

Dr Oliver Mathematics

Worked Examples

Quadratic Inequality 3

From: Edexcel GCSE Mathematics 2022 Paper 2H (Calculator)

1. Find algebraically the set of values of x for which

(5)

$$x^2 - 49 > 0 \text{ and } 5x^2 - 31x - 72 > 0.$$

Solution

$x^2 - 49 > 0$:

Difference of two squares:

$$\begin{aligned} x^2 - 49 > 0 &\Rightarrow (x)^2 - 7^2 > 0 \\ &\Rightarrow (x - 7)(x + 7) > 0 \end{aligned}$$

	$x < -7$	$x = -7$	$-7 < x < 7$	$x = 7$	$x > 7$
$x + 7$	-	0	+	+	+
$x - 7$	-	-	-	0	+
$(x + 7)(x - 7)$	+	0	-	0	+

$$\Rightarrow x < -7 \text{ or } x > 7.$$

$5x^2 - 31x - 72 > 0$:

$$\left. \begin{array}{l} \text{add to:} \qquad \qquad \qquad -31 \\ \text{multiply to: } (+5) \times (-72) = -360 \end{array} \right\} \text{hmm...}$$

Oh, well: we will try quadratic formula:

$a = 5$, $b = -31$, and $c = -72$:

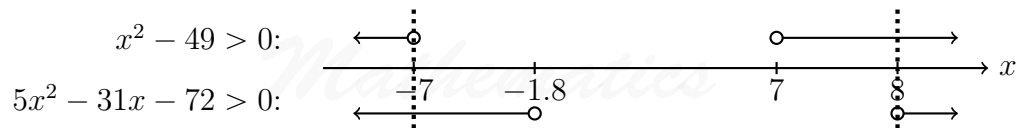
$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{31 \pm \sqrt{(-31)^2 - 4(5)(-72)}}{2 \times 5} \\
 &= \frac{31 \pm \sqrt{2401}}{10} \\
 &= \frac{31 \pm 49}{10} \\
 &= -1.8, 8.
 \end{aligned}$$

	$x < -1.8$	$x = -1.8$	$-1.8 < x < 8$	$x = 8$	$x > 8$
$x + 1.8$	-	0	+	+	+
$x - 8$	-	-	-	0	+
$5(x + 1.8)(x - 8)$	+	0	-	0	+

Hence,

$$5x^2 - 31x - 72 > 0 \Rightarrow x < -1.8 \text{ or } x > 8.$$

Now, draw a number line and look at the areas where they are both positive:



Finally, the solution set is

$$\underline{x < -7 \text{ or } x > 8.}$$