# Dr Oliver Mathematics
# Applied Mathematics: Mechanics or Statistics
# Section B
# 2005 Paper
# 1 hour

The total number of marks available is 32.
You must write down all the stages in your working.

1. Differentiate, and simplify as appropriate,
   (a) $\mathrm{f}(x) = \exp(\tan \frac{1}{2}x)$, where $-\pi < x < \pi$, (3)
   (b) $\mathrm{g}(x) = (x^3 + 1)\ln(x^3 + 1)$, where $x > 0$. (3)

2. Given that (3)
$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix},$$
   show that
$$\mathbf{A}^2 - \mathbf{A} = k\mathbf{I}$$
   for a suitable value of $k$, where $\mathbf{I}$ is the $2 \times 2$ unit matrix.

3. A curve is defined by the parametric equations
$$x = 5t^2 - 5 \text{ and } y = 3t^3.$$
   (a) Find the value of $t$ corresponding to the point $(0, -3)$. (2)
   (b) Calculate the gradient of the curve at this point. (3)

4. Expand and simplify (3)
$$\left(2a - \frac{3}{a}\right)^4.$$

5. (a) Express (3)
$$\frac{x^2 + 3}{x(1 + x^2)}$$
   in partial fractions.
   (b) Hence obtain (3)
$$\int_{\frac{1}{2}}^{1} \frac{x^2 + 3}{x(1 + x^2)}\,\mathrm{d}x.$$

6. (a) Given the differential equation (4)

$$\sin x \frac{\mathrm{d}y}{\mathrm{d}x} - 2y \cos x = 0,$$

find the general solution, expressing $y$ explicitly in terms of $x$.

(b) Find the general solution of (5)

$$\sin x \frac{\mathrm{d}y}{\mathrm{d}x} - 2y \cos x = 3 \sin^3 x.$$