

Dr Oliver Mathematics
AQA GCSE Mathematics
2012 June Paper 2: Calculator
2 hours

The total number of marks available is 105.
You must write down all the stages in your working.

1. Andy thinks of a number. (3)

He multiplies it by 4.
He then subtracts 6.
His answer is 7.2.

What number did he think of?

Solution

We will call the number x . So,

$$4x - 6 = 7.2 \Rightarrow 4x = 13.2$$
$$\Rightarrow \underline{x = 3.3}.$$

2. Ellie drives 169 miles from Sheffield to London. (4)

She drives at an average speed of 65 miles per hour.
She leaves Sheffield at 6:30 am.

Does she arrive in London before 9:00 am?

You **must** show your working.

Solution

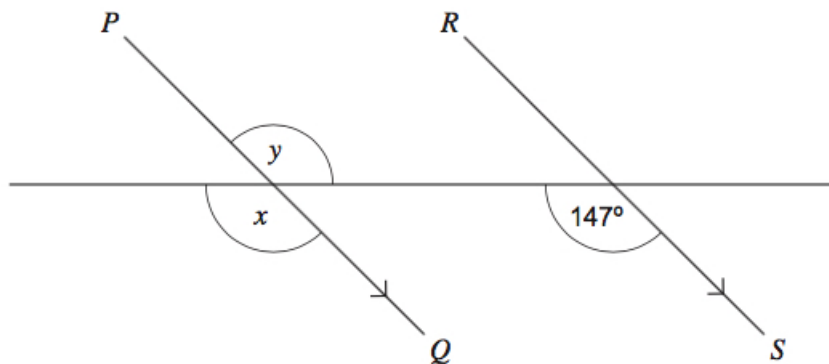
$$\text{Time taken} = \frac{169}{65}$$
$$= 2.6 \text{ hours}$$
$$= 2 \text{ hours } 36 \text{ minutes.}$$

Now,

$$6 : 30 + 2 : 36 = 9 : 06;$$

hence, she does not arrive in London until 9:06 am.

3. PQ and RS are parallel.



- (a) Write down the value of x . (2)
Give a reason for your answer.

Solution

$$\underline{x = 147^\circ} \text{ (corresponding angles)}$$

- (b) Write down the value of y . (2)
Give a reason for your answer.

Solution

$$\underline{y = 147^\circ} \text{ (vertically opposite angles)}$$

4. Ben sees these adverts to hire the same car. (6)

Hire Deal

No charge for mileage

Normal price £78 each day

Offer Now $\frac{1}{3}$ off

Price includes VAT

Best Cars

£36 each day

15p for each mile

Prices exclude VAT

VAT is 20%

Ben wants to hire the car for 10 days.
He expects to drive 600 miles.

Should he choose Hire Deal or Best Cars to get the cheaper deal?
You **must** show your working.

Solution

Hire Deal:

$$\text{Total cost} = 78 \times 10 \times \frac{2}{3} = £520.$$

Best Cars:

$$\begin{aligned} \text{Total cost} &= 1.2[(36 \times 10) + (600 \times 0.15)] \\ &= 1.2 \times 450 \\ &= £540. \end{aligned}$$

Hence, Ben should choose Hire Cars and he will save £20.

5. Work out the value of

$$15(3n + 8)$$

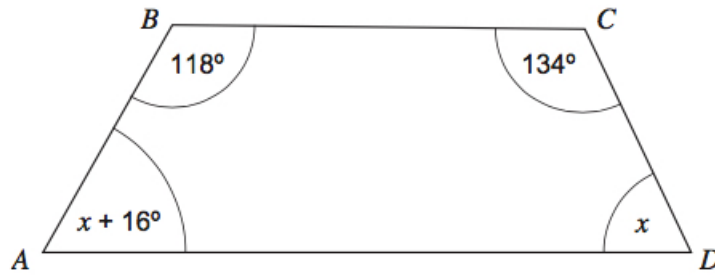
(2)

when $n = 13$.

Solution

$$15(3 \times 13 + 8) = \underline{705}.$$

6. $ABCD$ is a quadrilateral.



Not drawn accurately

(a) Work out the value of x . (2)

Solution

$$\begin{aligned} 118 + 134 + x + (x + 16) &= 360 \Rightarrow 2x + 268 = 360 \\ &\Rightarrow 2x = 92 \\ &\Rightarrow \underline{x = 46}. \end{aligned}$$

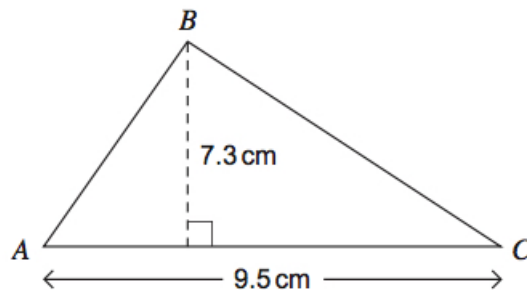
(b) Is BC parallel to AD ? (2)
Give a reason for your answer.

Solution

$$134 + 46 = 180;$$

hence, BC parallel to AD (interior angles).

7. The diagram shows a triangle ABC . (3)



Not drawn accurately

Work out the area of the triangle.
Give your answer to 1 decimal place.

Solution

$$\begin{aligned}\text{Area} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 7.3 \times 9.5 \\ &= 34.675 \text{ (exact!)} \\ &= \underline{\underline{34.7}} \text{ (1 dp).}\end{aligned}$$

8. Solve

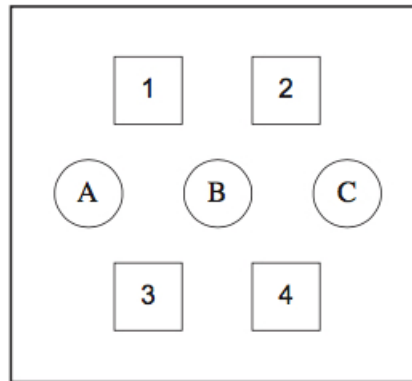
$$4(3x - 7) = 20.$$

(3)

Solution

$$\begin{aligned}4(3x - 7) = 20 &\Rightarrow 3x - 7 = 5 \\ &\Rightarrow 3x = 12 \\ &= \underline{\underline{x = 4}}.\end{aligned}$$

9. The diagram shows a door lock.



(3)

The code (number, letter, number) is entered by pressing a button from each row in turn (top row, middle row, bottom row).

Sarah knows that the code begins with 1.

She presses 1 and then enters the rest of the code at random.

Work out the probability that she enters the correct code.

Solution

$$\begin{aligned} \text{Probability} &= \frac{1}{3} \times \frac{1}{2} \\ &= \underline{\underline{\frac{1}{6}}} \end{aligned}$$

10. Use trial and improvement to find a solution to the equation

(4)

$$x^3 - 3x = 45.$$

The first step is shown in the table.

Give your solution to 1 decimal place.

x	$x^3 - 3x$	Comment
3	18	Too small

Solution

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.

F(X)= and you type in $X^3 - 3X$; then you press $\boxed{=}$.

Start? and you enter 3; then you press $\boxed{=}$.

End? and you enter 4; then you press $\boxed{=}$.

Step? and enter 0.05 – 1 decimal place divided by 2; then you press $\boxed{=}$.

x	$x^3 - 3x$	Comment
3	18	Too small
3.8	43.472 ...	Too small
3.85	45.516 ...	Too large

Clearly,

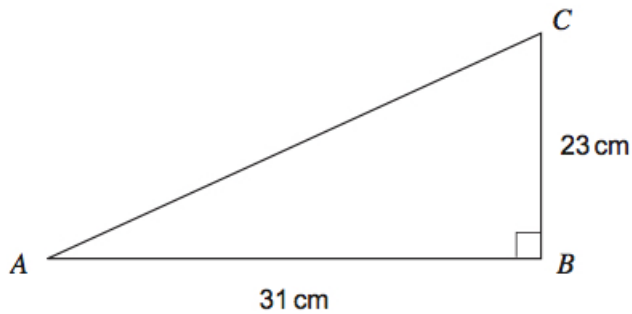
$$3.8 < x < 3.85$$

and the answer is

$$\underline{\underline{x = 3.8 \text{ (1 dp)}}}.$$

11. Work out the length AC .

(3)



Not drawn accurately

Solution

Pythagoras' Theorem:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \Rightarrow AC^2 = 31^2 + 23^2 \\ &\Rightarrow AC^2 = 1490 \\ &\Rightarrow AC = 38.60051813 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{AC = 38.6 \text{ (3 sf)}}}. \end{aligned}$$

12. A gym owner wants to know the number of hours that people exercise.

(2)

Write a question that he can use in his survey.

Include a response section.

Solution

A suitable question with a time frame, e.g., "Did you exercise today/last week/last month? If so, how long was the workout? Tick the appropriate box."

At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, Did Not Exercise, $0 \text{ hr} < \text{Workout} \leq 0.5 \text{ hr}$, $0.5 \text{ hr} < \text{Workout} \leq 1 \text{ hr}$, $1 \text{ hr} < \text{Workout} \leq 2 \text{ hr}$, $\text{Workout} \geq 2 \text{ hr}$.

13. (a) Solve the inequality

$$3x - 5 \geq 16.$$

(2)

Solution

$$\begin{aligned} 3x - 5 \geq 16 &\Rightarrow 3x \geq 21 \\ &\Rightarrow \underline{\underline{x \geq 7}}. \end{aligned}$$

- (b) The values $-1, 0, 1, 2,$ and 3 satisfy one of the inequalities below.

(1)

Circle the correct inequality.

$$-2 < 2y \leq 6$$

$$-2 \leq 2y \leq 6$$

$$-2 \leq 2y < 6$$

Solution

$$-2 < 2y \leq 6$$

$$\underline{\underline{-2 \leq 2y \leq 6}}$$

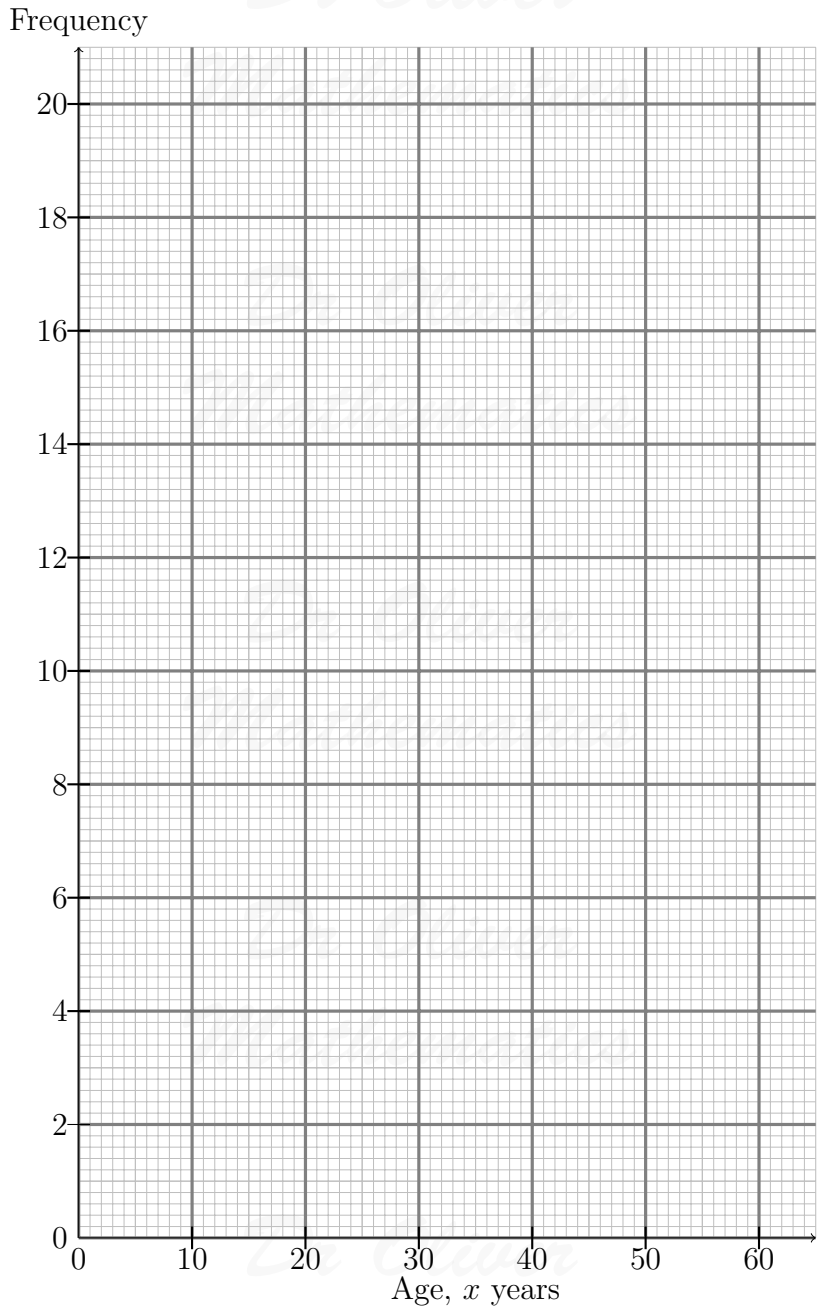
$$-2 \leq 2y < 6$$

14. The table shows information about the ages of people in a club.

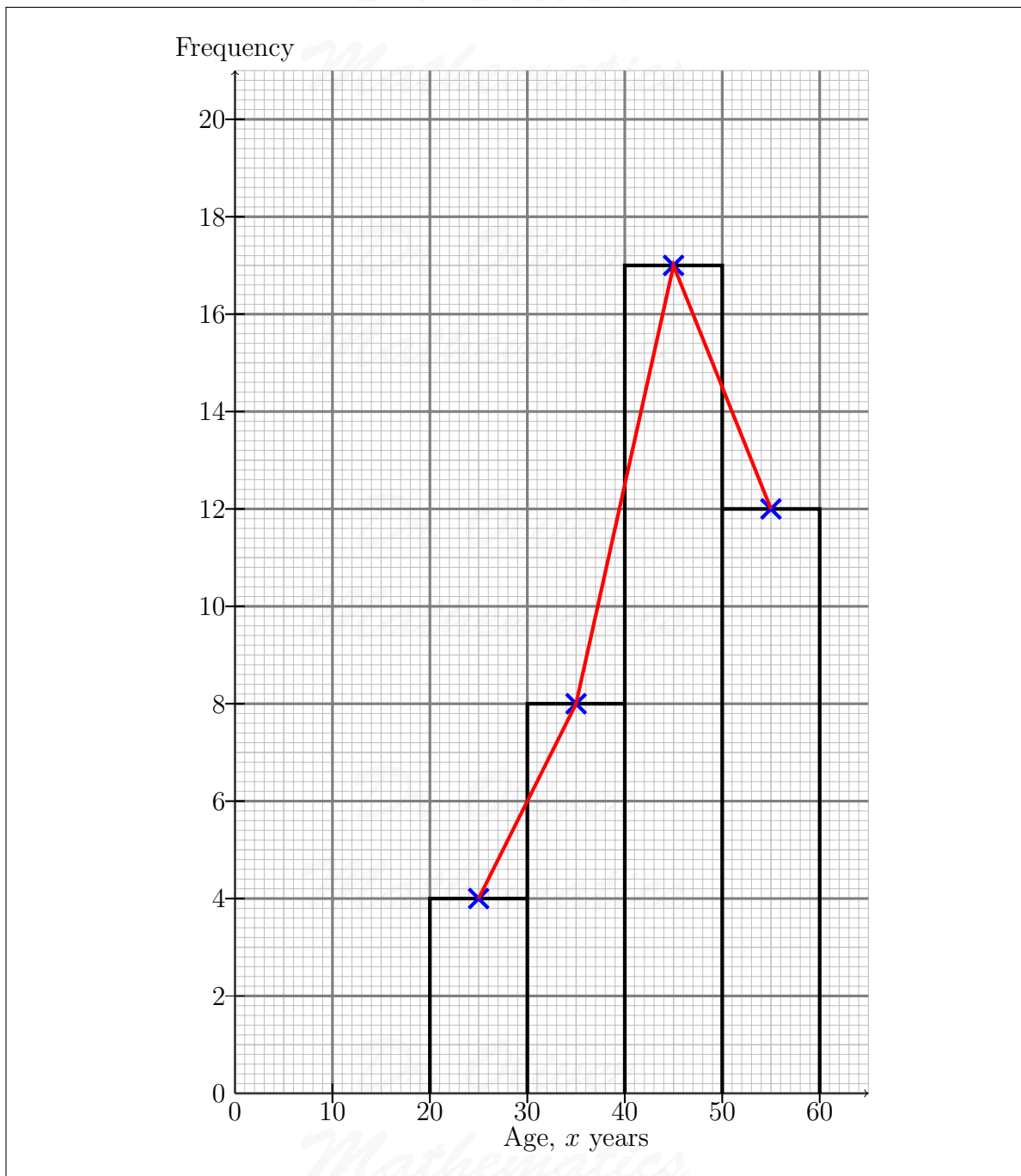
(2)

Age, x years	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Frequency	4	8	17	12

Draw a frequency polygon to represent the data.

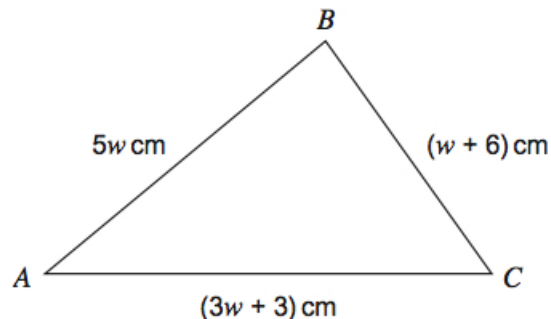


Solution



15. The diagram shows a triangle ABC .
 $AB = AC$.

(4)



Not drawn accurately

Show that the triangle is equilateral.

Solution

$$\begin{aligned}
 AB = AC &\Rightarrow 5w = 3w + 3 \\
 &\Rightarrow 2w = 3 \\
 &\Rightarrow w = 1\frac{1}{2};
 \end{aligned}$$

hence, $AB = 7\frac{1}{2} \text{ cm}$, $AC = 7\frac{1}{2} \text{ cm}$, and $BC = 7\frac{1}{2} \text{ cm}$. So, the triangle is equilateral.

16. Here is a pattern for the numbers 1, 8, and 17.

(2)

$$\begin{aligned}
 1^3 = 1 &\quad \text{and} \quad 1 = 1 \\
 8^3 = 512 &\quad \text{and} \quad 5 + 1 + 2 = 8 \\
 17^3 = 4913 &\quad \text{and} \quad 4 + 9 + 1 + 3 = 17.
 \end{aligned}$$

Find a number between 25 and 30 that follows this pattern.

Solution

Number	Add Them Up
$25^3 = 15\,625$	$1 + 5 + 6 + 2 + 5 = 19$
$26^3 = 17\,576$	$1 + 7 + 5 + 7 + 6 = 26$
$27^3 = 19\,683$	$1 + 9 + 6 + 8 + 3 = 27$
$28^3 = 21\,952$	$2 + 1 + 9 + 5 + 2 = 19$
$29^3 = 24\,389$	$2 + 4 + 3 + 8 + 9 = 26$
$30^3 = 27\,000$	$2 + 7 + 0 + 0 + 0 = 9$

So, either 26 or 27.

17. A car is advertised for £3 000.
The car will be in a sale next month.
Tom can afford to pay £2 500.

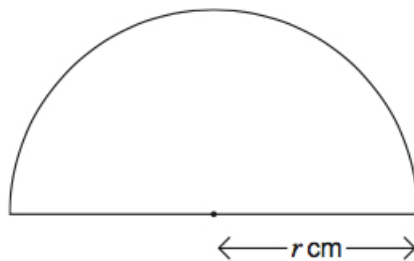
(3)

By what percentage will the price have to be reduced so that he can afford the car?

Solution

$$\begin{aligned}\text{Percentage} &= \frac{3\,000 - 2\,500}{3\,000} \times 100\% \\ &= \underline{\underline{16\frac{2}{3}\%}}.\end{aligned}$$

18. The diagram shows a semi-circular shape.



Not drawn
accurately

- (a) Circle the correct expression for the perimeter of the shape.

(1)

$2\pi r$ $\pi r + 2r$ $\frac{1}{2}\pi r^2$ πr

Solution

$$\begin{aligned}\text{Perimeter} &= \text{curved part} + \text{straight line} \\ &= \frac{1}{2} \times 2\pi r + 2r \\ &= \underline{\underline{\pi r + 2r}}.\end{aligned}$$

The perimeter of the shape is 11.6 cm.

(b) Calculate r .

Give your answer to a suitable degree of accuracy.

(4)

Solution

$$\text{Perimeter} = 11.6 \Rightarrow (\pi + 2)r = 11.6$$

$$\Rightarrow r = \frac{11.6}{\pi + 2}$$

$$\Rightarrow r = 2.256\ 110\ 272 \text{ (FCD)}$$

$$\Rightarrow r = \underline{\underline{2.26 \text{ (3 sf) or } 2.3 \text{ (2 sf)}}}$$

19. Bags of sugar are weighed.

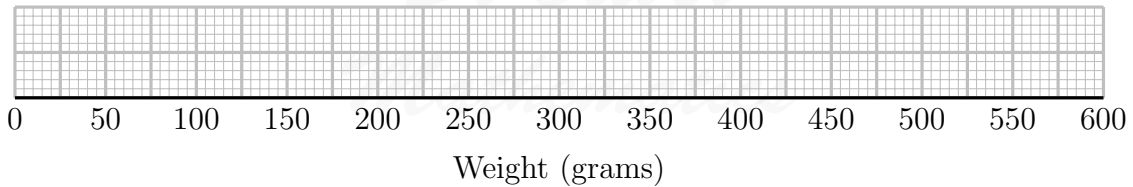
The results are summarised in the table.

All measurements are in grams.

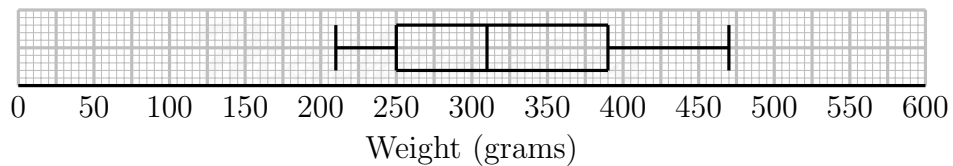
Minimum	Lower Quartile	Median	Upper Quartile	Maximum
210	250	310	390	470

(a) Draw a box plot to show this information.

(2)



Solution



An extra 10 grams of sugar is added to each of the bags.

(b) Tick the correct box to show how each of the following will change.

(3)

	Decrease	No Change	Increase
Range	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Median	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Lower quartile	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Solution
 Range: No Change
 Median: Increase
 Lower Quartile: Increase

20. (a) Complete the table of values for (2)

$$y = 2x^2 - 3.$$

x	-2	-1	0	1	2
y	5			-1	

Solution

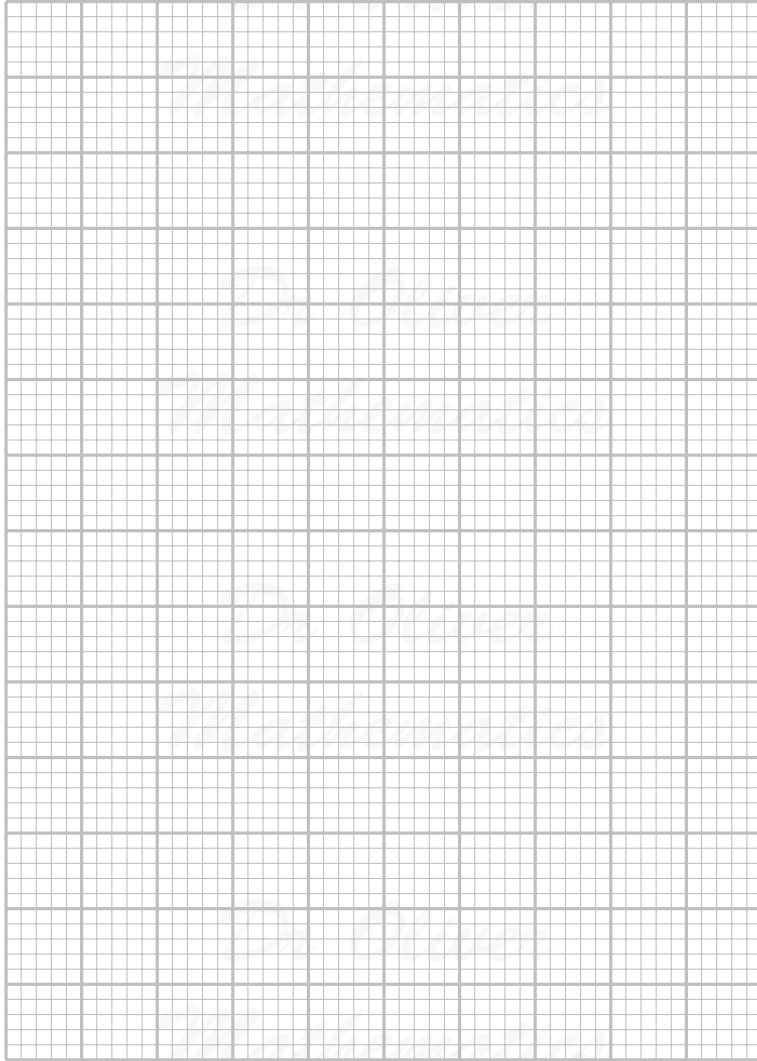
x	-2	-1	0	1	2
y	5	<u>-1</u>	<u>-3</u>	-1	<u>5</u>

(b) Draw the graph of (4)

$$y = 2x^2 - 3$$

for values of $-2 \leq x \leq 2$.

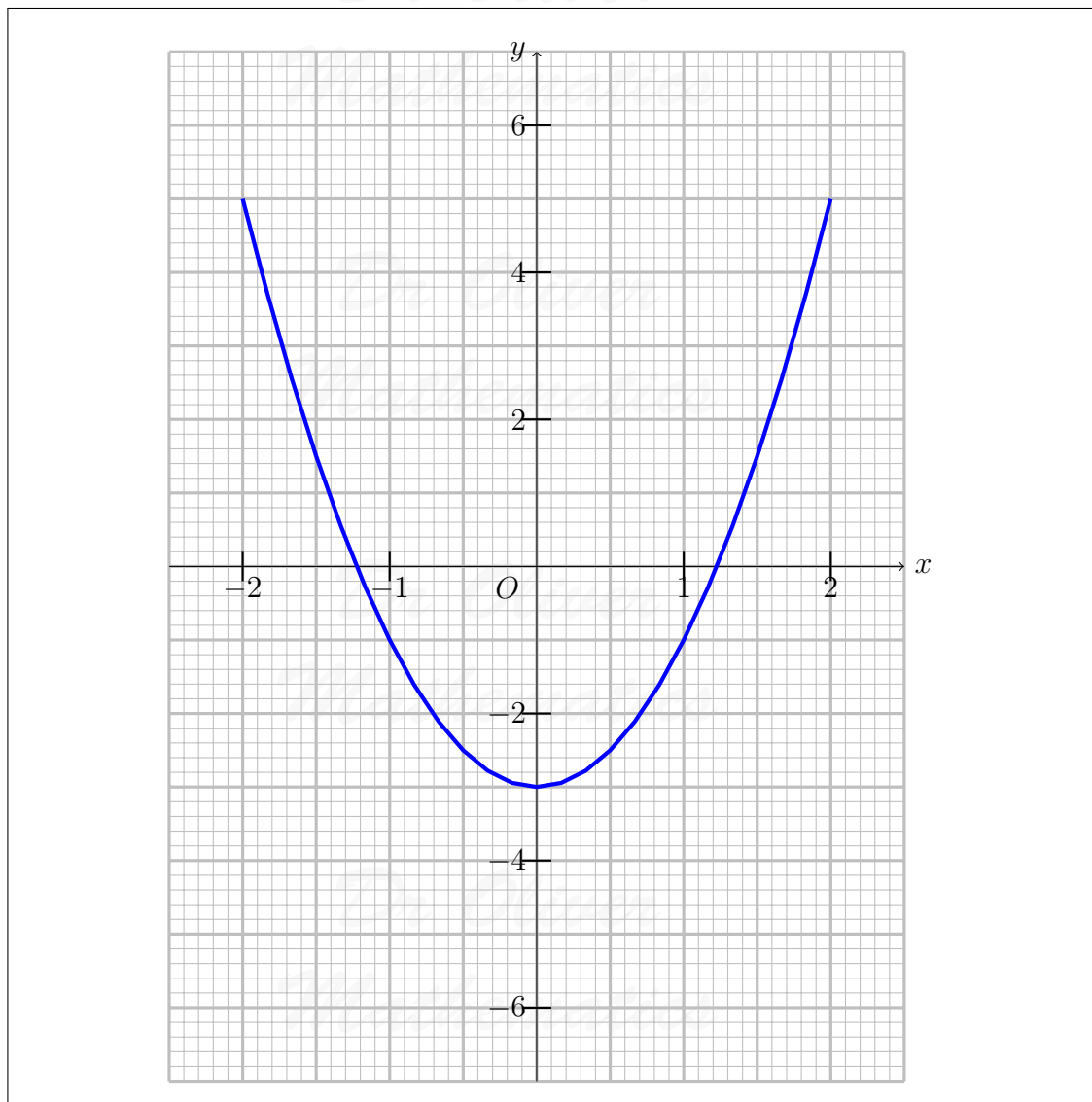
Dr Oliver



Solution

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Mathematics*

*Dr Oliver
Mathematics*



21. Amy and Kate each catch three fish.

The weight of each fish, to the nearest tenth of a kilogram, is shown.

(4)

Amy	6.8 kg	4.3 kg	5.2 kg
Kate	8.2 kg	3.4 kg	4.5 kg

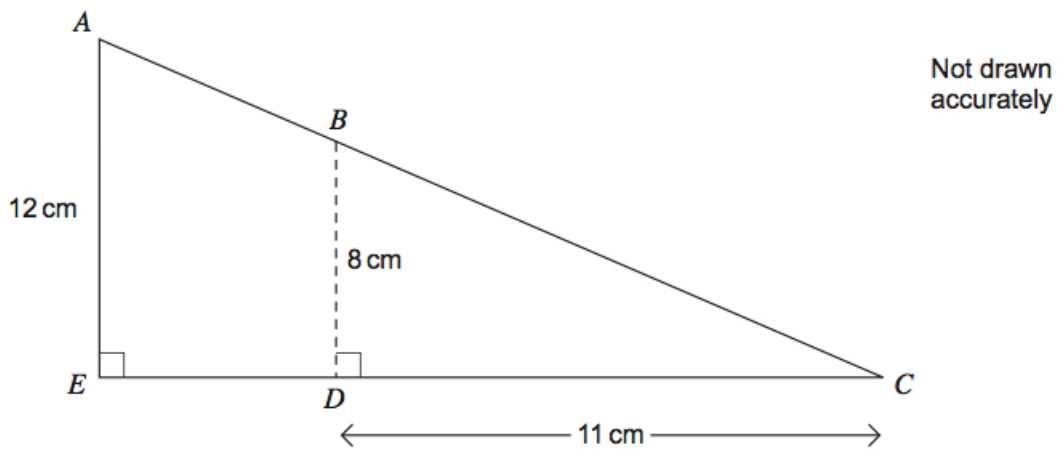
Kate says that the total weight of her fish is more than the total weight of Amy's fish. Show that this could be true.

Solution

	Lower Bound	Upper Bound
Amy	16.15	16.45
Kate	15.95	16.25

Amy's lower bound is less than Kate's upper bound; hence, this could be true.

22. The diagram shows a triangle cut into a smaller triangle and a trapezium. (5)



Work out the area of the trapezium $ABDE$.

Solution

$$\begin{aligned} \frac{AE}{BD} &= \frac{EC}{DC} \Rightarrow \frac{12}{8} = \frac{EC}{11} \\ &\Rightarrow EC = \frac{11 \times 12}{8} \\ &\Rightarrow EC = 16.5 \\ &\Rightarrow ED = 5.5 \text{ cm.} \end{aligned}$$

Hence,

$$\begin{aligned} \text{area of the trapezium } ABDE &= \frac{1}{2} \times (8 + 12) \times 5.5 \\ &= \underline{\underline{55 \text{ cm}^2}}. \end{aligned}$$

23. Two ordinary fair dice are thrown. (3)
 One dice shows a number greater than 3.
 The other dice shows a number less than 3.

Put these statements in order, starting with the least likely.

-
- A** Both dice show an even number.
B Both dice show an odd number.
C One dice shows an odd number and one dice shows an even number.
-

You **must** show your working.

Solution

$$P(A) = \frac{1}{2} \times \frac{2}{3}$$

$$= \frac{1}{3}$$

$$P(B) = \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{1}{6}$$

$$P(C) = \left(\frac{1}{2} \times \frac{2}{3}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right)$$

$$= \frac{1}{3} + \frac{1}{6}$$

$$= \frac{1}{2}$$

Hence, the order is B, A, C.

24. Expand and simplify (3)
 $(3x + y)(2x - 5y)$.

Solution

	$3x$	$+y$
$2x$	$6x^2$	$+2xy$
$-5y$	$-15xy$	$-10y^2$

Hence,

$$(3x + y)(2x - 5y) = \underline{\underline{6x^2 - 13xy - 10y^2}}$$

25. Solve the quadratic equation

$$6x^2 + 2x - 5 = 0.$$

(3)

Give your answers to 2 decimal places.

Solution

$a = 6$, $b = 2$, and $c = -5$:

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-2 \pm \sqrt{2^2 - 4 \times 6 \times (-5)}}{2 \times 6} \\&= \frac{-2 \pm \sqrt{124}}{12} \\&= -1.094\,627\,394, 0.761\,294\,060\,5 \text{ (FCD)} \\&= \underline{\underline{-1.09, 0.76}} \text{ (2 dp).}\end{aligned}$$

26. Jack is making spheres out of clay.

A box of clay contains 25 packs.

Each pack is a cuboid measuring 10 cm by 10 cm by 4 cm.

(a) How many spheres of radius 6 cm can Jack make from a **box** of clay?

(6)

Solution

$$\begin{aligned}\text{Spheres} &= \frac{25 \times 10 \times 10 \times 4}{\frac{4}{3} \times \pi \times 6^3} \\&= 11.052\dots;\end{aligned}$$

hence, he can make 11 such spheres.

A **pack** of clay has a mass of 500 grams.

(b) Work out the density of the clay.

(2)

Solution

$$\begin{aligned} \text{Density} &= \frac{\text{mass}}{\text{volume}} \\ &= \frac{500}{10 \times 10 \times 4} \\ &= \underline{\underline{1.25 \text{ g/cm}^3}}. \end{aligned}$$

27. Prove that

$$\frac{3n-1}{n} - \frac{3n+1}{n-2} \equiv \frac{2-8n}{n(n-2)}.$$

(4)

Solution

$$\begin{aligned} \frac{3n-1}{n} - \frac{3n+1}{n-2} &\equiv \frac{(3n-1)(n-2)}{n(n-2)} - \frac{n(3n+1)}{n(n-2)} \\ &\equiv \frac{(3n-1)(n-2) - n(3n+1)}{n(n-2)} \\ &\equiv \frac{(3n^2 - 7n + 2) - (3n^2 + n)}{n(n-2)} \\ &\equiv \frac{2 - 8n}{n(n-2)}, \end{aligned}$$

as required.

28. A bag contains 4 blue, 4 red, and 4 white counters.

Two counters are chosen at random without replacement.

(4)

What is the probability that the counters are different colours?

Solution

Well, there 3 ways and, hence,

$$\begin{aligned} P(\text{different colours}) &= 3 \times \frac{4}{12} \times \frac{8}{11} \\ &= \underline{\underline{\frac{24}{33}}}. \end{aligned}$$