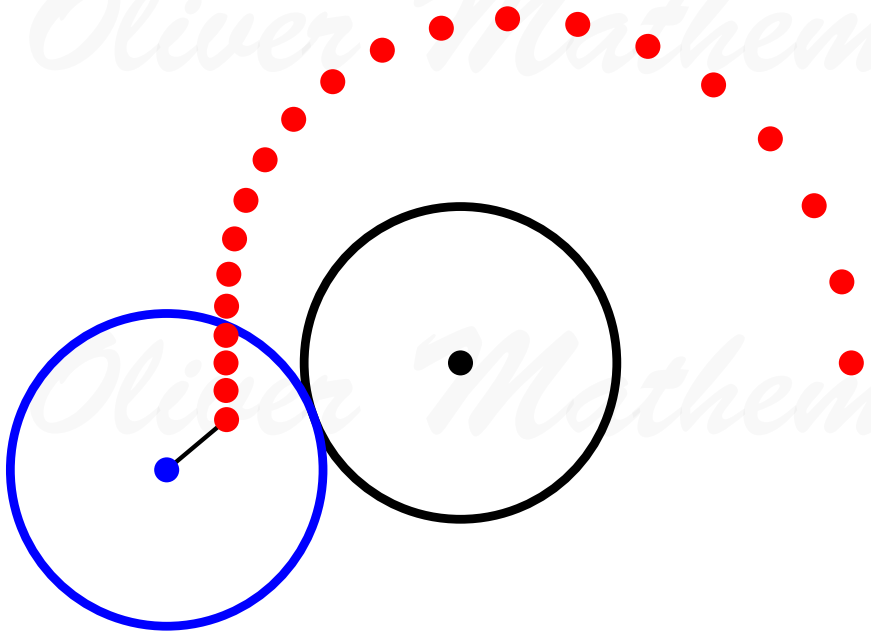

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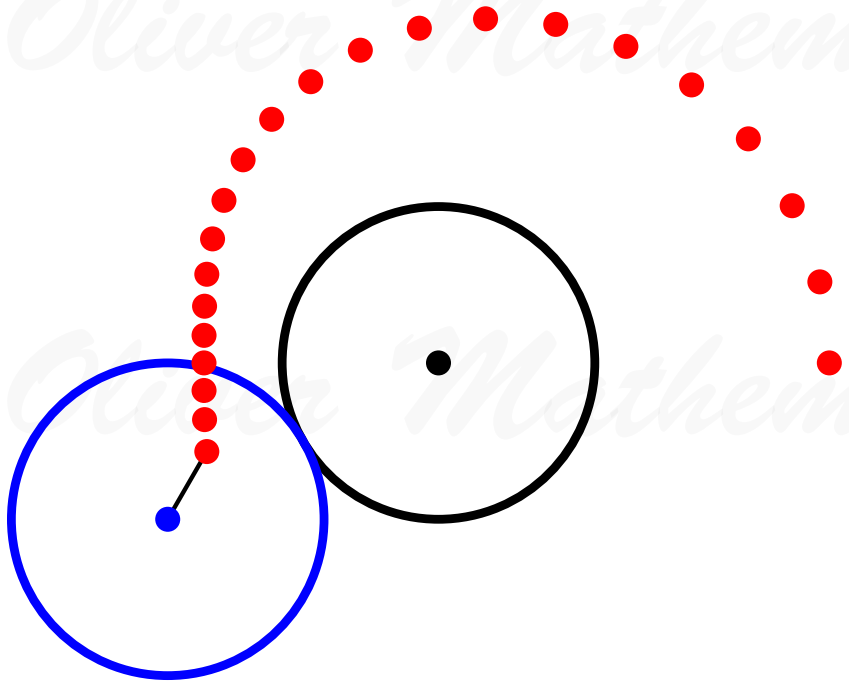

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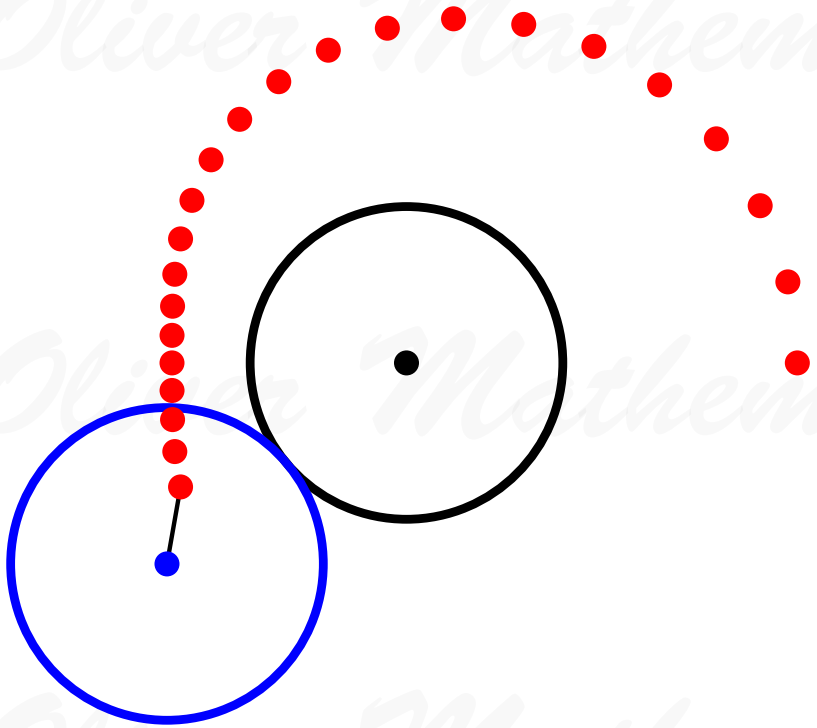

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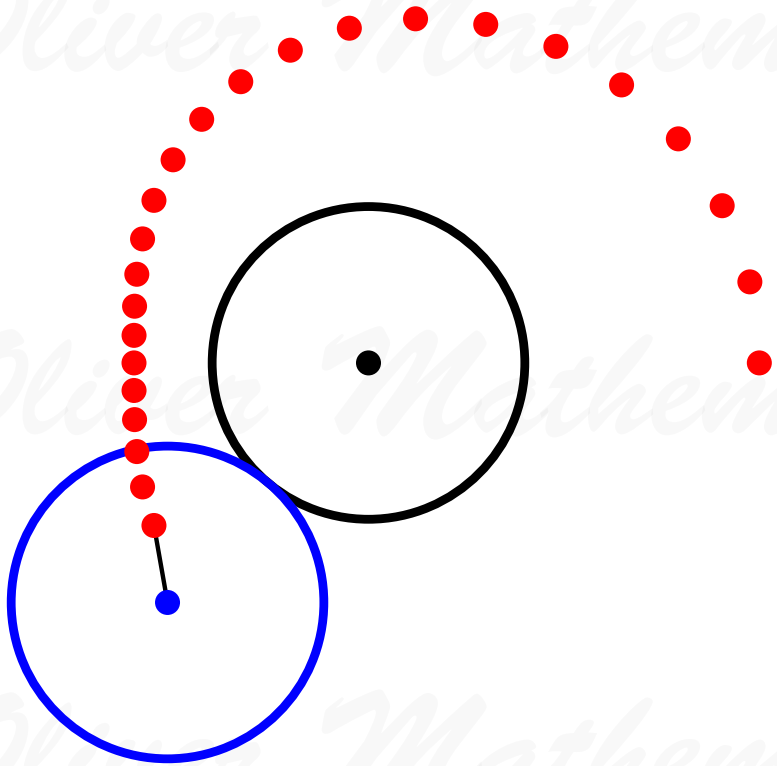

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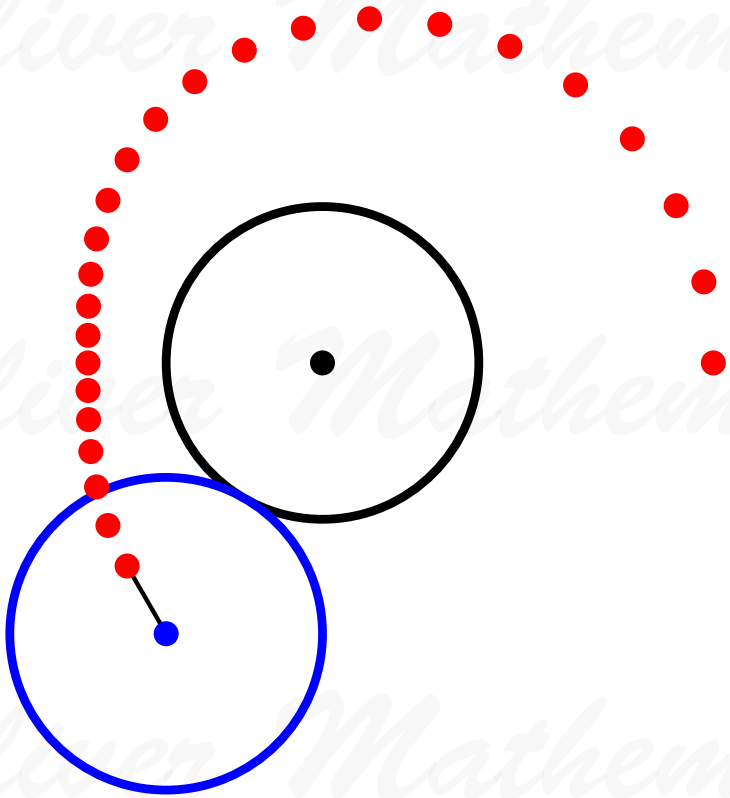

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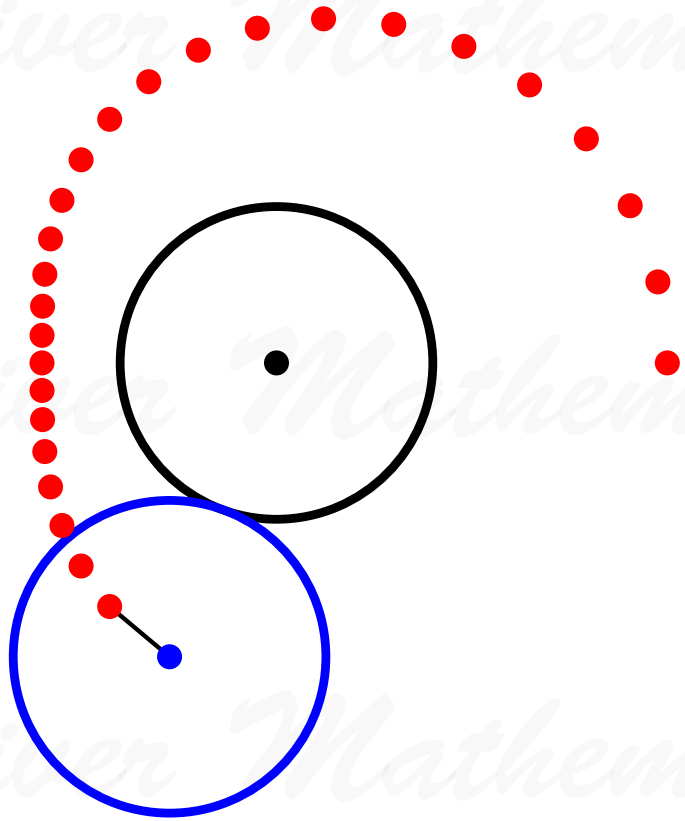

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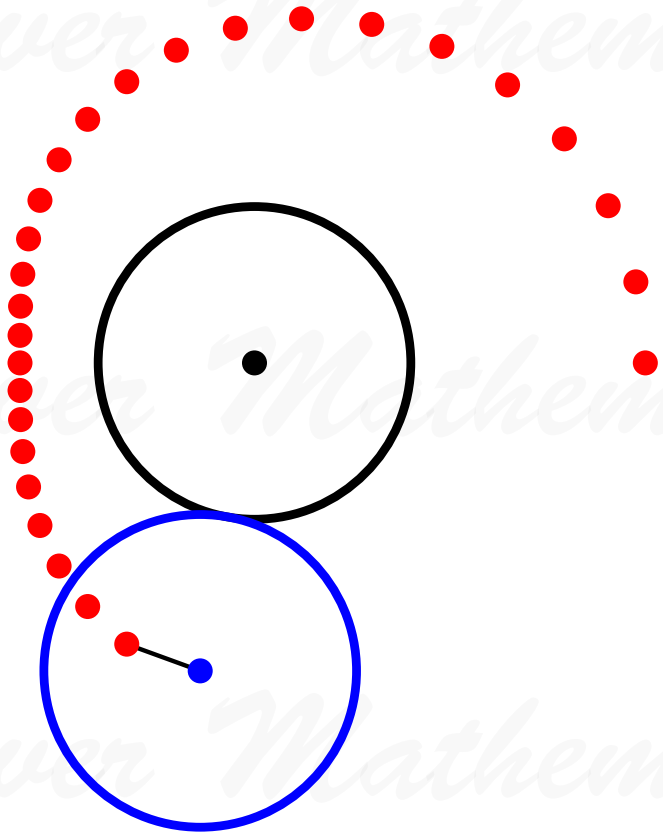

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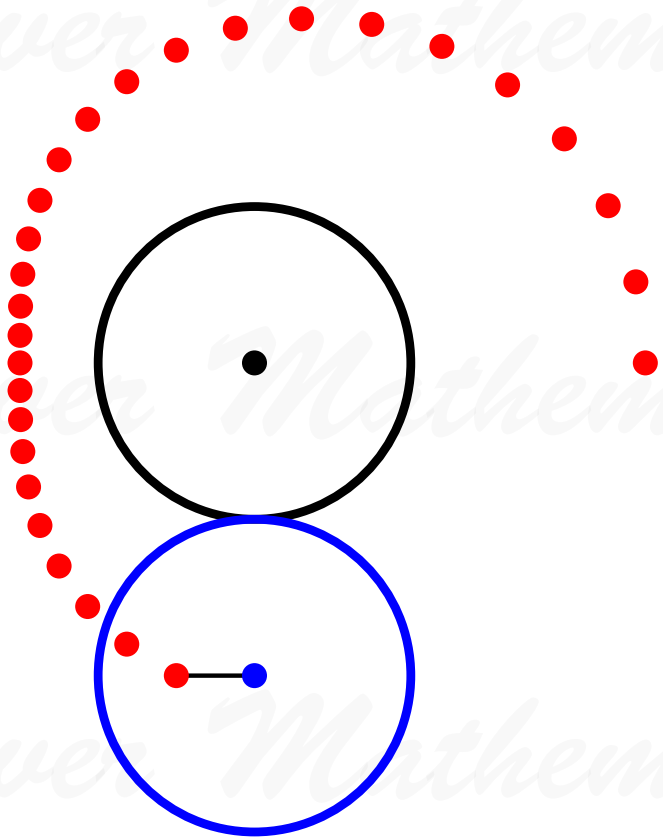

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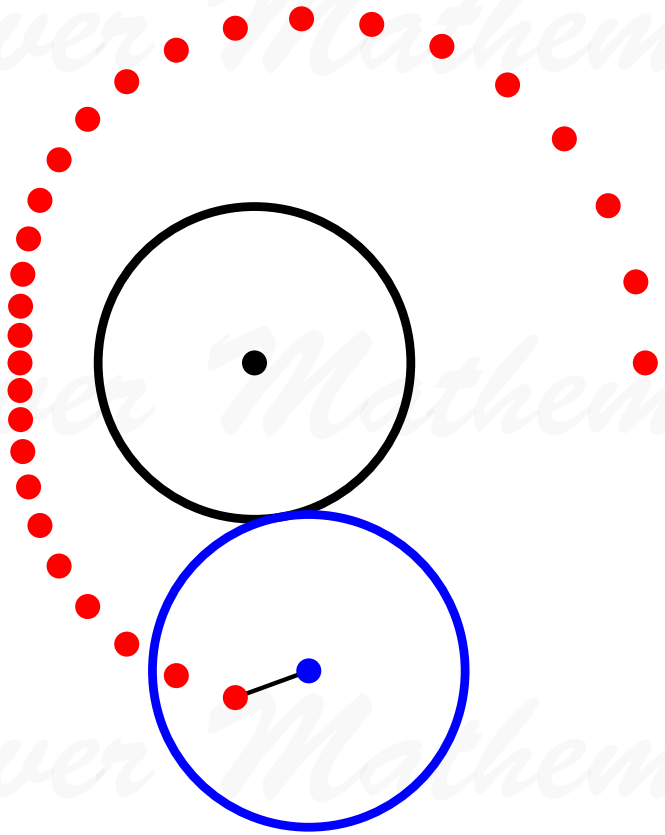

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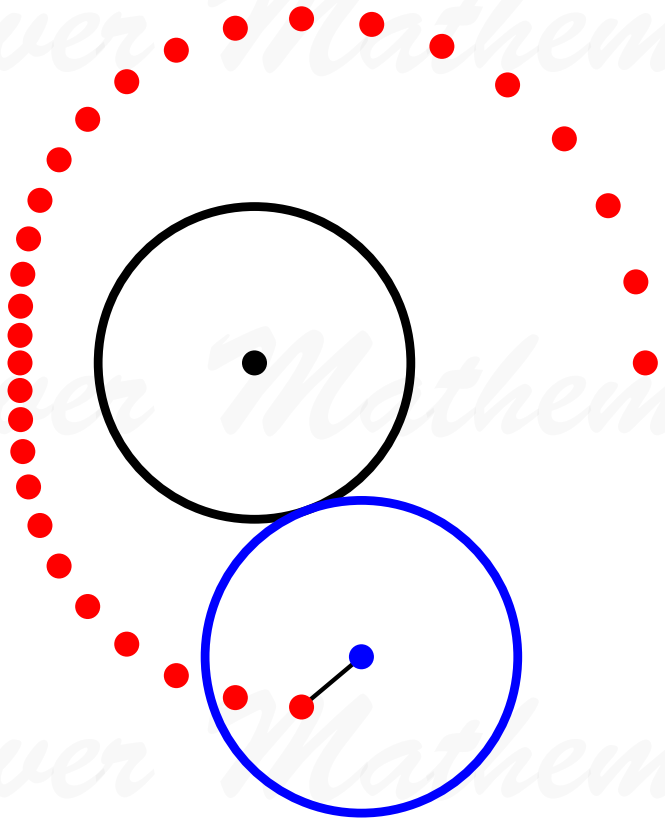

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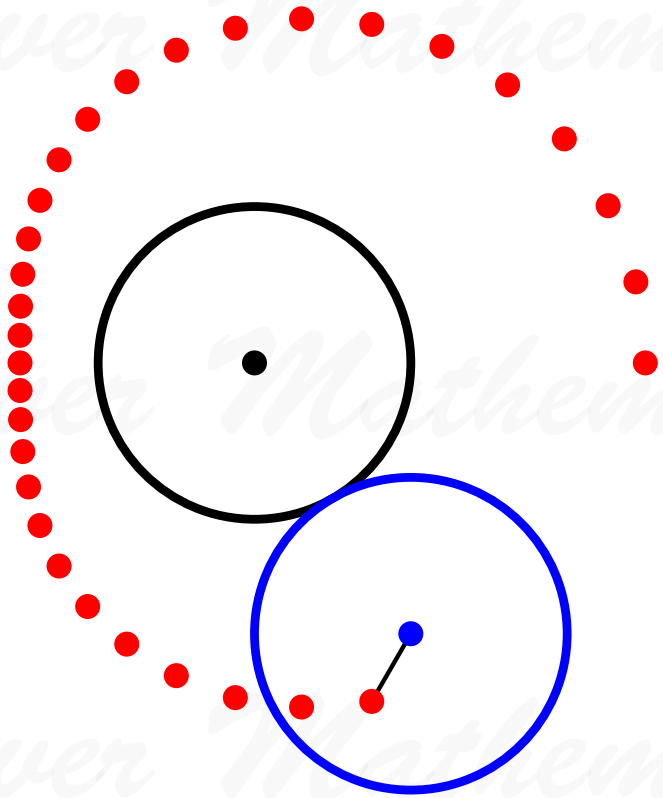

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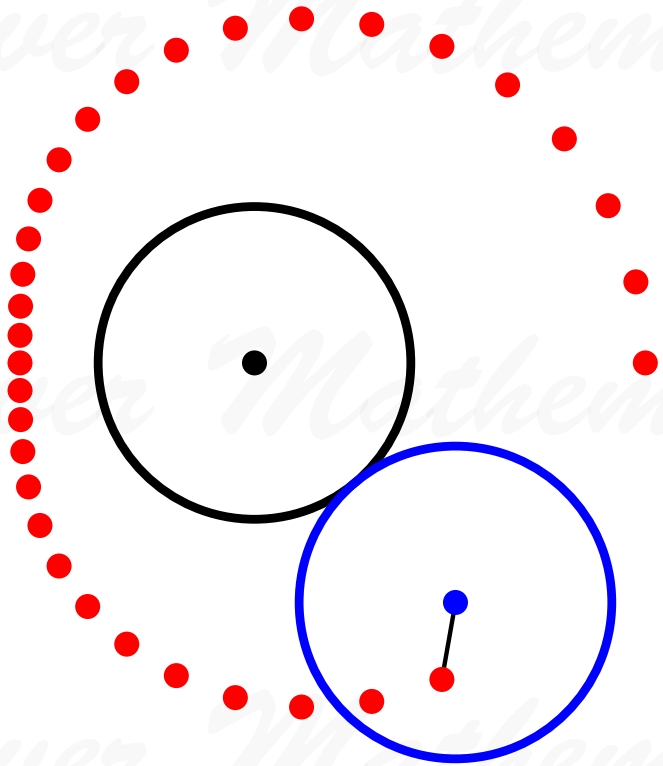

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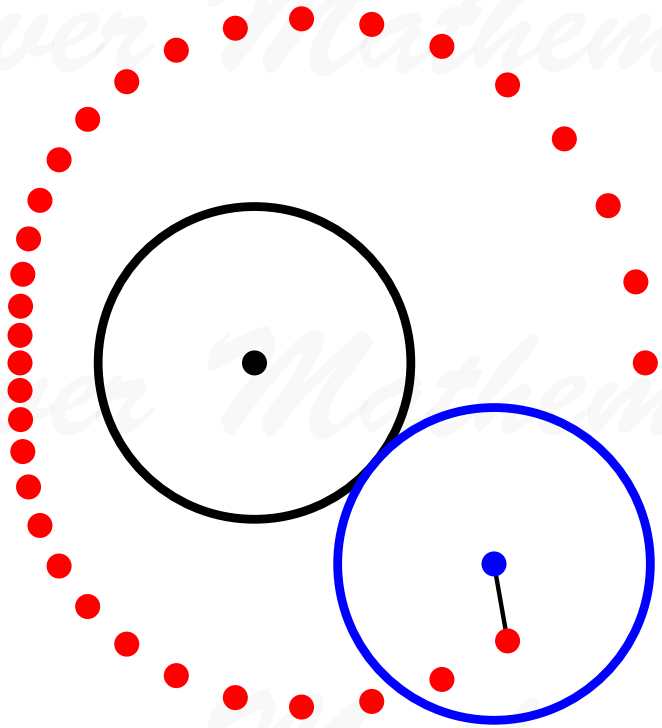

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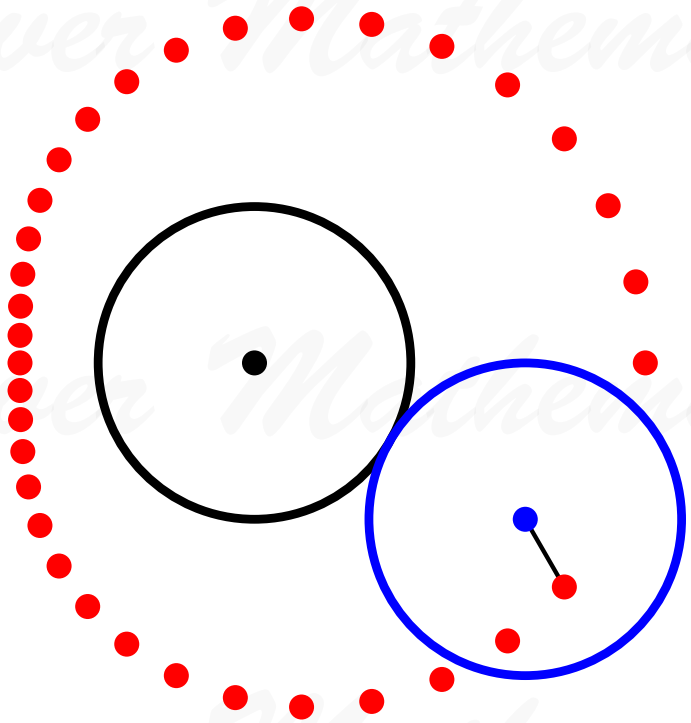

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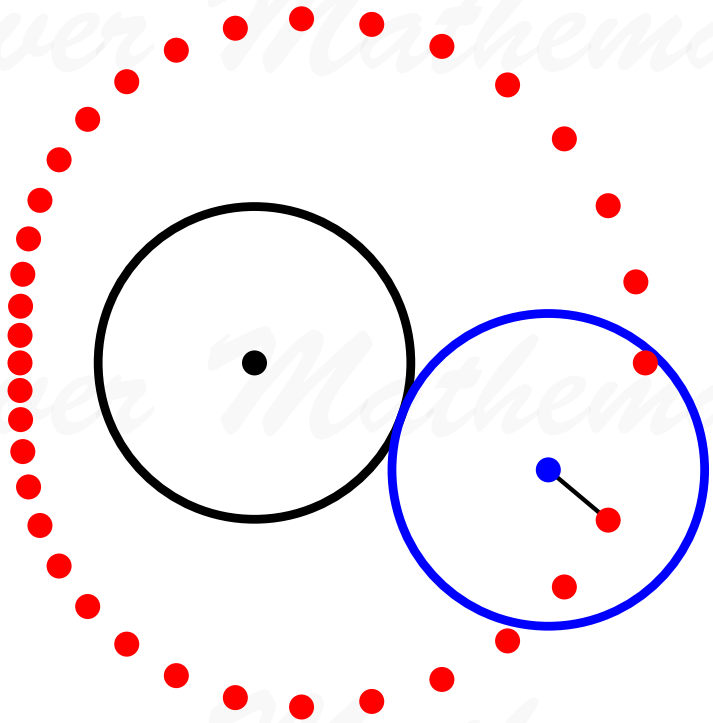

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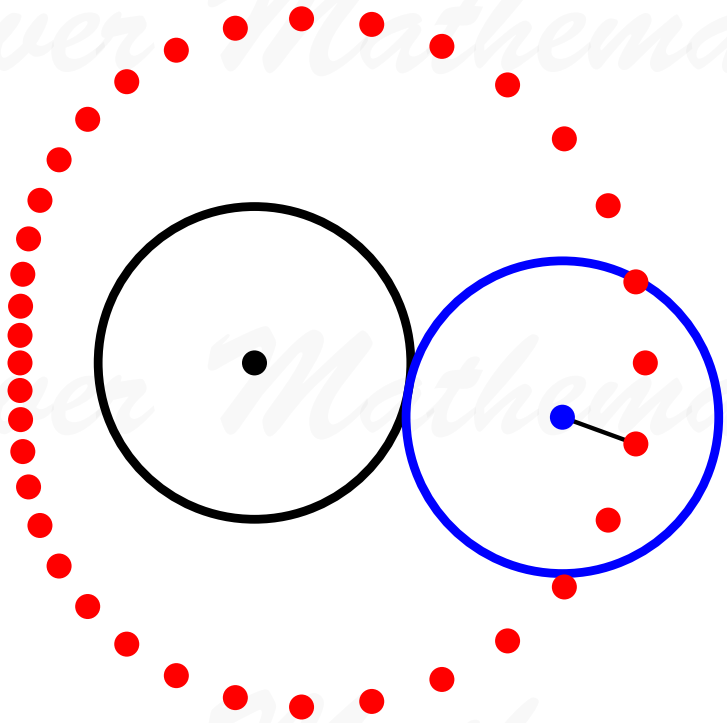

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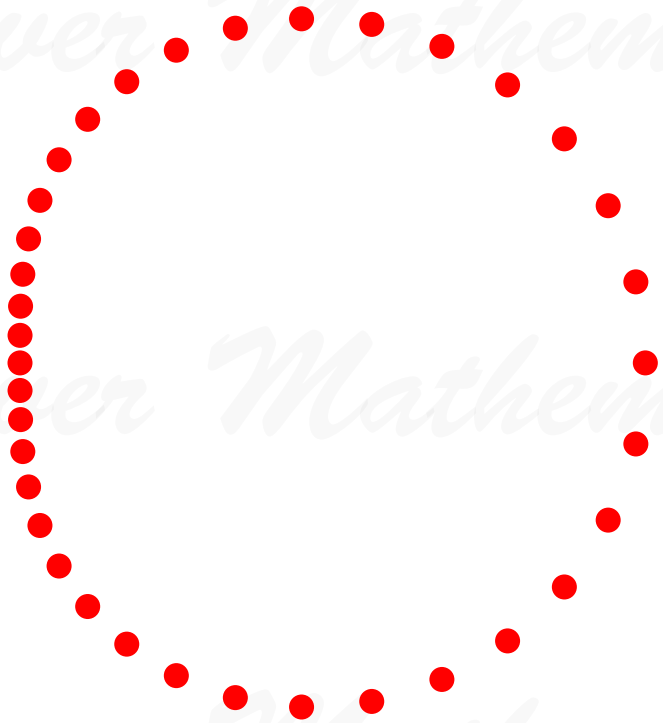

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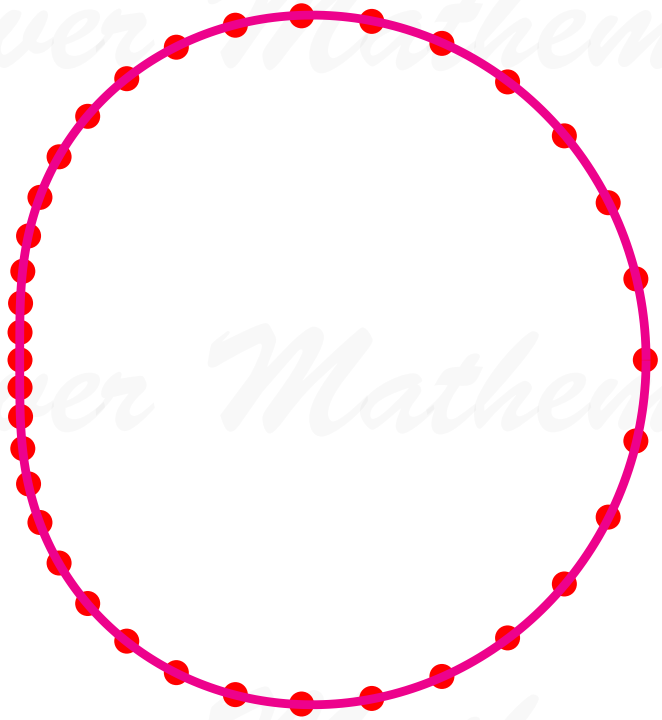

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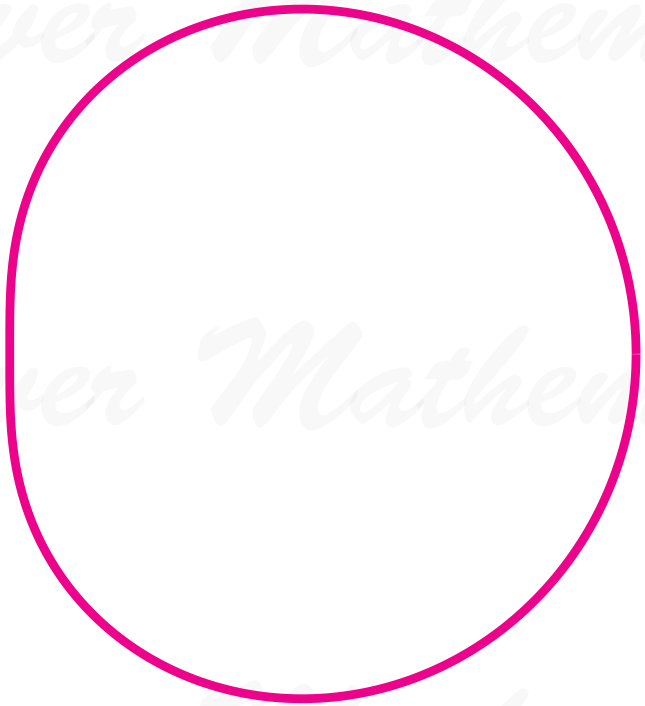

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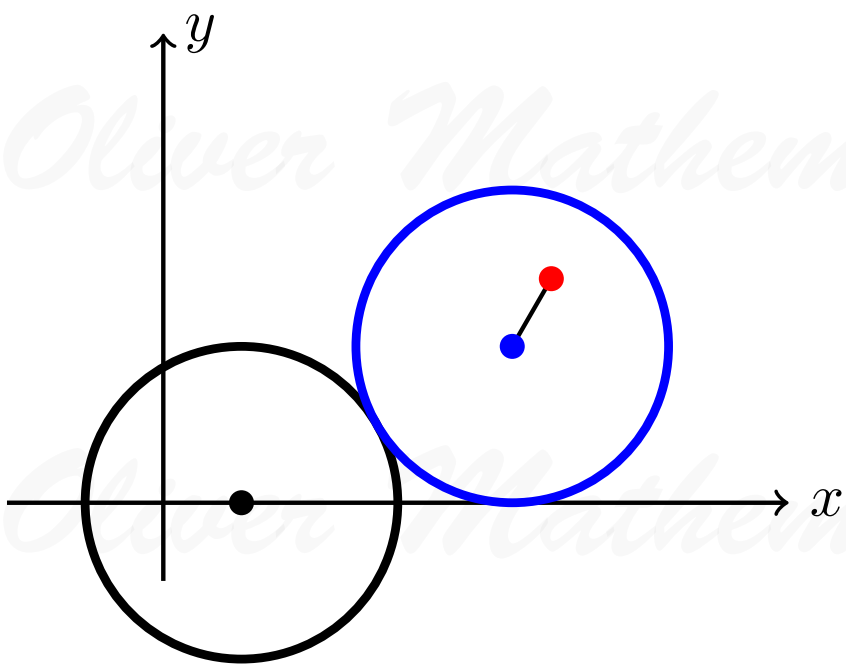
The equation of the curve

Suppose that the black circle has centre $(0.5, 0)$.



$$x = 0.5 + 2\cos\theta + 0.5\cos 2\theta$$

and

$$y = 2\sin\theta + 0.5\sin 2\theta$$

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The equation of the curve

$$\begin{aligned} x &= 0.5 + 2\cos\theta + 0.5\cos 2\theta \\ &= 0.5 + 2\cos\theta + 0.5\left(2\cos^2\theta - 1\right) \\ &= 0.5 + 2\cos\theta + \cos^2\theta - 0.5 \\ &= 2\cos\theta + \cos^2\theta \\ &= \cos\theta\left(2 + \cos\theta\right) \end{aligned}$$

and

$$\begin{aligned} y &= 2\sin\theta + 0.5\sin 2\theta \\ &= 2\sin\theta + 0.5\left(2\sin\theta\cos\theta\right) \\ &= 2\sin\theta + \sin\theta\cos\theta \\ &= \sin\theta\left(2 + \cos\theta\right) \end{aligned}$$

and hence the polar equation of the curve is $r = 2 + \cos\theta$.