Dr Oliver Mathematics
Further Mathematics
Series
Past Examination Questions

This booklet consists of 19 questions across a variety of examination topics.
The total number of marks available is 130.

$$\sum_{r=1}^{n} 1 = n$$
$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)$$
$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$
$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2.$$

1. Calculate (3)

$$\sum_{r=1}^{40} (3r-1)^2.$$

2. Show that (4)

$$\sum_{r=n}^{2n} r^2 = \tfrac{1}{6}n(n+1)(an+b),$$

where a and b are constants to be found.

3. (a) Show that (4)

$$\sum_{r=1}^{n} (r^2 - r - 1) = \tfrac{1}{3}n(n^2-4).$$

(b) Hence, or otherwise, find the value of (2)

$$\sum_{r=10}^{20} (r^2 - r - 1).$$

4. (a) Show, using the formulae for $\sum r$ and $\sum r^2$, that (5)

$$\sum_{r=1}^{n} (6r^2 + 4r - 1) = n(n+2)(2n+1).$$

(b) Hence, or otherwise, find the value of (2)

$$\sum_{r=11}^{20}(6r^2+4r-1).$$

5. (a) Show that (5)

$$\sum_{r=1}^{n} r(r+2)(r+4) = \tfrac{1}{4}n(n+1)(n+4)(n+5).$$

(b) Hence evaluate (2)

$$\sum_{r=21}^{30} r(r+2)(r+4).$$

6. (a) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$, show that (5)

$$\sum_{r=1}^{n}(r^3+3r+2) = \tfrac{1}{4}n(n+2)(n^2+7).$$

(b) Hence evaluate (2)

$$\sum_{r=15}^{25}(r^3+3r+2).$$

7. (a) Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that (5)

$$\sum_{r=1}^{n}(r+2)(r+3) = \tfrac{1}{3}n(n^2+an+b),$$

where a and b are integers to be found.

(b) Hence show that (3)

$$\sum_{r=n+1}^{2n}(r+2)(r+3) = \tfrac{1}{3}n(7n^2+27n+26).$$

8. (a) Use the results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$, and $\sum_{r=1}^{n} r^3$, to prove that (5)

$$\sum_{r=1}^{n} r(r+1)(r+5) = \tfrac{1}{4}n(n+1)(n+2)(n+7).$$

(b) Hence, or otherwise, find the value of (2)

$$\sum_{r=20}^{50} r(r+1)(r+5).$$

9. (a) Use the results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that (6)

$$\sum_{r=1}^{n}(2r-1)^2 = \tfrac{1}{3}n(2n-1)(2n+1),$$

for all positive integers n.

(b) Hence show that (4)

$$\sum_{r=n+1}^{3n}(2r-1)^2 = \tfrac{2}{3}n(an^2+b),$$

where a and b are integers to be found.

10. (a) Using the result (3)

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2,$$

show that

$$\sum_{r=1}^{n}(r^3-2) = \tfrac{1}{4}n(n^3+2n^2+n-8).$$

(b) Calculate the exact value of (3)

$$\sum_{r=20}^{50}(r^3-2).$$

11. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to show that (5)

$$\sum_{r=1}^{n}(r^3+6r-3) = \tfrac{1}{4}n^2(n^2+2n+13),$$

for all positive integers n.

(b) Hence find the exact value of (2)

$$\sum_{r=16}^{30}(r^3+6r-3).$$

12. Show, using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, that (5)

$$\sum_{r=1}^{n} 3(2r-1)^2 = n(2n-1)(2n+1),$$

for all positive integers n.

13. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that (6)

$$\sum_{r=1}^{n}(r+2)(r+3) = \tfrac{1}{3}n(n^2+9n+26),$$

for all positive integers n.

(b) Hence show that (4)

$$\sum_{r=n+1}^{3n}(r+2)(r+3) = \tfrac{2}{3}n(an^2+bn+c),$$

where a, b, and c are integers to be found.

14. Given that (4)

$$\sum_{r=1}^{n} r(2r-1) = \tfrac{1}{6}n(n+1)(4n-1),$$

show that

$$\sum_{r=n+1}^{3n} r(2r-1) = \tfrac{1}{3}n(an^2+bn+c),$$

where a, b, and c are integers to be found.

15. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that (6)

$$\sum_{r=1}^{n}(2r-1)^2 = \tfrac{1}{3}n(4n^2-1),$$

for all positive integers n.

(b) Hence show that (3)

$$\sum_{r=2n+1}^{4n}(2r-1)^2 = an(bn^2-1),$$

where a and b are integers to be found.

16. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to show that (5)

$$\sum_{r=1}^{n} r(r^2-3) = \tfrac{1}{4}n(n+1)(n+3)(n-2).$$

(b) Calculate the value of (3)

$$\sum_{r=10}^{50} r(r^2-3).$$

17. (a) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that (5)

$$\sum_{r=1}^{n}(r+1)(r+4) = \tfrac{1}{3}n(n+4)(n+5),$$

for all positive integers n.

(b) Hence show that (3)

$$\sum_{r=n+1}^{2n}(r+1)(r+4) = \tfrac{1}{3}n(n+1)(an+b),$$

where a and b are integers to be found.

18. (a) Using the formula for $\sum_{r=1}^{n} r^2$ write down, in terms of n only, an expression for (1)

$$\sum_{r=1}^{3n} r^2.$$

(b) Show that, for all integers n, where $n > 0$, (4)

$$\sum_{r=2n+1}^{3n} r^2 = \tfrac{1}{6}n(an^2+bn+c),$$

where the values of the constants a, b, and c are to be found.

19. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that (5)

$$\sum_{r=1}^{n}(3r^2+8r+3) = \tfrac{1}{2}n(2n+5)(n+3),$$

or all positive integers n.

Given that

$$\sum_{r=1}^{12}[3r^2 + 8r + 3 + k(2^{r-1})] = 3\,520,$$

(b) find the exact value of the constant k. (4)