Dr Oliver Mathematics
Further Mathematics
Rational Inequalities
Past Examination Questions

This booklet consists of 20 questions across a variety of examination topics.
The total number of marks available is 162.

1. (a) Sketch the graph of $y = |x - 2a|$, given that $a > 0$. (2)

   (b) Solve $|x - 2a| > 2x + a$, where $a > 0$. (3)

2. (a) On the same diagram, sketch the curve with equation $y = |x^2 - 4|$ and the line with equation $y = |2x - 1|$, showing the coordinates of the points where the curve meets the axes. (4)

   (b) Solve $|x^2 - 4| = |2x - 1|$, giving your answer in surd form as appropriate. (5)

   (c) Hence, or otherwise, find the set of values of x for which $|x^2 - 4| > |2x - 1|$. (3)

3. (a) Use algebra to find the exact solutions of the equation (6)

$$|2x^2 + x - 6| = 6 - 3x.$$

   (b) On the same diagram, sketch the curve with equation $y = |2x^2 + x - 6|$ and the line with equation $y = 6 - 3x$. (3)

   (c) Find the set of values of x for which (3)

$$|2x^2 + x - 6| > 6 - 3x.$$

4. Figure 1 shows a sketch of the curve with equation

$$y = \frac{x^2 - 1}{|x + 2|}, \ x \neq -2.$$

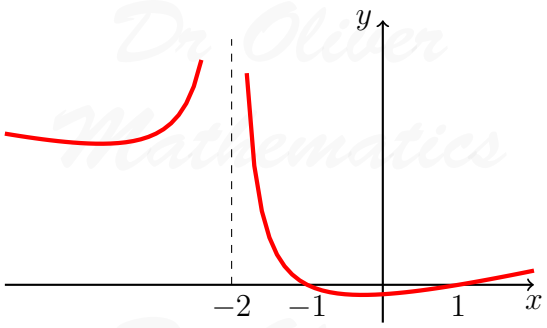

Figure 1: $y = \dfrac{x^2 - 1}{|x + 2|}$

The curve crosses the x-axis at $x = 1$ and $x = -1$ and the line $x = -2$ is an asymptote of the curve.

(a) Use algebra to solve the equation (6)

$$\frac{x^2 - 1}{|x + 2|} = 3(1 - x).$$

(b) Hence, or otherwise, find the set of values of x for which (3)

$$\frac{x^2 - 1}{|x + 2|} < 3(1 - x).$$

5. Find the set of values of x for which (7)

$$\frac{x + 1}{2x - 3} < \frac{1}{x - 3}.$$

6. (a) Simplify the expression (4)

$$\frac{(x + 3)(x + 9)}{x - 1} - (3x - 5),$$

giving your answer in the form

$$\frac{a(x + b)(x + c)}{x - 1},$$

where a, b, c are integers.

(b) Hence, or otherwise, solve the inequality (4)

$$\frac{(x + 3)(x + 9)}{x - 1} > (3x - 5).$$

7. (a) Find, in the simplest surd form where appropriate, the exact values of x for which (5)

$$\frac{x}{2} + 3 = \left|\frac{4}{x}\right|.$$

(b) Sketch, on the same axes, the line with equation $y = \frac{x}{2} + 3$ and the graph of $y = \left|\frac{4}{x}\right|$, $x \neq 0$. (3)

(c) Find the set of values of x for which (2)

$$\frac{x}{2} + 3 > \left|\frac{4}{x}\right|.$$

8. Find the set of values of x for which (6)

$$\frac{x^3 + 5x - 12}{x - 3} > 4.$$

9. (a) On the same diagram, sketch the graph of $y = x + 2$ and the graph of $y = \left|\frac{1}{x-2}\right|$. (6)
Indicate on your sketch the coordinates of any points at which the graphs cross the axes, and state the equations of any asymptotes.

(b) Find the set of values of x for which (6)

$$x + 2 < \left|\frac{1}{x - 2}\right|.$$

10. Figure 2 shows the graph of $y = 10 + 3x - x^2$ and the graph of $y = |3x - 1|$.

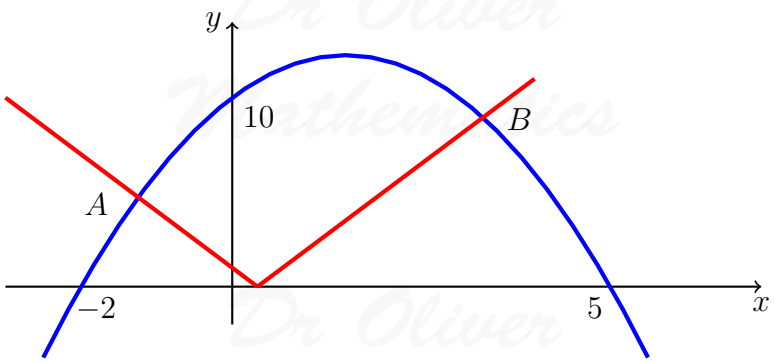

Figure 2: $y = 10 + 3x - x^2$ and $y = |3x - 1|$

The graphs intersect at the points A and B.

(a) Use algebra to find the exact x-coordinates of A and B. (5)

(b) Find the set of values of x for which (2)

$$10 + 3x - x^2 > |3x - 1|.$$

(c) Find the set of values of x for which $|10 + 3x - x^2| < |3x - 1|$. (3)

11. (a) Find the set of values of x for which (6)

$$x + 4 > \frac{2}{x + 3}.$$

(b) Deduce, or otherwise find, the set of values of x for which (1)

$$x + 4 > \frac{2}{|x + 3|}.$$

12. Find the set of values of x for which (7)

$$\frac{3}{x + 3} > \frac{x - 4}{x}.$$

13. Find the set of values of x for which (5)

$$\left|x^2 - 4\right| > 3x.$$

14. Use algebra to find the set of values for which (7)

$$\frac{6x}{3 - x} > \frac{1}{x + 1}.$$

15. (a) Use algebra to find the exact solutions of the equation (6)

$$\left|2x^2 + 6x - 5\right| = 5 - 2x.$$

(b) On the same diagram, sketch the curve with equation $y = |2x^2 + 6x - 5|$ and the line with equation $y = 5 - 2x$, showing the x-coordinate of the points where the line crosses the curve. (3)

(c) Find the set of values of x for which (3)

$$\left|2x^2 + 6x - 5\right| > 5 - 2x.$$

16. Using algebra, find the set of values of x for which (5)

$$3x - 5 < \frac{2}{x}.$$

17. Use algebra to find the set of values of x for which (6)

$$\left|3x^2 - 19x + 20\right| < 2x + 2.$$

18. (a) Use algebra to find the set of values of x for which (6)

$$x + 2 > \frac{12}{x + 3}.$$

	$x < -2$	$-2 < x < -\sqrt{2}$	$-\sqrt{2} < x < -1$	$-1 < x < \sqrt{2}$	$x > \sqrt{2}$
$x + 2$	$-$	$+$	$+$	$+$	$+$
$x + \sqrt{2}$	$-$	$-$	$+$	$+$	$+$
$x + 1$	$-$	$-$	$-$	$+$	$+$
$x - \sqrt{2}$	$-$	$-$	$-$	$-$	$+$
$\dfrac{x^2 - 2}{(x+1)(x+2)}$	$+$	$-$	$+$	$-$	$+$

Table 1: $\dfrac{x^2 - 2}{(x+1)(x+2)}$

(b) Hence, or otherwise, find the set of values of x for which (1)

$$x + 2 > \frac{12}{|x + 3|}.$$

19. Use algebra to find the set of values of x for which (6)

$$\frac{x}{x + 1} < \frac{2}{x + 2}.$$

20. Use algebra to find the set of values of x for which (9)

$$\frac{x - 2}{2(x + 2)} \leqslant \frac{12}{x(x + 2)}.$$