# Dr Oliver Mathematics Synthetic Division

In this note, we present an alternative to the factor theorem.

1. Divide 
$$\frac{x^2 + x - 6}{x - 2}$$
.

### Solution

First, we have x-2 as the divisor and we want '2' in the top-left. Secondly, we have  $x^2+x-6$  and we want

$$1 \quad 1 \quad -6$$

(the coefficient of  $x^2$ , the coefficient of x, and the constant term) to complete the first line:

Now, drag the 1 down.

What is  $2 \times 1$ ? 2.

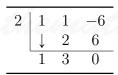
$$\begin{array}{c|cccc}
2 & 1 & 1 & -6 \\
\downarrow & 2 & \\
\hline
1 & & & \\
\end{array}$$

Add: 1 + 2 = 3.

What is  $2 \times 3$ ? 6.

2	1	1	-6
	$\downarrow$	2	6
,	1	3	

Add: (-6) + 6 = 0.



So,

are the coefficients of of my new polynomial (x + 3) and 0 is the remainder. Hence,

$$\frac{x^2 + x - 6}{x - 2} = \frac{(x + 3)(x - 2)}{x - 2}$$
$$= \underline{x + 3}.$$

2. Divide  $\frac{x^3 - 5x^2 - x + 5}{x - 1}$ .

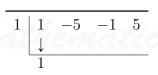
#### Solution

First, we have x-1 as the divisor and we want '1' in the top-left. Secondly, we have  $x^3-5x^2-x+5$  and we want

$$1 - 5 - 1 5$$

(the coefficient of  $x^3$ , the coefficient of  $x^2$ , the coefficient of x, and the constant term) to complete the first line:

First, drag the 1 down.



What is  $1 \times 1$ ? 1.

$$\begin{array}{c|ccccc}
1 & 1 & -5 & -1 & 5 \\
\downarrow & 1 & & & \\
\hline
1 & & & & & \\
\end{array}$$

Add the -5 + 1 = -4.

$$\begin{array}{c|ccccc}
1 & 1 & -5 & -1 & 5 \\
\downarrow & 1 & & & \\
\hline
1 & -4 & & & & \\
\end{array}$$

What is  $1 \times (-4)$ ? -4.

Add: -1 + (-4) = -5.

What is  $1 \times (-5)$ ? -5.

Add: 5 + (-5) = 0.

Hence,

$$\frac{x^3 - 5x^2 - x + 5}{x - 1} = \frac{(x - 1)(x^2 - 4x - 5)}{x - 1}$$
$$= x^2 - 4x - 5.$$

3. Divide  $\frac{2x^2 - 3x + 1}{2x - 1}$ .

#### Solution

First, we have 2x - 1 as the divisor and we want  $(x \pm a)$  so divide by 2:

$$\frac{2x^2 - 3x + 1}{2x - 1} = \frac{x^2 - \frac{3}{2}x + \frac{1}{2}}{x - \frac{1}{2}}$$

and proceed.

Now, drag the 1 down.

$$\begin{array}{c|cccc} \frac{1}{2} & 1 & -\frac{3}{2} & \frac{1}{2} \\ \downarrow & & 1 & & \end{array}$$

What is  $\frac{1}{2} \times 1$ ?  $\frac{1}{2}$ .

Add: 
$$-\frac{3}{2} + \frac{1}{2} = -1$$
.

What is  $\frac{1}{2} \times (-1)$ ?  $-\frac{1}{2}$ .

Add:  $\frac{1}{2} + \left(-\frac{1}{2}\right) = 0$ .

Hence,

$$\frac{2x^2 - 3x + 1}{2x - 1} = \underline{\underline{x - 1}}.$$

4. Divide  $\frac{6x^2 + 11x + 4}{3x + 4}$ .

## Solution

First,

$$\frac{6x^2 + 11x + 4}{3x + 4} = \frac{2x^2 + \frac{11}{3}x + \frac{4}{3}}{x + \frac{4}{3}}$$

and we divide by  $(x + \frac{4}{3}) = (x - (-\frac{4}{3}))$ .

Hence,  $\frac{6x^2 + 11x + 4}{3x + 4} = \underbrace{2x + 1}_{}.$ 

What about those questions that have multiple divisors?

5. Divide  $\frac{x^3 + 3x^2 - 10x - 24}{(x+2)(x-3)}.$ 

#### Solution

Well, we can do

$$\frac{x^3 + 3x^2 - 10x - 24}{x + 2}$$

and then divide by (x-3) or vice versa.

Hence,

$$\frac{x^3 + 3x^2 - 10x - 24}{(x+2)(x-3)} = \underline{\underline{x+4}}.$$

Alternatively:

and the answer is

$$\frac{x^3 + 3x^2 - 10x - 24}{(x+2)(x-3)} = \underline{x+4}.$$

What about those questions that have a remainder?

6. Divide  $\frac{2x^3 - 5x + 14}{x + 3}$ .

#### Solution

Now,

$$x + 3 = x - (-3)$$

and we want -3 in the top-left.

$$2x^3 - 5x + 14 = 2x^3 + 0x^2 - 5x + 14$$

and we want

$$2 \quad 0 \quad -5 \quad 14,$$

completing the first line:

First, drag the 2 down.

What is  $(-3) \times 2? -6$ .

Add: 0 + (-6) = -6.

-3	2	0	-5	14
al	1	-6		
	2	-6		

What is  $(-3) \times (-6)$ ? 18.

Add: -5 + 18 = 13.

What is  $(-3) \times 13? -39$ .

Finally, add: 14 + (-39) = -25.

Hence,