

Dr Oliver Mathematics

Three Ratios on a Straight Line

In this note, we will explore three ratios that lie in order on a straight line.

1. The points A , B , C , and D lie in order on a straight line.

$$AB : BD = 1 : 3.$$

$$AC : CD = 1 : 1.$$

Work out $AB : BC : CD$.

Solution

$$\begin{aligned} AB : BD = 1 : 3 &\Rightarrow 3AB = BD \\ &\Rightarrow 3AB = BC + CD \quad (1) \end{aligned}$$

and

$$\begin{aligned} AC : CD = 1 : 1 &\Rightarrow AC = CD \\ &\Rightarrow AB + BC = CD \\ &\Rightarrow AB = CD - BC \\ &\Rightarrow 3AB = 3CD - 3BC \quad (2). \end{aligned}$$

Subtract (1) – (2):

$$\begin{aligned} 0 &= 4BC - 2CD \Rightarrow 4BC = 2CD \\ &\Rightarrow 2BC = CD \\ &\Rightarrow \boxed{BC : CD = 1 : 2.} \end{aligned}$$

Next,

$$\begin{aligned} AB &= CD - BC \Rightarrow AB = 2BC - BC \\ &\Rightarrow AB = BC \\ &\Rightarrow \boxed{AB : BC = 1 : 1.} \end{aligned}$$

Finally,

$$AB : BC : CD = \underline{\underline{1 : 1 : 2.}}$$

2. The points A , B , C , and D lie in order on a straight line.

$$AB : BD = 1 : 2.$$

$$AC : CD = 3 : 2.$$

Work out $AB : BC : CD$.

Solution

$$\begin{aligned} AB : BD = 1 : 2 &\Rightarrow 2AB = BD \\ &\Rightarrow 2AB = BC + CD \quad (1) \end{aligned}$$

and

$$\begin{aligned} AC : CD = 3 : 2 &\Rightarrow 2AC = 3CD \\ &\Rightarrow 2AB + 2BC = 3CD \\ &\Rightarrow 2AB = 3CD - 2BC \quad (2). \end{aligned}$$

Subtract (1) – (2):

$$\begin{aligned} 0 &= 3BC - 2CD \Rightarrow 3BC = 2CD \\ &\Rightarrow BC : CD = 2 : 3 \\ &\Rightarrow \boxed{BC : CD = 4 : 6}. \end{aligned}$$

Next,

$$\begin{aligned} 2AB = BC + CD &\Rightarrow 2AB = BC + \frac{3}{2}BC \\ &\Rightarrow 2AB = \frac{5}{2}BC \\ &\Rightarrow 4AB = 5BC \\ &\Rightarrow \boxed{AB : BC = 5 : 4}. \end{aligned}$$

Finally,

$$AB : BC : CD = \underline{5 : 4 : 6}.$$

3. The points A , B , C , and D lie in order on a straight line.

$$AB : BD = 2 : 3.$$

$$AC : CD = 3 : 1.$$

Work out $AB : BC : CD$.

Solution

$$\begin{aligned} AB : BD = 2 : 3 &\Rightarrow 3AB = 2BD \\ &\Rightarrow 3AB = 2BC + 2CD \quad (1) \end{aligned}$$

and

$$\begin{aligned}AC : CD = 3 : 1 &\Rightarrow AC = 3CD \\ &\Rightarrow AB + BC = 3CD \\ &\Rightarrow AB = 3CD - BC \\ &\Rightarrow 3AB = 9CD - 3BC \quad (2).\end{aligned}$$

Subtract (2) – (1):

$$\begin{aligned}0 = 7CD - 5BC &\Rightarrow 5BC = 7CD \\ &\Rightarrow \boxed{BC : CD = 7 : 5}.\end{aligned}$$

Next,

$$\begin{aligned}AB = 3CD - BC &\Rightarrow AB = \frac{15}{7}BC - BC \\ &\Rightarrow AB = \frac{8}{7}BC \\ &\Rightarrow 7AB = 8BC \\ &\Rightarrow \boxed{AB : BC = 8 : 7}.\end{aligned}$$

Finally,

$$AB : BC : CD = \underline{\underline{8 : 7 : 5}}.$$

4. The points A , B , C , and D lie in order on a straight line.

$$AB : BD = 1 : 2.$$

$$AC : CD = 7 : 5.$$

Work out $AB : BC : CD$.

Solution

$$\begin{aligned}AB : BD = 1 : 2 &\Rightarrow 2AB = BD \\ &\Rightarrow 2AB = BC + CD \\ &\Rightarrow 10AB = 5BC + 5CD \quad (1)\end{aligned}$$

and

$$\begin{aligned}AC : CD = 7 : 5 &\Rightarrow 5AC = 7CD \\ &\Rightarrow 5AB + 5BC = 7CD \\ &\Rightarrow 5AB = 7CD - 5BC \\ &\Rightarrow 10AB = 14CD - 10BC \quad (2).\end{aligned}$$

Subtract (1) – (2):

$$\begin{aligned}0 &= 15BC - 9CD \Rightarrow 15BC = 9CD \\ &\Rightarrow 5BC = 3CD \\ &\Rightarrow \boxed{BC : CD = 3 : 5.}\end{aligned}$$

Next,

$$\begin{aligned}2AB &= BC + CD \Rightarrow 2AB = BC + \frac{5}{3}BC \\ &\Rightarrow 2AB = \frac{8}{3}BC \\ &\Rightarrow 6AB = 8BC \\ &\Rightarrow 3AB = 4BC \\ &\Rightarrow \boxed{AB : BC = 4 : 3.}\end{aligned}$$

Finally,

$$AB : BC : CD = \underline{4 : 3 : 5}.$$

5. The points A , B , C , and D lie in order on a straight line.

$$AB : BD = 3 : 5.$$

$$AC : CD = 5 : 1.$$

Work out $AB : BC : CD$.

Solution

$$\begin{aligned}AB : BD = 3 : 5 &\Rightarrow 5AB = 3BD \\ &\Rightarrow 5AB = 3BC + 3CD \quad (1)\end{aligned}$$

and

$$\begin{aligned}AC : CD = 5 : 1 &\Rightarrow AC = 5CD \\ &\Rightarrow AB + BC = 5CD \\ &\Rightarrow AB = 5CD - BC \\ &\Rightarrow 5AB = 25CD - 5BC \quad (2).\end{aligned}$$

Subtract (1) – (2):

$$\begin{aligned}0 &= 8BC - 22CD \Rightarrow 8BC = 22CD \\ &\Rightarrow 4BC = 11CD \\ &\Rightarrow \boxed{BC : CD = 11 : 4.}\end{aligned}$$

Next,

$$\begin{aligned}AB &= 5CD - BC \Rightarrow AB = \frac{20}{11}BC - BC \\&\Rightarrow AB = \frac{9}{11}BC \\&\Rightarrow 11AB = 9BC \\&\Rightarrow \boxed{AB : BC = 9 : 11.}\end{aligned}$$

Finally,

$$AB : BC : CD = \underline{9 : 11 : 4}.$$

6. The points A , B , C , and D lie in order on a straight line.

$$AB : BD = 7 : 13.$$

$$AC : CD = 2 : 1.$$

Work out $AB : BC : CD$.

Solution

$$\begin{aligned}AB : BD = 7 : 13 &\Rightarrow 13AB = 7BD \\&\Rightarrow 13AB = 7BC + 7CD \quad (1)\end{aligned}$$

and

$$\begin{aligned}AC : CD = 2 : 1 &\Rightarrow AC = 2CD \\&\Rightarrow AB + BC = 2CD \\&\Rightarrow AB = 2CD - BC \\&\Rightarrow 13AB = 26CD - 13BC \quad (2).\end{aligned}$$

Subtract (1) – (2):

$$\begin{aligned}0 &= 20BC - 19CD \Rightarrow 20BC = 19CD \\&\Rightarrow \boxed{BC : CD = 19 : 20.}\end{aligned}$$

Next,

$$\begin{aligned}AB &= 2CD - BC \Rightarrow AB = \frac{20}{19}BC - BC \\&\Rightarrow AB = \frac{21}{19}BC \\&\Rightarrow 19AB = 21BC \\&\Rightarrow \boxed{AB : BC = 21 : 19.}\end{aligned}$$

Finally,

$$AB : BC : CD = \underline{21 : 19 : 20}.$$