Dr Oliver Mathematics
Advanced Subsidiary Paper 22: Mechanics
June 2022: Calculator
1 hour 15 minutes

The total number of marks available is 30.
You must write down all the stages in your working.
Inexact answers should be given to three significant figures unless otherwise stated.
(It goes with Paper 21: Statistics)

1. The point A is 1.8 m vertically above horizontal ground.

   At time $t = 0$, a small stone is projected vertically upwards with speed U ms^{-1} from the point A.

   At time $t = T$ seconds, the stone hits the ground.

   The speed of the stone as it hits the ground is 10 ms^{-1}.

   In an initial model of the motion of the stone as it moves from A to where it hits the ground,

   - the stone is modelled as a particle moving freely under gravity and
   - the acceleration due to gravity is modelled as having magnitude 10 ms^{-2}.

   Using the model,

   (a) find the value of U, (3)

   Solution
   ($\uparrow$) $s = 1.8$, $u = U$, $v = -10$, $a = -10$, $t = T$: we use $v^2 = u^2 + 2as$:

   $$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow (-10)^2 = U^2 + 2(-10)(1.8) \\ &\Rightarrow 100 = U^2 - 36 \\ &\Rightarrow U^2 = 64 \\ &\Rightarrow \underline{\underline{U = 8}}. \end{aligned}$$

   (b) find the value of T. (2)

Solution

We use $v = u + at$:

$$\begin{aligned} v = u + at &\Rightarrow -10 = 8 - 10T \\ &\Rightarrow 10T = 18 \\ &\Rightarrow \underline{\underline{T = 1.8}}. \end{aligned}$$

(c) Suggest one refinement, apart from including air resistance, that would make the model more realistic. (1)

Solution

E.g., spin on the stone, allow for wind effects, use a more accurate value for gravity.

In reality the stone will not move freely under gravity and will be subject to air resistance.

(d) Explain how this would affect your answer to part (a). (1)

Solution

E.g., it will make it larger.

2. A train travels along a straight horizontal track from station P to station Q.

In a model of the motion of the train, at time $t = 0$ the train starts from rest at P, and moves with constant acceleration until it reaches its maximum speed of 25 ms^{-1}.

The train then travels at this constant speed of 25 ms^{-1} before finally moving with constant deceleration until it comes to rest at Q.

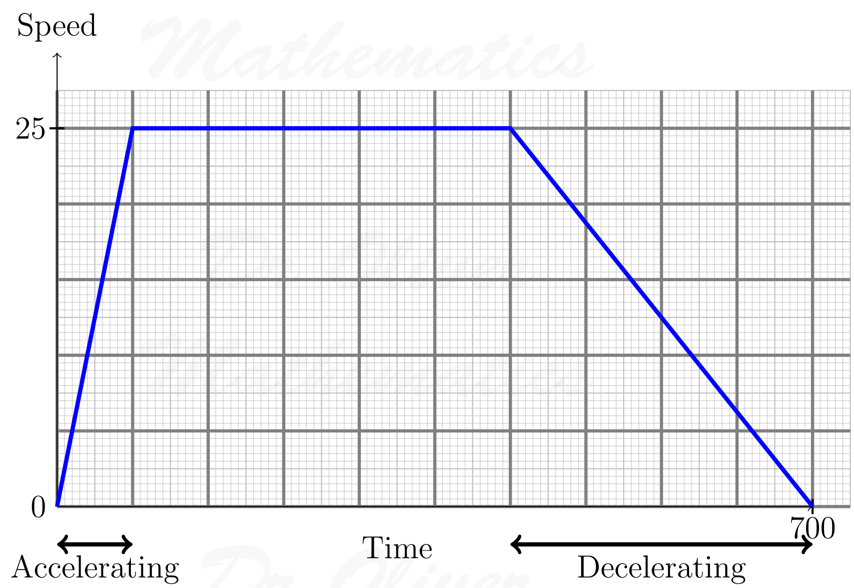
The time spent decelerating is four times the time spent accelerating.

The journey from P to Q takes 700 s.

Using the model,

(a) sketch a speed-time graph for the motion of the train between the two stations P and Q. (1)

Solution



The distance between the two stations is 15 km.

Using the model,

(b) show that the time spent accelerating by the train is 40 s, (3)

Solution

Let the period of acceleration be T seconds. Then the period of deceleration will be $4T$ seconds and the period of constant velocity is

$$700 - T - 4T = (700 - 5T) \text{ seconds.}$$

The distance between the two stations is

$$15 \text{ km} = 15\,000 \text{ m}$$

and the area is the distance travelled.

$$\begin{aligned} 15\,000 = \tfrac{1}{2}[700 + (700 - 5T)](25) &\Rightarrow 15\,000 = \tfrac{1}{2}(1\,400 - 5T)(25) \\ &\Rightarrow 30\,000 = (1\,400 - 5T)(25) \\ &\Rightarrow 1\,200 = 1\,400 - 5T \\ &\Rightarrow 5T = 200 \\ &\Rightarrow \underline{\underline{T = 40}}, \end{aligned}$$

as required.

(c) find the acceleration, in ms^{-2}, of the train, (1)

Solution

$$\text{Acceleration} = \frac{25}{40} = \underline{\underline{0.625 \text{ ms}^{-2}}}.$$

(d) find the speed of the train 572 s after leaving P. (2)

Solution

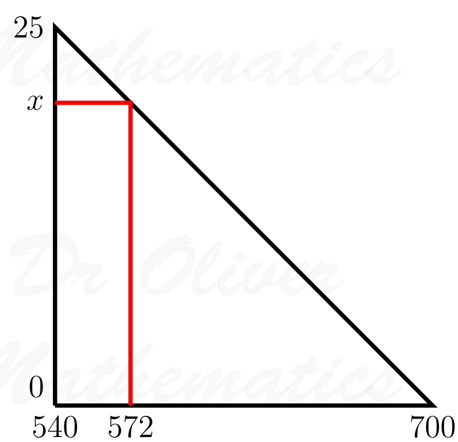
Well,

$$40 + 500 = 540$$

so the train is slowing down and we want

$$572 - 540 = 32 \text{ s}$$

after that:



Use similar triangles:

$$\frac{x}{700-572} = \frac{25}{700-540} \Rightarrow \frac{x}{128} = \frac{25}{160}$$
$$\Rightarrow x = \frac{128 \times 25}{160}$$
$$\Rightarrow x = 20;$$

hence, the speed of the train is $\underline{\underline{20 \text{ ms}^{-1}}}$.

(e) State one limitation of the model which could affect your answers to parts (b) and (c). (1)

Solution
E.g., the train cannot instantaneously change acceleration, the train will not move with constant acceleration, the train will not move with constant speed.

3. A fixed point O lies on a straight line.

A particle P moves along the straight line.

At time t seconds, $t \geqslant 0$, the distance, s metres, of P from O is given by

$$s = \tfrac{1}{3}t^3 - \tfrac{5}{2}t^2 + 6t.$$

(a) Find the acceleration of P at each of the times when P is at instantaneous rest. (6)

Solution
Well,

$$\begin{aligned} s = \tfrac{1}{3}t^3 - \tfrac{5}{2}t^2 + 6t &\Rightarrow v = t^2 - 5t + 6 \\ &\Rightarrow a = 2t - 5. \end{aligned}$$

Now,

$$v = 0 \Rightarrow t^2 - 5t + 6 = 0$$

$$\left.\begin{array}{ll} \text{add to:} & -5 \\ \text{multiply to:} & +6 \end{array}\right\} -2, -3$$

$$\begin{aligned} &\Rightarrow (t-2)(t-3) = 0 \\ &\Rightarrow t - 2 = 0 \text{ or } t - 3 = 0 \\ &\Rightarrow t = 2 \text{ or } t = 3. \end{aligned}$$

Now,

$$t = 2 \Rightarrow a = 2(2) - 5 = \underline{\underline{\underline{-1 \text{ ms}^{-2}}}}$$

and

$$t = 3 \Rightarrow a = 2(3) - 5 = \underline{\underline{\underline{1 \text{ ms}^{-2}}}}.$$

(b) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$. (3)

Solution

We need three separate times: $0 \leqslant s \leqslant 2$, $2 \leqslant s \leqslant 3$, and $3 \leqslant s \leqslant 4$.

$\underline{0 \leqslant s \leqslant 2}$:

$$\begin{aligned}\left[\tfrac{1}{3}t^3 - \tfrac{5}{2}t^2 + 6t\right]_{t=0}^{2} &= (\tfrac{8}{3} - 10 + 12) - (0 - 0 + 0) \\ &= 4\tfrac{2}{3}.\end{aligned}$$

$\underline{2 \leqslant s \leqslant 3}$:

$$\begin{aligned}\left[\tfrac{1}{3}t^3 - \tfrac{5}{2}t^2 + 6t\right]_{t=2}^{3} &= (9 - \tfrac{45}{2} + 18) - (\tfrac{8}{3} - 10 + 12) \\ &= 4\tfrac{1}{2} - 4\tfrac{2}{3} \\ &= -\tfrac{1}{6}.\end{aligned}$$

$\underline{3 \leqslant s \leqslant 4}$:

$$\begin{aligned}\left[\tfrac{1}{3}t^3 - \tfrac{5}{2}t^2 + 6t\right]_{t=3}^{4} &= (\tfrac{64}{3} - 40 + 24) - (9 - \tfrac{45}{2} + 18) \\ &= 5\tfrac{1}{3}|4\tfrac{1}{2} \\ &= \tfrac{5}{6}.\end{aligned}$$

Hence,

$$\begin{aligned}\text{distance travelled} &= |4\tfrac{2}{3}| + |-\tfrac{1}{6}| + |\tfrac{5}{6}| \\ &= 4\tfrac{2}{3} + \tfrac{1}{6} + \tfrac{5}{6} \\ &= \underline{\underline{5\tfrac{2}{3}\text{ m}}}.\end{aligned}$$

4. In Figure 1, a vertical rope PQ has its end Q attached to the top of a small lift cage.

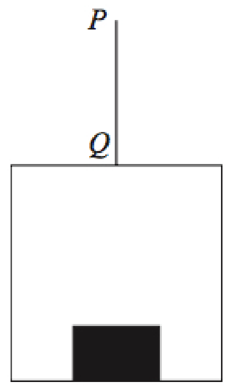


Figure 1: a vertical rope PQ

The lift cage has mass 40 kg and carries a block of mass 10 kg, as shown in the figure.

The lift cage is raised vertically by moving the end P of the rope vertically upwards with constant acceleration 0.2 ms^{-2}.

The rope is modelled as being light and inextensible and air resistance is ignored.

Using the model,

(a) find the tension in the rope PQ, (3)

Solution

Let the tension in PQ be T N.
Then $F = ma$:

$$\begin{aligned} T - (40g + 10g) = 50 \times 0.2 &\Rightarrow T - 50g = 10 \\ &\Rightarrow T = 50g + 10 \\ &\Rightarrow \underline{\underline{T = 500 \text{ N}}}. \end{aligned}$$

(b) find the magnitude of the force exerted on the block by the lift cage. (3)

Solution

Let the force exerted on the block by the lift cage be S N.

Then $F = ma$:

$$\begin{aligned} S - 10g = 10 \times 0.2 &\Rightarrow S - 10g = 2 \\ &\Rightarrow S = 10g + 2 \\ &\Rightarrow \underline{\underline{S = 100 \text{ N}}}. \end{aligned}$$