Dr Oliver Mathematics
Further Mathematics
Collisions
Past Examination Questions

This booklet consists of 27 questions across a variety of examination topics.
The total number of marks available is 351.

1. A smooth sphere P of mass $2m$ is moving in a straight line with speed u on a smooth horizontal table. Another smooth sphere Q of mass m is at rest on the table. The sphere P collides directly with Q. The coefficient of restitution between P and Q is $\frac{1}{3}$. The spheres are modelled as particles.

   (a) Show that, immediately after the collision, the speeds of P and Q are $\frac{5}{9}u$ and $\frac{8}{9}u$ respectively. (7)

   After the collision, Q strikes a fixed vertical wall which is perpendicular to the direction of motion of P and Q. The coefficient of restitution between Q and the wall is e. When P and Q collide again, P is brought to rest.

   (b) Find the value of e. (7)

   (c) Explain why there must be a third collision between P and Q. (1)

2. A smooth sphere A of mass m is moving with speed u on a smooth horizontal table when it collides directly with another smooth sphere B of mass $3m$, which is at rest on the table. The coefficient of restitution between A and B is e. The spheres have the same radius and are modelled as particles.

   (a) Show that the speed of B immediately after the collision is $\frac{1}{4}(1+e)u$. (5)

   (b) Find the speed of A immediately after the collision. (2)

   Immediately after the collision the total kinetic energy of the spheres is $\frac{1}{6}mu^2$.

   (c) Find the value of e. (6)

   (d) Hence show that A is at rest after the collision. (1)

3. Two small smooth spheres, P and Q, of equal radius, have masses $2m$ and $3m$ respectively. The sphere P is moving with speed $5u$ on a smooth horizontal table when it collides directly with Q, which is at rest on the table. The coefficient of restitution between P and Q is e.

   (a) Show that the speed of Q immediately after the collision is $2(1+e)u$. (5)

   After the collision, Q hits a smooth vertical wall which is at the edge of the table and perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f, $0 < f \leqslant 1$.

(b) Show that, when $e = 0.4$, there is a second collision between P and Q. (3)

Given that $e = 0.8$ and there is a second collision between P and Q,

(c) find the range of possible values of f. (3)

4. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on a smooth horizontal table. The particle P collides with a particle Q of mass $2m$ moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q after the collision is $\frac{1}{5}u(9e + 4)$. (5)

As a result of the collision, the direction of motion of P is reversed.

(b) Find the range of possible values of e. (5)

Given that the magnitude of the impulse of P on Q is $\frac{32}{5}mu$,

(c) find the value of e. (4)

5. Two small spheres A and B have mass $3m$ and $2m$ respectively. They are moving towards each other in opposite directions on a smooth horizontal plane, both with speed $2u$, when they collide directly. As a result of the collision, the direction of motion of B is reversed and its speed is unchanged.

(a) Find the coefficient of restitution between the spheres. (7)

Subsequently, B collides directly with another small sphere C of mass $5m$ which is at rest. The coefficient of restitution between B and C is $\frac{3}{5}$.

(b) Show that, after B collides with C, there will be no further collisions between the spheres. (7)

6. A particle A of mass $2m$ is moving with speed $3u$ in a straight line on a smooth horizontal table. The particle collides directly with a particle B of mass m moving with speed $2u$ in the opposite direction to A. Immediately after the collision, the speed of B is $\frac{8}{3}u$ and the direction of motion of B is reversed.

(a) Calculate the coefficient of restitution between A and B. (6)

(b) Show that the kinetic energy lost in the collision is $7mu^2$. (3)

After the collision B strikes a fixed vertical wall that is perpendicular to the direction of motion of B. The magnitude of the impulse of the wall on B is $\frac{14}{3}mu$.

(c) Calculate the coefficient of restitution between B and the wall. (4)

7. Two particles A and B move on a smooth horizontal table. The mass of A is m, and the mass of B is $4m$. Initially A is moving with speed u when it collides directly with B, which is at rest on the table. As a result of the collision, the direction of motion of A is reversed. The coefficient of restitution between the particles is e.

(a) Find expressions for the speed of $A$and the speed of B immediately after the collision. (7)

In the subsequent motion, B strikes a smooth vertical wall and rebounds. The wall is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{4}{5}$. Given that there is a second collision between A and B,

(b) show that $\frac{1}{4} < e < \frac{9}{16}$. (5)

Given that $e = \frac{1}{2}$,

(c) find the total kinetic energy lost in the first collision between A and B. (3)

8. A particle P of mass m is moving in a straight line on a smooth horizontal table. Another particle Q of mass km is at rest on the table. The particle P collides directly with Q. The direction of motion of P is reversed by the collision. After the collision, the speed of P is v and the speed of Q is $3v$. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(a) Find, in terms of v only, the speed of P before the collision. (3)

(b) Find the value of k. (3)

After being struck by P, the particle Q collides directly with a particle R of mass $11m$ which is at rest on the table. After this second collision, Q and R have the same speed and are moving in opposite directions. Show that

(c) the coefficient of restitution between Q and R is $\frac{3}{4}$, (4)

(d) there will be a further collision between P and Q. (2)

9. Two small spheres P and Q of equal radius have masses m and $5m$ respectively. They lie on a smooth horizontal table. Sphere P is moving with speed u when it collides directly with sphere Q which is at rest. The coefficient of restitution between the spheres is e, where $e > \frac{1}{5}$.

(a) (i) Show that the speed of P immediately after the collision is $\frac{1}{6}u(5e - 1)$. (6)

(ii) Find an expression for the speed of Q immediately after the collision, giving your answer in the form λu, where λ is in terms of e.

Three small spheres A, B, and C of equal radius lie at rest in a straight line on a smooth horizontal table, with B between A and C. The spheres A and C each have mass $5m$, and the mass of B is m. Sphere B is projected towards C with speed u. The coefficient of restitution between each pair of spheres is $\frac{4}{5}$.

(b) Show that, after B and C have collided, there is a collision between B and A. (3)

(c) Determine whether, after B and A have collided, there is a further collision between B and C. (4)

10. A particle P of mass $2m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. A particle Q of mass $3m$ is moving with speed u in the same direction as P. The particles collide directly. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(a) Show that the speed of Q immediately after the collision is $\frac{8}{5}u$. (5)

(b) Find the total kinetic energy lost in the collision. (5)

After the collision between P and Q, the particle Q collides directly with a particle R of mass m which is at rest on the plane. The coefficient of restitution between Q and R is e.

(c) Calculate the range of values of e for which there will be a second collision between P and Q. (7)

11. A particle A of mass $4m$ is moving with speed $3u$ in a straight line on a smooth horizontal table. The particle A collides directly with a particle B of mass $3m$ moving with speed $2u$ in the same direction as A. The coefficient of restitution between A and B is e. Immediately after the collision the speed of B is $4eu$.

(a) Show that $e = \frac{3}{4}$. (5)

(b) Find the total kinetic energy lost in the collision. (4)

12. A particle P of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{1}{5}(9e + 4)u$. (5)

The speed of P immediately after the collision is $\frac{1}{2}u$.

(b) Show that $e = \frac{1}{4}$. (4)

The collision between P and Q takes place at the point A. After the collision Q hits a smooth fixed vertical wall which is at right-angles to the direction of motion of Q. The distance from A to the wall is d.

(c) Show that P is a distance $\frac{3}{5}d$ from the wall at the instant when Q hits the wall. (4)

Particle Q rebounds from the wall and moves so as to collide directly with particle P at the point B. Given that the coefficient of restitution between Q and the wall is $\frac{1}{5}$,

(d) find, in terms of d, the distance of the point B from the wall. (4)

13. Particles A, B, and C of masses $4m$, $3m$, and m respectively, lie at rest in a straight line on a smooth horizontal plane with B between A and C. Particles A and B are projected towards each other with speeds u ms^{-1} and v ms^{-1} respectively, and collide directly. As a result of the collision, A is brought to rest and B rebounds with speed kv ms^{-1}. The coefficient of restitution between A and B is $\frac{3}{4}$.

(a) Show that $u = 3v$. (6)

(b) Find the value of k. (2)

Immediately after the collision between A and B, particle C is projected with speed $2v$ ms^{-1} towards B so that B and C collide directly.

(c) Show that there is no further collision between A and B. (4)

14. Two particles, P, of mass $2m$, and Q, of mass m, are moving along the same straight line on a smooth horizontal plane. They are moving in opposite directions towards each other and collide. Immediately before the collision the speed of P is $2u$ and the speed of Q is u. The coefficient of restitution between the particles is e, where $e < 1$. Find, in terms of u and e, (7)

(a) the speed of P immediately after the collision,

(b) the speed of Q immediately after the collision.

15. A small ball A of mass $3m$ is moving with speed u in a straight line on a smooth horizontal table. The ball collides directly with another small ball B of mass m moving with speed u towards A along the same straight line. The coefficient of restitution between A and B is $\frac{1}{2}$. The balls have the same radius and can be modelled as particles.

(a) Find (7)

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$.

(b) Find the speed of B immediately after hitting the wall. (2)

The first collision between A and B occurred at a distance $4a$ from the wall. The balls collide again T seconds after the first collision.

(c) Show that $T = \dfrac{112a}{15u}$. (6)

16. A particle P of mass m kg is moving with speed 6 ms^{-1} in a straight line on a smooth horizontal floor. The particle strikes a fixed smooth vertical wall at right angles and rebounds. The kinetic energy lost in the impact is 64 J. The coefficient of restitution between P and the wall is $\frac{1}{3}$.

(a) Show that $m = 4$. (6)

After rebounding from the wall, P collides directly with a particle Q which is moving towards P with speed 3 ms^{-1}. The mass of Q is 2 kg and the coefficient of restitution between P and Q is $\frac{1}{3}$.

(b) Show that there will be a second collision between P and the wall. (7)

17. A particle P of mass m is moving in a straight line on a smooth horizontal surface with speed $4u$. The particle P collides directly with a particle Q of mass $3m$ which is at rest on the surface. The coefficient of restitution between P and Q is e. The direction of motion of P is reversed by the collision. (8)

Show that $e > \frac{1}{3}$.

18. Three identical particles, A, B, and C, lie at rest in a straight line on a smooth horizontal table with B between A and C. The mass of each particle is m. Particle A is projected towards B with speed u and collides directly with B. The coefficient of restitution between each pair of particles is $\frac{2}{3}$.

(a) Find, in terms of u, (7)

(i) the speed of A after this collision,

(ii) the speed of B after this collision

(b) Show that the kinetic energy lost in this collision is $\frac{5}{36}mu^2$. (4)

After the collision between A and B, particle B collides directly with C.

(c) Find, in terms of u, the speed of C immediately after this collision between B and C. (4)

19. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. The particle P collides directly with a particle Q of mass $4m$ moving on the plane with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Find the speed of Q immediately after the collision. (6)

Given that the direction of motion of P is reversed by the collision,

(b) find the range of possible values of e. (5)

20. A particle A of mass m is moving with speed u on a smooth horizontal floor when it collides directly with another particle B, of mass $3m$, which is at rest on the floor. The coefficient of restitution between the particles is e. The direction of motion of A is reversed by the collision.

(a) Find, in terms of e and u, (7)

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

After being struck by A the particle B collides directly with another particle C, of mass $4m$, which is at rest on the floor. The coefficient of restitution between B and C is $2e$. Given that the direction of motion of B is reversed by this collision,

(b) find the range of possible values of e, (6)

(c) determine whether there will be a second collision between A and B. (3)

21. Three particles P, Q, and R lie at rest in a straight line on a smooth horizontal table with Q between P and R. The particles P, Q, and R have masses $2m$, $3m$, and $4m$ respectively. Particle P is projected towards Q with speed u and collides directly with it. The coefficient of restitution between each pair of particles is e.

(a) Show that the speed of Q immediately after the collision with P is $\frac{2}{5}(1+e)u$. (6)

After the collision between P and Q there is a direct collision between Q and R. Given that $e = \frac{3}{4}$, find

(b) (i) the speed of Q after this collision, (6)
(ii) the speed of R after this collision.

Immediately after the collision between Q and R, the rate of increase of the distance between P and R is V.

(c) Find V in terms of u. (3)

22. Two particles P and Q, of masses $2m$ and m respectively, are on a smooth horizontal table. Particle Q is at rest and particle P collides directly with it when moving with speed u. After the collision the total kinetic energy of the two particles is $\frac{3}{4}mu^2$. Find

(a) the speed of Q immediately after the collision, (10)

(b) the coefficient of restitution between the particles. (3)

23. A particle of mass m kg lies on a smooth horizontal surface. Initially the particle is at rest at a point O midway between a pair of fixed parallel vertical walls. The walls are 2 m apart. At time $t = 0$ the particle is projected from O with speed u ms^{-1} in a direction perpendicular to the walls. The coefficient of restitution between the particle and each wall is $\frac{2}{3}$. The magnitude of the impulse on the particle due to the first impact with a wall is λmu Ns.

(a) Find the value of λ. (3)

The particle returns to O, having bounced off each wall once, at time $t = 3$ seconds.

(b) Find the value of u. (6)

24. A particle P of mass $2m$ is moving in a straight line with speed $3u$ on a smooth horizontal table. A second particle Q of mass $3m$ is moving in the opposite direction to P along the same straight line with speed u. The particle P collides directly with Q. The direction of motion of P is reversed by the collision. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{1}{5}u(8e+3)$. (6)

(b) Find the range of possible values of e. (4)

The total kinetic energy of the particles before the collision is T. The total kinetic energy of the particles after the collision is kT. Given that $e = \frac{1}{2}$,

(c) find the value of k. (4)

25. Three identical particles P, Q, and R, each of mass m, lie in a straight line on a smooth horizontal plane with Q between P and R. Particles P and Q are projected directly towards each other with speeds $4u$ and $2u$ respectively, and at the same time particle R is projected along the line away from Q with speed $3u$. The coefficient of restitution between each pair of particles is e. After the collision between P and Q there is a collision between Q and R.

(a) Show that $e > \frac{2}{3}$. (7)

It is given that $e = \frac{3}{4}$.

(b) Show that there will not be a further collision between P and Q. (6)

26. Two particles A and B, of mass $2m$ and $3m$ respectively, are initially at rest on a smooth horizontal surface. Particle A is projected with speed $3u$ towards B. Particle A collides directly with particle B. The coefficient of restitution between A and B is $\frac{3}{4}$.

(a) Find (7)

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

After the collision B hits a fixed smooth vertical wall and rebounds. The wall is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is e. The magnitude of the impulse received by B when it hits the wall is $\frac{27}{4}mu$.

(b) Find the value of e. (3)

(c) Determine whether there is a further collision between A and B after B rebounds from the wall. (2)

27. Two particles A and B, of masses $3m$ and $4m$ respectively, lie at rest on a smooth horizontal surface. Particle B lies between A and a smooth vertical wall which is perpendicular to the line joining A and B. Particle B is projected with speed $5u$ in a direction perpendicular to the wall and collides with the wall. The coefficient of restitution between B and the wall is $\frac{3}{5}$.

(a) Find the magnitude of the impulse received by B in the collision with the wall. (3)

After the collision with the wall, B rebounds from the wall and collides directly with A. The coefficient of restitution between A and B is e.

(b) Show that, immediately after they collide, A and B are both moving in the same direction. (7)

The kinetic energy of B immediately after it collides with A is one-quarter of the kinetic energy of B immediately before it collides with A.

(c) Find the value of e. (4)