Dr Oliver Mathematics
Synthetic Division

In this note, we present an alternative to the factor theorem.

1. Divide $\dfrac{x^2+x-6}{x-2}$.

Solution

First, we have $x-2$ as the divisor and we want '2' in the top-left.
Secondly, we have x^2+x-6 and we want

$$1 \quad 1 \quad -6$$

(the coefficient of x^2, the coefficient of x, and the constant term) to complete the first line:

2	1	1	−6

Now, drag the 1 down.

2	1	1	−6
	↓		
	1		

What is 2×1? 2.

2	1	1	−6
	↓	2	
	1		

Add: $1 + 2 = 3$.

2	1	1	−6
	↓	2	
	1	3	

What is 2×3? 6.

$$\begin{array}{r|ccc} 2 & 1 & 1 & -6 \\ & \downarrow & 2 & 6 \\ \hline & 1 & 3 & \end{array}$$

Add: $(-6) + 6 = 0$.

$$\begin{array}{r|ccc} 2 & 1 & 1 & -6 \\ & \downarrow & 2 & 6 \\ \hline & 1 & 3 & 0 \end{array}$$

So,

$$1 \quad 3$$

are the coefficients of of my new polynomial $(x + 3)$ and 0 is the remainder. Hence,

$$\begin{aligned} \frac{x^2 + x - 6}{x - 2} &= \frac{(x + 3)(x - 2)}{x - 2} \\ &= \underline{\underline{x + 3}}. \end{aligned}$$

2. Divide $\dfrac{x^3 - 5x^2 - x + 5}{x - 1}$.

Solution

First, we have $x - 1$ as the divisor and we want '1' in the top-left.
Secondly, we have $x^3 - 5x^2 - x + 5$ and we want

$$1 \quad -5 \quad -1 \quad 5$$

(the coefficient of x^3, the coefficient of x^2, the coefficient of x, and the constant term) to complete the first line:

$$\begin{array}{r|cccc} 1 & 1 & -5 & -1 & 5 \\ & & & & \\ \hline & & & & \end{array}$$

First, drag the 1 down.

1	1	-5	-1	5
	$\downarrow$			
	1			

What is 1×1? 1.

1	1	-5	-1	5
	$\downarrow$	1		
	1			

Add the $-5 + 1 = -4$.

1	1	-5	-1	5
	$\downarrow$	1		
	1	-4		

What is $1 \times (-4)$? -4.

1	1	-5	-1	5
	$\downarrow$	1	-4	
	1	-4		

Add: $-1 + (-4) = -5$.

1	1	-5	-1	5
	$\downarrow$	1	-4	
	1	-4	-5	

What is $1 \times (-5)$? -5.

1	1	-5	-1	5
	$\downarrow$	1	-4	-5
	1	-4	-5	

Add: $5 + (-5) = 0$.

$$\begin{array}{r|cccc} & 1 & -5 & -1 & 5 \\ 1 & \downarrow & 1 & -4 & -5 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

Hence,

$$\begin{aligned} \frac{x^3 - 5x^2 - x + 5}{x - 1} &= \frac{(x-1)(x^2 - 4x - 5)}{x - 1} \\ &= \underline{\underline{x^2 - 4x - 5}}. \end{aligned}$$

3. Divide $\dfrac{2x^2 - 3x + 1}{2x - 1}$.

Solution

First, we have $2x - 1$ as the divisor and we want $(x \pm a)$ so divide by 2:

$$\frac{2x^2 - 3x + 1}{2x - 1} = \frac{x^2 - \frac{3}{2}x + \frac{1}{2}}{x - \frac{1}{2}}$$

and proceed.

$$\begin{array}{r|ccc} & 1 & -\frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & & & \\ \hline & & & \end{array}$$

Now, drag the 1 down.

$$\begin{array}{r|ccc} & 1 & -\frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \downarrow & & \\ \hline & 1 & & \end{array}$$

What is $\frac{1}{2} \times 1$? $\frac{1}{2}$.

$$\begin{array}{r|ccc} & 1 & -\frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \downarrow & \frac{1}{2} & \\ \hline & 1 & & \end{array}$$

Add: $-\frac{3}{2}+\frac{1}{2}=-1$.

$$\begin{array}{c|ccc} \frac{1}{2} & 1 & -\frac{3}{2} & \frac{1}{2} \\ & \downarrow & \frac{1}{2} & \\ \hline & 1 & -1 & \end{array}$$

What is $\frac{1}{2}\times(-1)$? $-\frac{1}{2}$.

$$\begin{array}{c|ccc} \frac{1}{2} & 1 & -\frac{3}{2} & \frac{1}{2} \\ & \downarrow & \frac{1}{2} & -\frac{1}{2} \\ \hline & 1 & -1 & \end{array}$$

Add: $\frac{1}{2}+(-\frac{1}{2})=0$.

$$\begin{array}{c|ccc} \frac{1}{2} & 1 & -\frac{3}{2} & \frac{1}{2} \\ & \downarrow & \frac{1}{2} & -\frac{1}{2} \\ \hline & 1 & -1 & 0 \end{array}$$

Hence,

$$\frac{2x^2-3x+1}{2x-1}=\underline{\underline{x-1}}.$$

4. Divide $\dfrac{6x^2+11x+4}{3x+4}$.

Solution

First,

$$\frac{6x^2+11x+4}{3x+4}=\frac{2x^2+\frac{11}{3}x+\frac{4}{3}}{x+\frac{4}{3}}$$

and we divide by $(x+\frac{4}{3})=(x-(-\frac{4}{3}))$.

$$\begin{array}{c|ccc} -\frac{4}{3} & 2 & \frac{11}{3} & \frac{4}{3} \\ & \downarrow & -\frac{8}{3} & -\frac{4}{3} \\ \hline & 2 & 1 & 0 \end{array}$$

Hence,

$$\frac{6x^2+11x+4}{3x+4} = \underline{\underline{2x+1}}.$$

What about those questions that have multiple divisors?

5. Divide $\dfrac{x^3+3x^2-10x-24}{(x+2)(x-3)}$.

Solution

Well, we can do

$$\frac{x^3+3x^2-10x-24}{x+2}$$

and then divide by $(x-3)$ or vice versa.

$$\begin{array}{r|cccc} -2 & 1 & 3 & -10 & -24 \\ & \downarrow & -2 & -2 & 24 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$$\begin{array}{r|ccc} 3 & 1 & 1 & -12 \\ & \downarrow & 3 & 12 \\ \hline & 1 & 4 & 0 \end{array}$$

Hence,

$$\frac{x^3+3x^2-10x-24}{(x+2)(x-3)} = \underline{\underline{x+4}}.$$

Alternatively:

$$\begin{array}{r|cccc} 3 & 1 & 3 & -10 & -24 \\ & \downarrow & 3 & 18 & 24 \\ \hline & 1 & 6 & 8 & 0 \end{array}$$

$$\begin{array}{r|ccc} -2 & 1 & 6 & 8 \\ & \downarrow & -2 & -8 \\ \hline & 1 & 4 & 0 \end{array}$$

and the answer is

$$\frac{x^3+3x^2-10x-24}{(x+2)(x-3)} = \underline{\underline{x+4}}.$$

What about those questions that have a remainder?

6. Divide $\dfrac{2x^3-5x+14}{x+3}$.

Solution

Now,

$$x+3 = x-(-3)$$

and we want -3 in the top-left.

$$2x^3-5x+14 = 2x^3+0x^2-5x+14$$

and we want

$$2 \quad 0 \quad -5 \quad 14,$$

completing the first line:

-3	2	0	-5	14

First, drag the 2 down.

-3	2	0	-5	14
	$\downarrow$			
	2			

What is $(-3)\times 2$? -6.

-3	2	0	-5	14
	$\downarrow$	-6		
	2			

Add: $0+(-6) = -6$.

-3	2	0	-5	14
	$\downarrow$	-6		
	2	-6		

What is $(-3) \times (-6)$? 18.

-3	2	0	-5	14
	$\downarrow$	-6	18	
	2	-6		

Add: $-5 + 18 = 13$.

-3	2	0	-5	14
	$\downarrow$	-6	18	
	2	-6	13	

What is $(-3) \times 13$? -39.

-3	2	0	-5	14
	$\downarrow$	-6	18	-39
	2	-6	13	

Finally, add: $14 + (-39) = -25$.

-3	2	0	-5	14
	$\downarrow$	-6	18	-39
	2	-6	13	-25

Hence,

$$\frac{2x^3 - 5x + 14}{x + 3} = \underline{\underline{2x^2 - 6x + 13 - \frac{25}{x + 3}}}.$$