

Dr Oliver Mathematics
GCSE Mathematics
2023 June Paper 3H: Calculator
1 hour 30 minutes

The total number of marks available is 80.

You must write down all the stages in your working.

1. (a) Simplify (1)

$$(m^2)^3.$$

- (b) Simplify (1)

$$x^5 \times x^8.$$

- (c) Expand (2)

$$4p(p^2 + 3p).$$

2. Jonny wants to know how much coffee he will need for 800 people at a meeting.

Each person who drinks coffee will drink 2 cups of coffee.

10.6 g of coffee is needed for each cup of coffee.

Jonny assumes 68% of the people will drink coffee.

- (a) Using this assumption, work out the amount of coffee Jonny needs. (4)

Give your answer correct to the nearest gram.

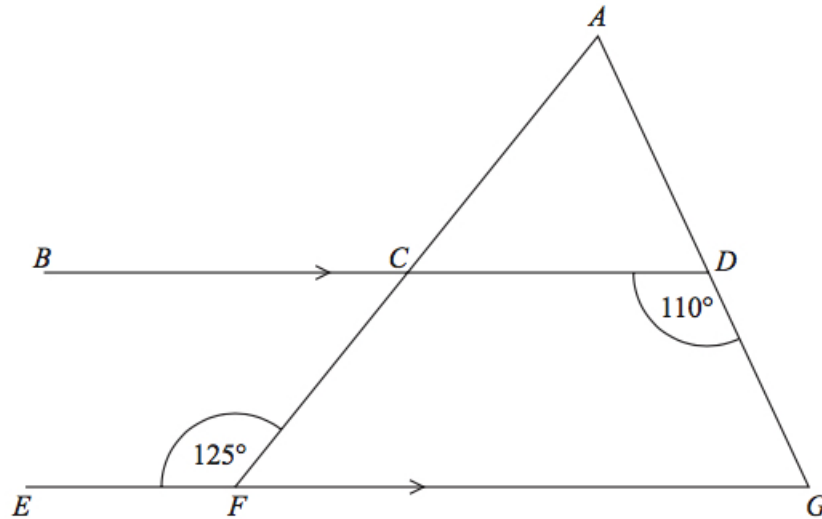
Jonny's assumption is wrong.

72% of the people will drink coffee.

- (b) How does this affect your answer to part (a)? (1)

3. ACF and ADG are straight lines. (5)

BCD and EFG are parallel lines.



Show that triangle ACD is isosceles.
Give a reason for each stage of your working.

4. It takes 14 hours for 5 identical pumps to fill a water tank. (2)

How many hours would it take 4 of these pumps to fill another water tank of the same size?

5. A and B are numbers such that

$$A = 2^2 \times 3^4 \times 7$$

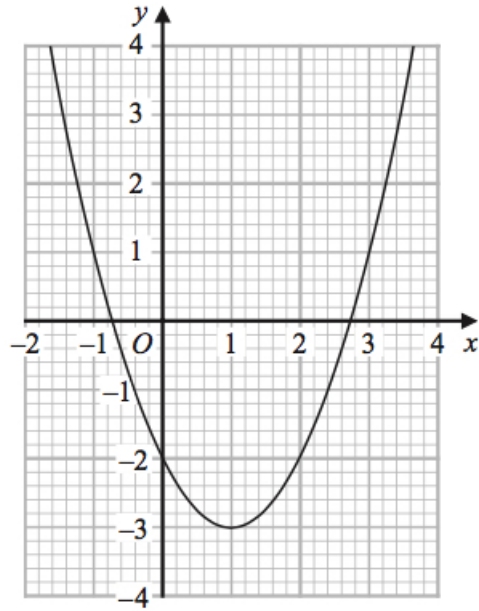
$$B = 3^2 \times 7^2.$$

- (a) Find the highest common factor (HCF) of A and B . (1)
 (b) Find the lowest common multiple (LCM) of A and B . (2)
6. Lava flows from a volcano at a constant rate of $11.9 \text{ m}^3/\text{s}$. (3)

How many days does it take for $67\,205\,600 \text{ m}^3$ of lava to flow from the volcano?
Give your answer correct to the nearest day.

7. Here is the graph of

$$y = x^2 - 2x - 2.$$



- (a) Write down the coordinates of the turning point on the graph of (1)
 $y = x^2 - 2x - 2.$

- (b) Write down an estimate for one of the roots of (1)
 $x^2 - 2x - 2 = 0.$

8. A solid cuboid is made of metal. (2)

The metal has a density of 9 g/cm^3 .
 The volume of the cuboid is 72 cm^3 .

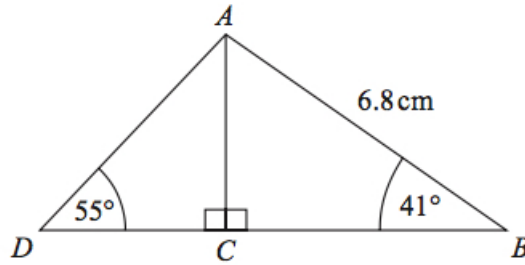
Work out the mass of the cuboid.

9. Some people were asked if they wanted a new television. (2)

70% of the people said yes.
 80% of the people who said yes wanted a television with a large screen.

What percentage of the people asked said they wanted a television with a large screen?

10. ABD is a triangle. (3)
 C is a point on BD .

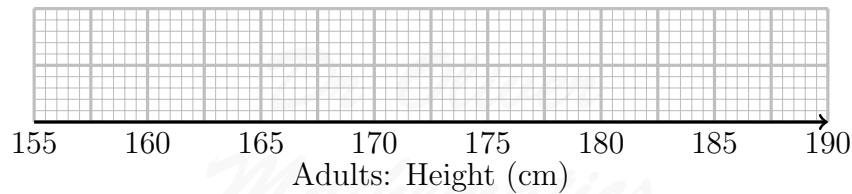


Work out the length of DC .
Give your answer correct to 1 decimal place.

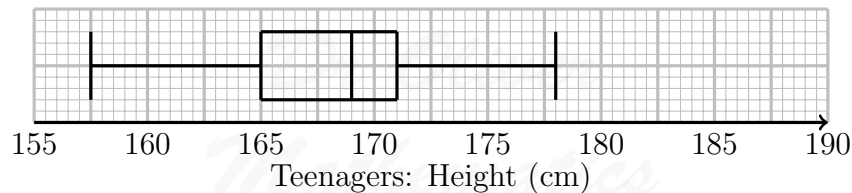
11. The table shows some information about the heights of a group of adults.

Least height	169 cm
Greatest height	186 cm
Median	177 cm
Lower quartile	174 cm
Upper quartile	180 cm

- (a) On the grid, draw a box plot for the information in the table. (3)



The box plot below shows the distribution of the heights of a group of teenagers.



- (b) Compare the distribution of the heights of the adults with the distribution of the heights of the teenagers. (2)
12. Show that (3)

$$(x - 1)(x + 3)(x - 5)$$

can be written in the form

$$ax^3 + bx^2 + cx + d,$$

where a , b , c , and d are integers.

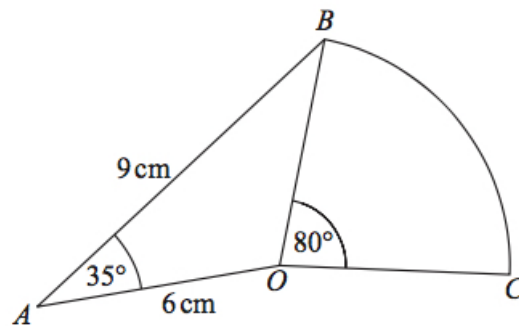
13. An expression for the n th term of the sequence of triangular numbers is (3)

$$\frac{1}{2}n(n + 1).$$

Prove that the sum of any two consecutive triangular numbers is a square number.

14. OAB is a triangle. (4)

OBC is a sector of a circle, centre O .



Calculate the area of OBC .

Give your answer correct to 3 significant figures.

15. (a) Factorise (1)

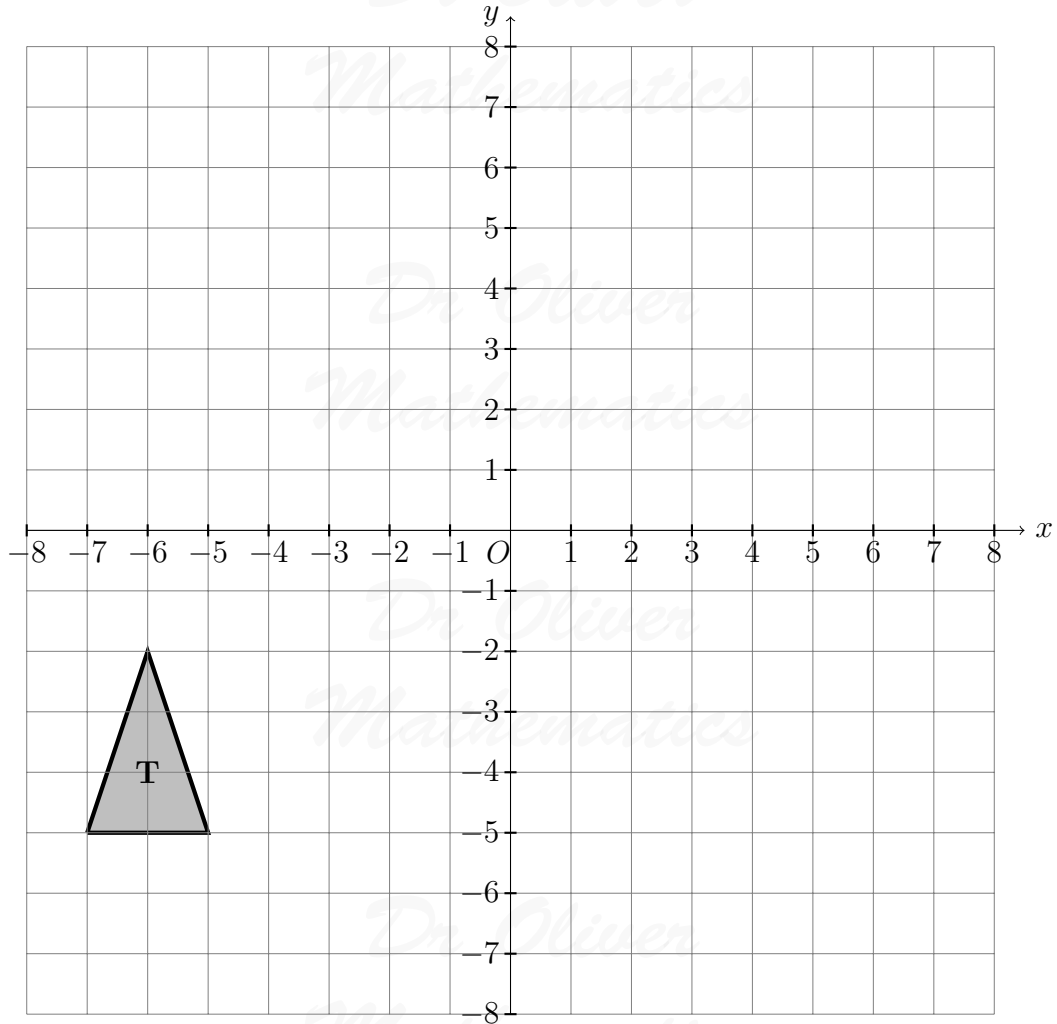
$$a^2 - b^2.$$

- (b) Show that (2)

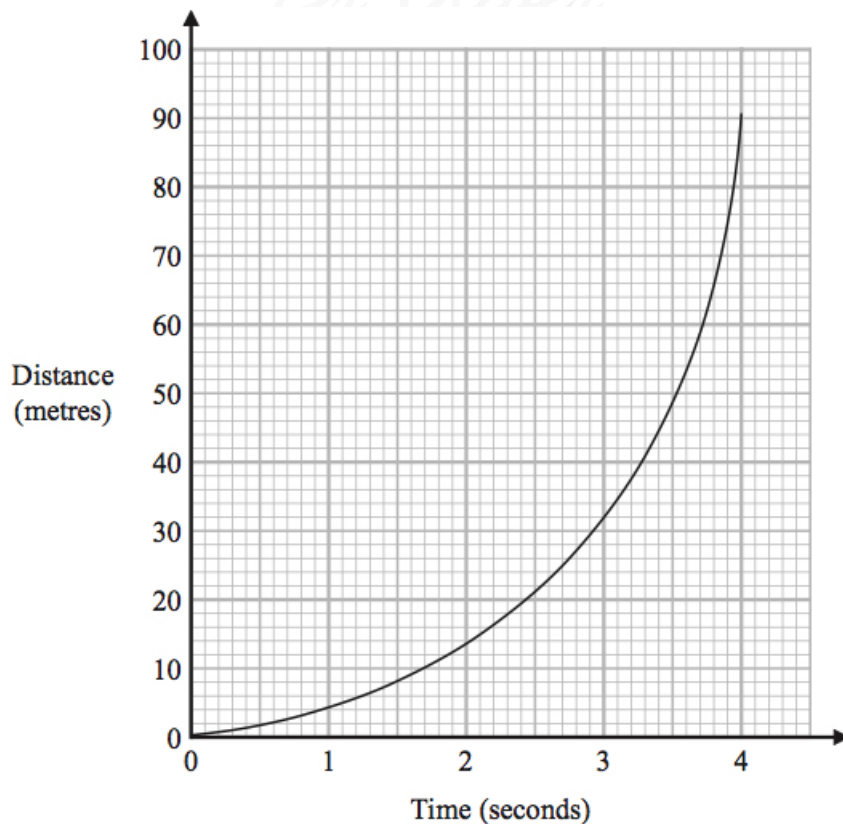
$$2^{40} - 1$$

is the product of two consecutive odd numbers.

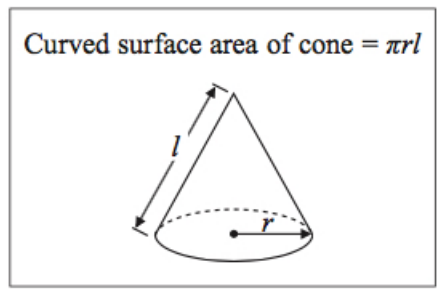
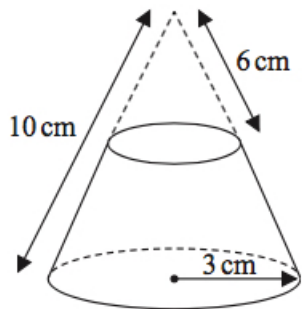
16. On the grid, enlarge triangle **T** by scale factor -2 with centre of enlargement $(-2, -2)$. (2)



17. Here is a distance-time graph.



- (a) Find an estimate of the gradient of the graph at time 2.5 seconds. (3)
You must show how you get your answer.
 - (b) What does the gradient of the graph represent? (1)
18. A solid frustum is made by removing a small cone from a large cone as shown in the diagram. (5)



- The slant height of the small cone is 6 cm.
- The slant height of the large cone is 10 cm.

- The radius of the base of the large cone is 3 cm.

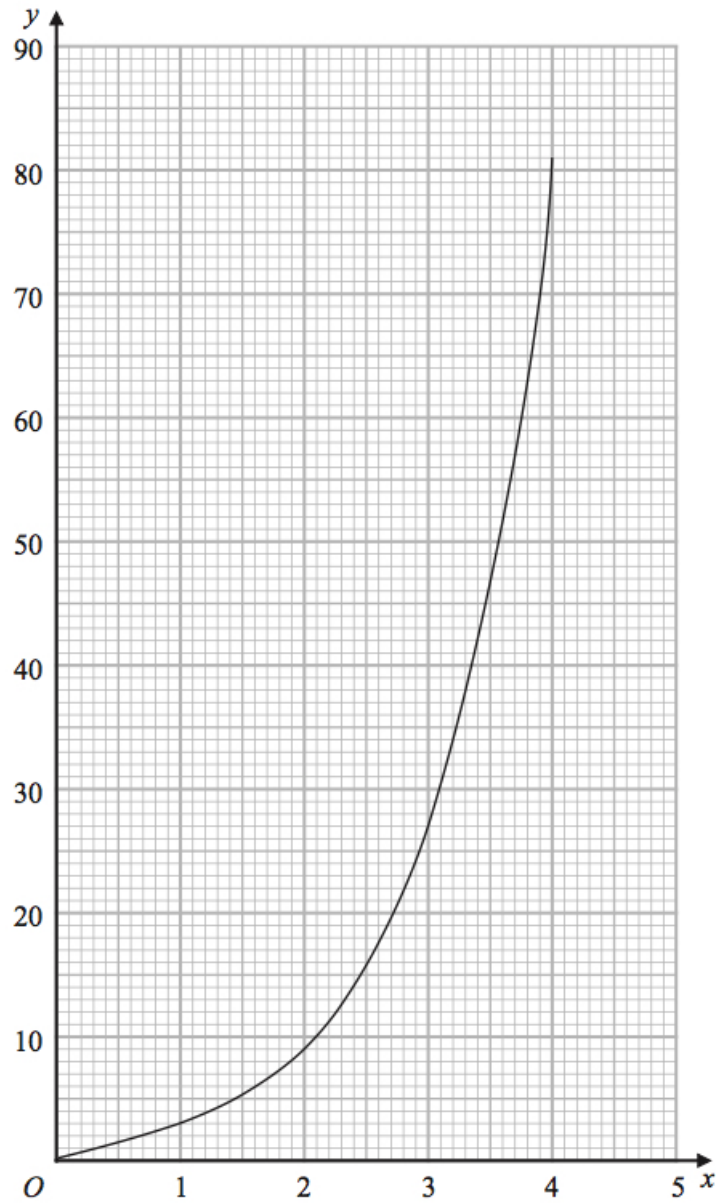
Calculate the total surface area of the frustum.
Give your answer correct to 3 significant figures.

19. Sana needs to draw the graph of

(1)

$$y = 3^x \text{ for } 0 \leq x \leq 4.$$

She draws the graph shown on the grid.



Write down one thing Sana has done wrong.

20. Prove algebraically that $0.1\dot{2}\dot{3}$ (3)

can be written as

$$\frac{61}{495}.$$

21. Solve (4)

$$\frac{1}{x+4} + \frac{3}{2-2x} = 1.$$

22. Given that the vector (3)

$$a \begin{pmatrix} 2 \\ 6 \end{pmatrix} + b \begin{pmatrix} 8 \\ 2 \end{pmatrix}$$

is parallel to the vector

$$\begin{pmatrix} 13 \\ 6 \end{pmatrix},$$

find an expression for b in terms of a .

23. A circle has equation (4)

$$x^2 + y^2 = 25.$$

The point P with coordinates $(-3, 4)$ lies on the circle.

Alex says that the tangent to the circle at P crosses the x -axis at the point $(-8, 0)$.

Is Alex correct?

You must show how you get your answer.

24. There is a total of y counters in a box. (5)

There are x pink counters and 5 blue counters in the box.

The rest of the counters are green.

$$x : y = 1 : 3.$$

Freda takes at random two counters from the box.

Find, in terms of x , an expression for the probability that Freda takes two counters of the same colour.

Give your answer as a fraction in the form

$$\frac{ax^2 + bx + c}{dx^2 + ex},$$

where a , b , c , d , and e are integers.