Dr Oliver Mathematics
OCR FMSQ Additional Mathematics
2012 Paper
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.
Final answers should be given correct to three significant figures where appropriate.

Section A

1. (a) Find the range of values of x satisfying (3)

$$x^2 - 4x + 3 \leqslant 0.$$

   (b) Show this range on the number line. (1)

2. A die has 6 faces numbered one to six. The die is biased so that when it is thrown the probability of obtaining a six is $\frac{1}{5}$.

   The die is thrown 5 times.

   Find the probability of obtaining

   (a) at least 1 six, (2)

   (b) exactly 3 sixes. (4)

3. The function

$$f(x) = x^3 + ax + 6$$

   is such that when $f(x)$ is divided by $(x - 3)$ the remainder is 12.

   (a) Show that the value of a is -7. (2)

   (b) Factorise $f(x)$. (3)

4. A car moves from rest with constant acceleration on a straight road. When the car passes a point A it is travelling at 10 ms^{-1} and when it passes a point B further along the road it is travelling at 16 ms^{-1}.

   The car takes 10 seconds to travel from A to B.

   Find

(a) the distance AB, (2)

(b) the constant acceleration. (2)

5. (a) Show that the equation (2)

$$3\cos^2\theta = \sin\theta + 1$$

can be written as

$$3\sin^2\theta + \sin\theta - 2 = 0.$$

(b) Solve this equation to find values of θ in the range $0^\circ < \theta < 360^\circ$ that satisfy (4)

$$3\cos^2\theta = \sin\theta + 1.$$

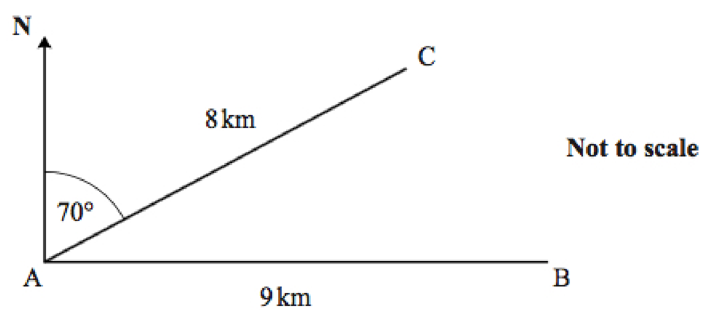
6. The equation of a curve is

$$y = 2x^3 - 9x^2 + 12x.$$

(a) Show that the curve has a stationary point where $x = 2$. (4)

(b) Determine whether the stationary value where $x = 2$ is a maximum or minimum. (2)

7. A yachtsman wishes to sail from a port, A, to another port, B, which is 9 km due East of A.
Because of the wind he is unable to sail directly East and sails 8 km on a bearing of 070° to point C.



Calculate

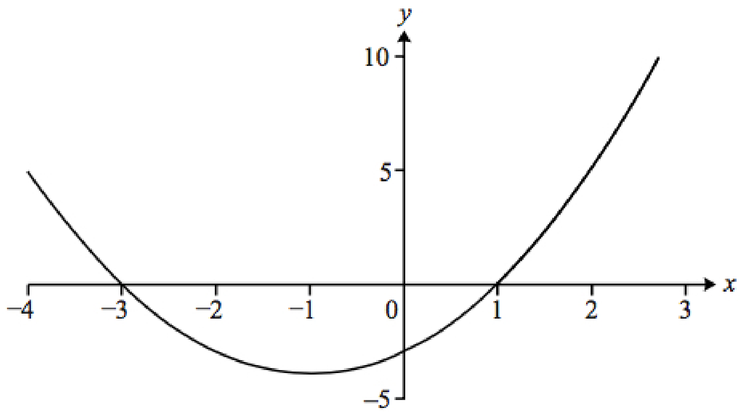
(a) the distance he is now from port B, (3)

(b) the angle ABC and hence the bearing on which he must sail to reach port B from point C, correct to the nearest degree. (4)

8. (a) Show that (3)

$$\int_0^2 (x^2 + 2x - 3)\,\mathrm{d}x = \tfrac{2}{3}.$$

The diagram shows part of the curve $y = x^2 + 2x - 3$.



Marc claims that the total area between the curve, the x-axis and the lines $x = 0$ and $x = 2$ is $\frac{2}{3}$.

(b) Explain why he is wrong. (1)

(c) Calculate the total area between the curve, the x-axis and the lines $x = 0$ and $x = 2$. (3)

9. The height above the ground of a seat on a fairground big wheel is h metres. At time t **minutes** after the wheel starts, h is given by

$$h = 7 - 5\cos(480t)^\circ.$$

(a) Write down the initial height above the ground of the seat (when $t = 0$). (1)

(b) Find the greatest height reached by the seat. (2)

(c) Calculate the time of the first occasion when the seat is 9 metres above the ground. Give your answer correct to the nearest second. (4)

Section B

10. $A(1, 10)$, $B(8, 9)$, and $C(7, 2)$ are three points.

(a) Find the coordinates of the midpoint, M, of AC. (1)

(b) Find the equation of the circle with AC as diameter. (4)

(c) Show that B lies on this circle. (1)

(d) Prove that AM and BM are perpendicular. (3)

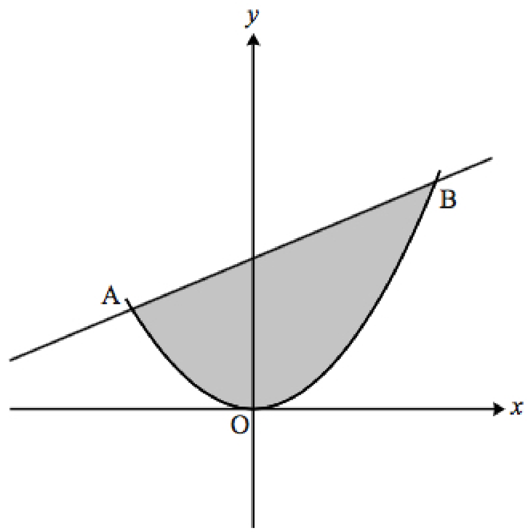
BD is a diameter of this circle.

(e) Find the coordinates of D. (3)

11. The shaded region in the diagram shows a wooden shape. The curve has equation

$$y = \frac{1}{2}x^2$$

and the coordinates of A are $(-2, 2)$.



The line AB is the normal to the curve at the point A.

(a) Find the equation of the line AB. (5)

(b) Find the coordinates of the point B where the line AB meets the curve again. (3)

(c) Find the shaded area. (4)

12. The Highway Code gives a table of shortest stopping distances (d feet) for a vehicle travelling at v miles per hour.

The formula used for this table is given by

$$d = av^2 + bv.$$

Two entries in the table are given below.

v mph	d feet
30	75
60	240

(a) By forming and solving a pair of simultaneous equations in a and b, show that the formula is (5)

$$d = \tfrac{1}{20}v^2 + v.$$

(b) Find the difference between the stopping distances for a car travelling at 65 mph and a car travelling at 70 mph. (3)

Many drivers maintain a distance of 50 feet or less when driving on a motorway.

(c) Use the formula in part (a) to find the speed at which the shortest stopping distance is 50 feet. (4)

13. (a) Find the coefficients a, b, and c in the expansion (3)

$$(2+h)^3 \equiv 8 + ah + bh^2 + ch^3.$$

The graph of the equation $y = x^3$ passes through the points P and Q which have x-coordinates 2 and $2+h$ respectively.

(b) Show that the gradient of the chord PQ is (3)

$$\frac{(2+h)^3 - 8}{h}.$$

(c) Express (2)

$$\frac{(2+h)^3 - 8}{h}$$

as a quadratic function of h.

As the value of h decreases, the point Q gets closer and closer to the point P on the curve. As h gets closer to 0 the chord PQ gets closer to being the tangent to the curve at P.

(d) Deduce the value of the gradient of the tangent at P. (1)

Kareen uses the same method to deduce the value of the gradient of the tangent at the point $(2, 16)$ on the curve $y = x^4$.

The first three lines of her working are given below.

Take P to be the point $(2, 16)$.
Take Q to be the point $(2+h, (2+h)^4)$.
The gradient of the chord PQ is given by

$$\frac{(2+h)^4 - 16}{h} =$$

(e) Complete Kareen's working. (3)