Dr Oliver Mathematics
AQA Further Maths Level 2
June 2015 Paper 2
2 hours

The total number of marks available is 105.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.

1. A circle, centre $(0, 0)$, has circumference 12π. (2)

   Work out the equation of the circle.

Solution

The r be the radius. Then

$$2\pi r = 12\pi \Rightarrow r = 6$$

and the equation of the circle is

$$(x-0)^2 + (y-0)^2 = 6^2 \Rightarrow \underline{\underline{x^2 + y^2 = 36}}.$$

2. (2)

$$a : b : c = 5 : 3 : 2.$$

   Work out

$$4a - c : 3b.$$

   Give your answer in its simplest form.

Solution

$$\begin{aligned} 4a - c : 3b &\Rightarrow (4 \times 5) - 2 : 3 \times 3 \\ &\Rightarrow 20 - 2 : 9 \\ &\Rightarrow 18 : 9 \\ &\Rightarrow \underline{\underline{2 : 1}}. \end{aligned}$$

3. The distance between the points $(2, 5p)$ and $(2, -10)$ is 30 units. (3)

Work out the **two** possible values of p.

Solution

$$\begin{aligned}(2-2)^2 + [5p - (-10)]^2 = 30^2 &\Rightarrow (5p+10)^2 = 30^2 \\ &\Rightarrow 5p + 10 = \pm 30 \\ &\Rightarrow 5p = -10 \pm 30 \\ &\Rightarrow p = -2 \pm 6 \\ &\Rightarrow \underline{\underline{p = -8 \text{ or } p = 4}}.\end{aligned}$$

4. The first term of a sequence is $1 - a$.

The term-to-term rule of a sequence is

add $2a$ then multiply by 3.

(a) Show that the second term is $3 + 3a$. (1)

Solution

$$\begin{aligned}\text{2nd term} &= 3[(1-a) + 2a] \\ &= 3(1+a) \\ &= \underline{\underline{3 + 3a}},\end{aligned}$$

as required.

The third term is 16.

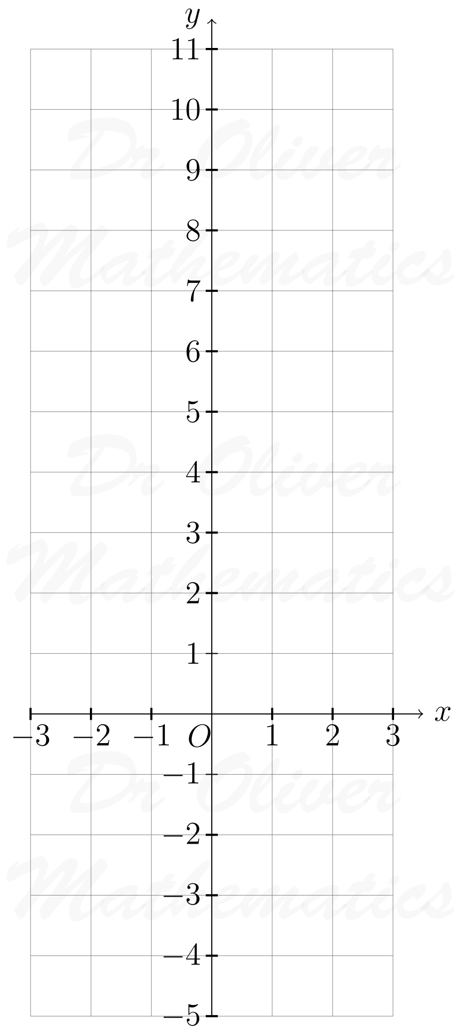
(b) Work out the value of a. (3)

Solution

$$\begin{aligned}\text{3rd term} = 16 &\Rightarrow 3[(3+3a) + 2a] = 16 \\ &\Rightarrow 3(3 + 5a) = 16 \\ &\Rightarrow 9 + 15a = 16 \\ &\Rightarrow 15a = 7 \\ &\Rightarrow \underline{\underline{a = \tfrac{7}{15}}}.\end{aligned}$$

5. A straight line L is parallel to the straight line $y = 1 - 2x$ and passes through $(3, -1)$. (4)

On the grid below, draw the straight line L for values of x from -3 to 3.



Solution

Well, the equation of the line is

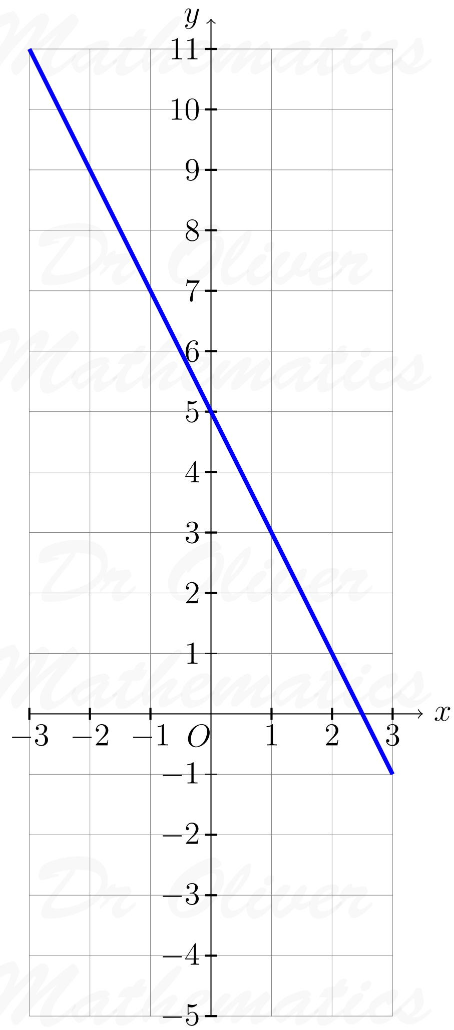
$$y = -2x + c,$$

for some c. Now,

$$-1 = -2(3) + c \Rightarrow c = 5$$

and the equation is

$$y = -2x + 5.$$



6. Write (2)

$$\frac{15x^8 - 18x^7}{3x^2}$$

in the form $ax^n - nx^a$, where a and n are integers.

Solution

$$\begin{aligned}\frac{15x^8 - 18x^7}{3x^2} &= \frac{15x^8}{3x^2} - 18x^7 3x^2 \\ &= \underline{\underline{5x^6 - 6x^5}};\end{aligned}$$

hence, $\underline{\underline{a = 6}}$ and $\underline{\underline{n = 5}}$.

7. (3)

$$y = \tfrac{2}{3}x^6 - 8x^3.$$

Work out the rate of change of y with respect to x when $x = -1$.

Solution

$$y = \tfrac{2}{3}x^6 - 8x^3 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4x^5 - 24x^2$$

and

$$\begin{aligned} x = -1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= 4[(-1)^5] - 24[(-1)^2] \\ \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= -4 - 24 \\ \Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = -28}}. \end{aligned}$$

8.

$$\mathrm{f}(x) = x^4.$$

The domain of $\mathrm{f}(x)$ is $x \geqslant 2$

(a) Work out the range of $\mathrm{f}(x)$. (1)

Solution

The range of $\mathrm{f}(x)$ is

$$\mathrm{f}(x) \geqslant 2^4 \Rightarrow \underline{\underline{\mathrm{f}(x) \geqslant 16}}.$$

$$\mathrm{g}(x) = x^2 - 1.$$

The domain of $\mathrm{g}(x)$ is $-2 \leqslant x \leqslant 3$.

(b) Work out the range of $\mathrm{g}(x)$. (2)

Solution

$\mathrm{g}(-2) = 3$, $\mathrm{g}(0) = -1$, and $\mathrm{g}(3) = 8$ and so

$$\underline{\underline{-1 \leqslant \mathrm{g}(x) \leqslant 8}}.$$

$$h(x) = 5x - 3.$$

The **range** of h(x) is $-2 < x < 1$.

(c) Work out the domain of h(x). (2)

Solution

$$\begin{aligned} 5x - 3 = -2 &\Rightarrow 5x = 1 \\ &\Rightarrow x = \tfrac{1}{5} \end{aligned}$$

and

$$\begin{aligned} 5x - 3 = 1 &\Rightarrow 5x = 4 \\ &\Rightarrow x = \tfrac{4}{5}. \end{aligned}$$

Hence, the range of h(x) is

$$\underline{\underline{\tfrac{1}{5} < x < \tfrac{4}{5}}}.$$

9. (a) Solve (3)

$$6(2y - 3) - 10 = 2y.$$

Solution

$$\begin{aligned} 6(2y - 3) - 10 = 2y &\Rightarrow (12y - 18) - 10 = 2y \\ &\Rightarrow 10y = 28 \\ &\Rightarrow \underline{\underline{y = 2.8}}. \end{aligned}$$

(b) Solve (3)

$$\frac{\sqrt{w + 4}}{2} = 6.$$

Solution

$$\begin{aligned} \frac{\sqrt{w + 4}}{2} = 6 &\Rightarrow \sqrt{w + 4} = 12 \\ &\Rightarrow w + 4 = 144 \\ &\Rightarrow \underline{\underline{w = 140}}. \end{aligned}$$

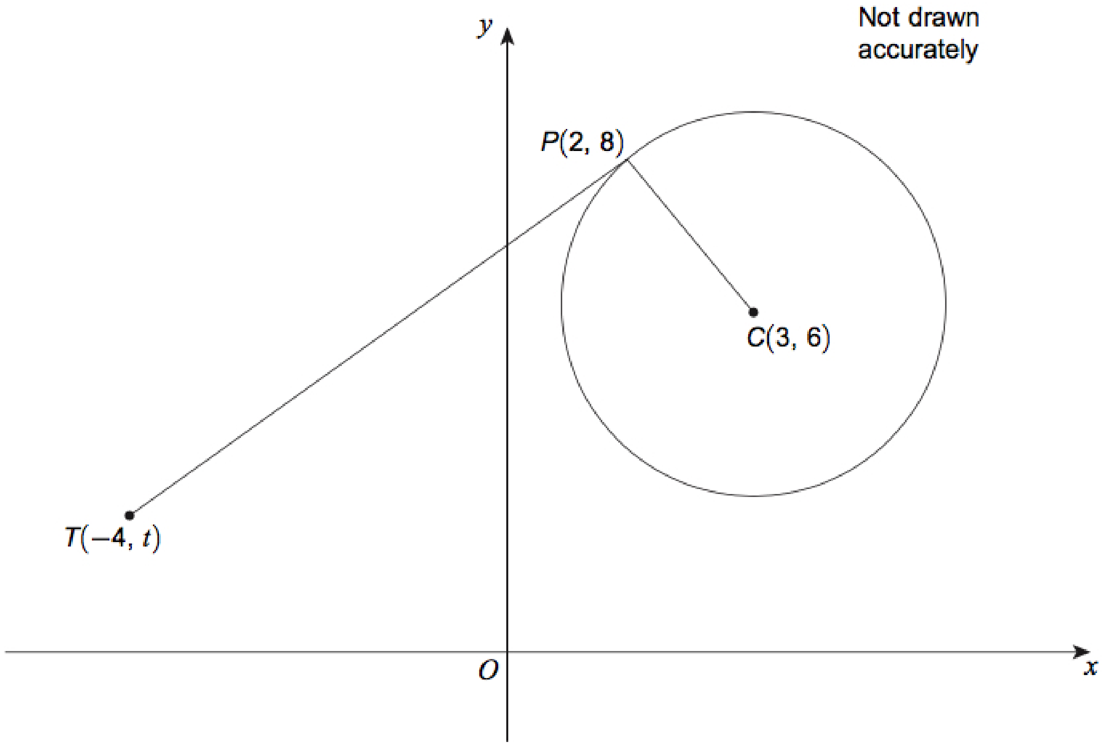
(c) Solve (2)

$$3m^{\frac{1}{5}} + 9 = 0.$$

Solution

$$\begin{aligned} 3m^{\frac{1}{5}} + 9 = 0 \Rightarrow 3m^{\frac{1}{5}} &= -9 \\ \Rightarrow m^{\frac{1}{5}} &= -3 \\ \Rightarrow m &= (-3)^5 \\ \Rightarrow \underline{\underline{m = -243}}. \end{aligned}$$

10. The diagram shows a circle, centre C. (4)
TP is a tangent to the circle at P.



Work out the value of t.

Solution

On account that is a radius,

$$[(3-2)^2+(6-8)^2]+[(2-(-4))^2+(8-t)^2]=(3-(-4))^2+(6-t)^2$$

$\times$	8	$-x$
8	64	$-8x$
$-x$	$-8x$	$+x^2$

$\times$	6	$-x$
6	36	$-6x$
$-x$	$-6x$	$+x^2$

$$\begin{aligned}
&\Rightarrow \quad (1+4)+[36+(64-16t+t^2)]=49+(36-12t+t^2)\\
&\Rightarrow \quad 105-16t+t^2=85-12t+t^2\\
&\Rightarrow \quad 20=4t\\
&\Rightarrow \quad \underline{\underline{t=5}}.
\end{aligned}$$

11. (a) Expand and simplify (3)

$$(3w+2y)(w-4y).$$

Solution

$\times$	$3w$	$+2y$
w	$3w^2$	$+2wy$
$-4y$	$-12wy$	$-8y^2$

$$(3w+2y)(w-4y)=\underline{\underline{3w^2-10wy-8y^2}}.$$

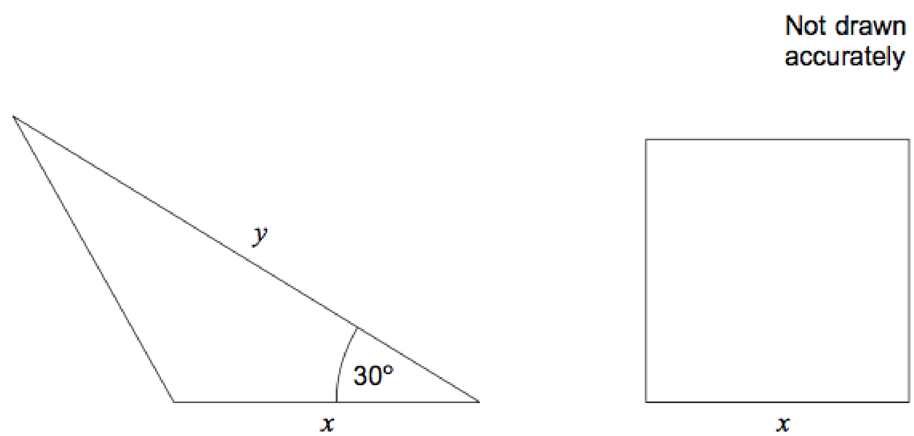
(b) Expand and simplify (3)

$$\frac{3}{x^2}\left(\frac{x}{3} + 3x^2 - 1\right).$$

Solution

$$\frac{3}{x^2}\left(\frac{x}{3} + 3x^2 - 1\right) = \underline{\underline{\frac{1}{x} + 9 - \frac{3}{x^2}}}.$$

12. The area of the triangle is equal to the area of the square. (2)
All dimensions are in centimetres.



Write y in terms of x.

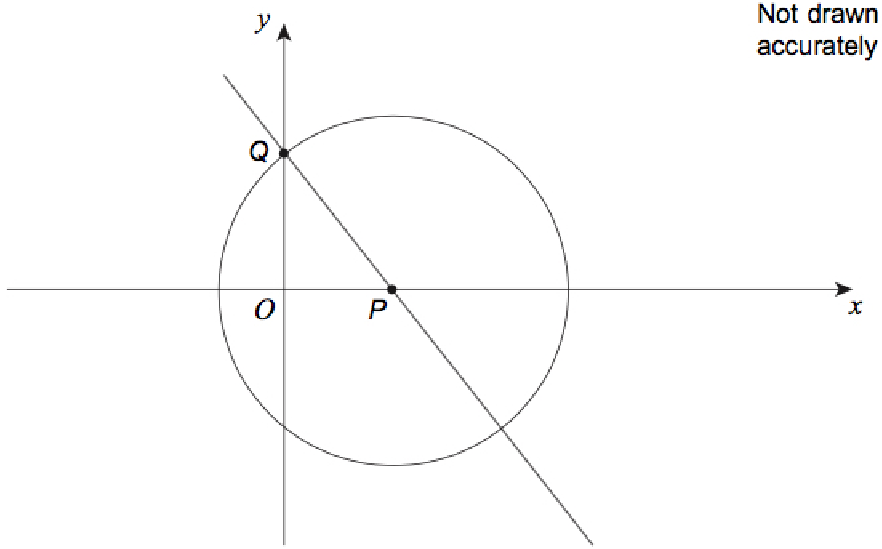
Solution

As the areas are equal,

$$\tfrac{1}{2}xy\sin 30^\circ = x^2 \Rightarrow y = \frac{x^2}{\tfrac{1}{2}x\sin 30^\circ}$$
$$\Rightarrow \underline{\underline{y = 4x}}.$$

13. The diagram shows a circle, centre P, and a straight line passing through points P and Q. (4)
Q lies on the y-axis and on the circumference of the circle.
The equation of the circle is

$$(x-3)^2 + y^2 = 25.$$



Work out the equation of the straight line through P and Q.
Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers.

Solution

Well, $P(3, 0)$ and

$$\begin{aligned} x = 0 &\Rightarrow (0-3)^2 + y^2 = 25 \\ &\Rightarrow 9 + y^2 = 25 \\ &\Rightarrow y^2 = 16 \\ &\Rightarrow y = \pm 4; \end{aligned}$$

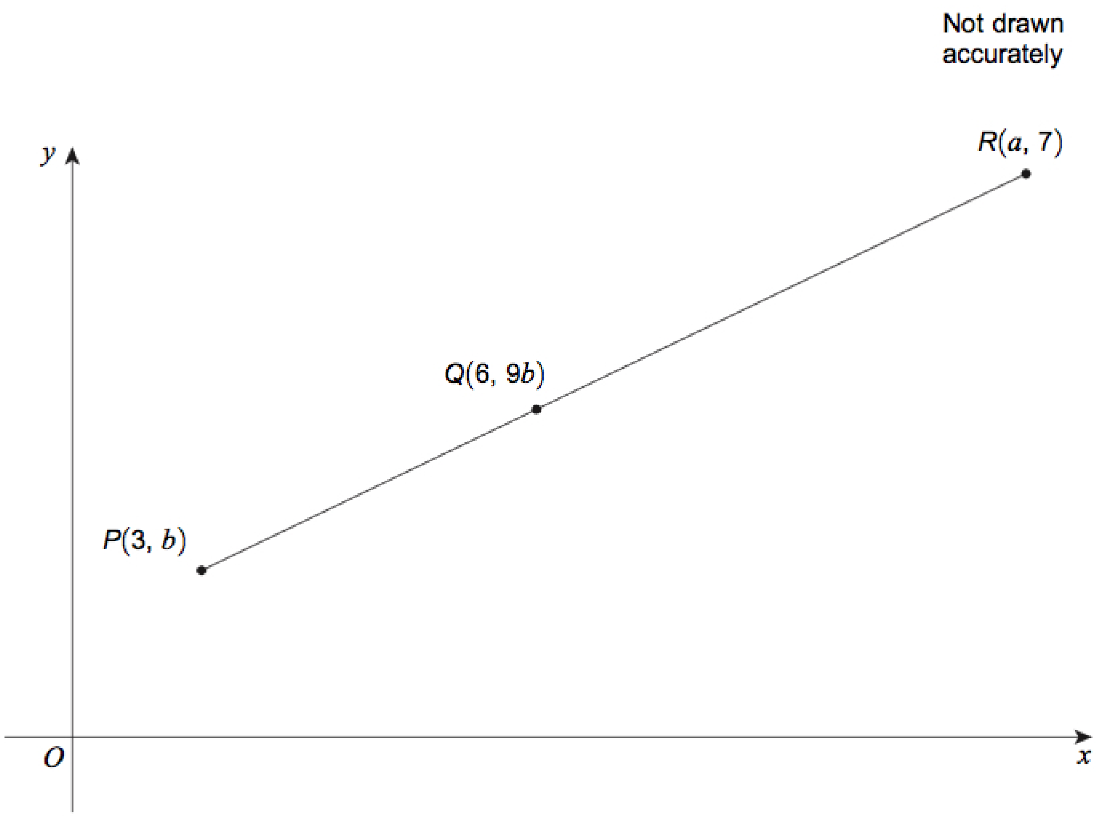
as Q is above the axis, $Q(0, 4)$. Now, the gradient of the line PQ is

$$\frac{0-4}{3-0} = -\tfrac{4}{3}$$

and the equation is

$$\begin{aligned} y - 0 = -\tfrac{4}{3}(x-3) &\Rightarrow 3y = -4(x-3) \\ &\Rightarrow 3y = -4x + 12 \\ &\Rightarrow \underline{\underline{4x + 3y - 12 = 0}}. \end{aligned}$$

14. PQR is a straight line.
$PQ : QR$ is $2 : 3$.



(a) Show that $a = 10 : 5$. (2)

Solution

$$
\begin{aligned}
PQ : QR = 2 : 3 &\Rightarrow PQ : PR = 2 : 5 \\
&\Rightarrow 6 - 3 : a - 3 = 2 : 5 \\
&\Rightarrow 3 : a - 3 = 2 : 5 \\
&\Rightarrow \frac{a-3}{3} = \frac{5}{2} \\
&\Rightarrow a - 3 = 7.5 \\
&\Rightarrow \underline{\underline{a = 10.5}},
\end{aligned}
$$

as required.

(b) Work out the value of b. (3)

Solution

$$\begin{aligned} PQ : PR = 2 : 5 &\Rightarrow 9b - b : 7 - b = 2 : 5 \\ &\Rightarrow 8b : 7 - b = 2 : 5 \\ &\Rightarrow \frac{7-b}{8b} = \frac{5}{2} \\ &\Rightarrow 7 - b = 20b \\ &\Rightarrow 21b = 7 \\ &\Rightarrow \underline{\underline{b = \tfrac{1}{3}}}. \end{aligned}$$

15. Use algebra to prove that the value of (3)

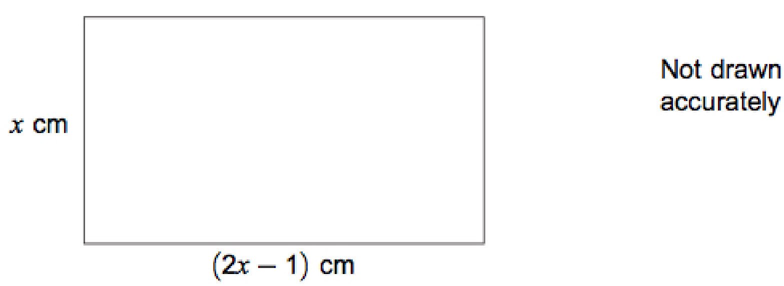
$$\frac{8c^2+16}{3c^2+6} + \frac{1}{3}$$

is an integer for all values of c.

Solution

$$\begin{aligned} \frac{8c^2+16}{3c^2+6} + \frac{1}{3} &= \frac{3(8c^2+16)}{3(3c^2+6)} + \frac{3c^2+6}{3(3c^2+6)} \\ &= \frac{(24c^2+48)+(3c^2+6)}{3(3c^2+6)} \\ &= \frac{27c^2+54}{9(c^2+2)} \\ &= \frac{27(c^2+2)}{9(c^2+2)} \\ &= \underline{\underline{3}}. \end{aligned}$$

16. The diagram shows a rectangle with area 9 cm^2. (5)



Set up and solve an equation to work out the value of x.
Give your answer to 3 significant figures.

Solution

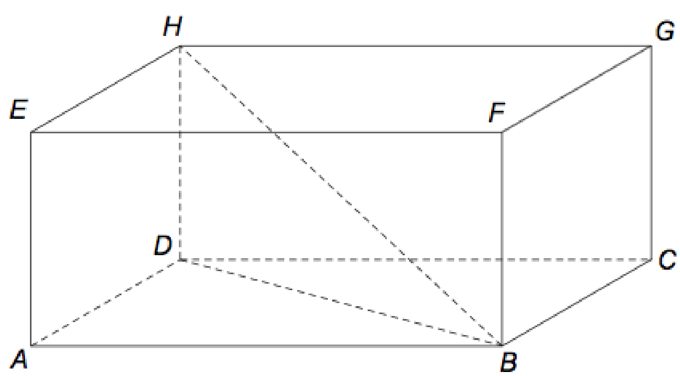
$$x(2x-1)=9 \Rightarrow 2x^2-x=9$$
$$\Rightarrow 2x^2-x-9=0$$

$a=2$, $b=-1$, and $c=-9$

$$\Rightarrow x=\frac{1\pm\sqrt{(-1)^2-4(2)(-9)}}{2(2)}$$
$$\Rightarrow x=\frac{1\pm\sqrt{73}}{4}$$
$$\Rightarrow x=-1.886\,000\,936,\ 2.386\,000\,936\ \text{(FCD)}.$$

Now, $x > 0.5$ (why?) so $\underline{\underline{x = 2.39\ \text{(FCD)}}}$.

17. $ABCDEFGH$ is a cuboid.



$HB = 34$ cm.
$HD = 16$ cm.
$AD = 18$ cm.

(a) Work out the length of AB. (3)

Solution

$$HB^2 = AB^2 + AD^2 + HD^2 \Rightarrow 34^2 = AB^2 + 18^2 + 16^2$$
$$\Rightarrow AB^2 = 576$$
$$\Rightarrow \underline{\underline{AB = 24 \text{ cm}}}.$$

(b) Work out the angle between HB and $ABCD$. (2)

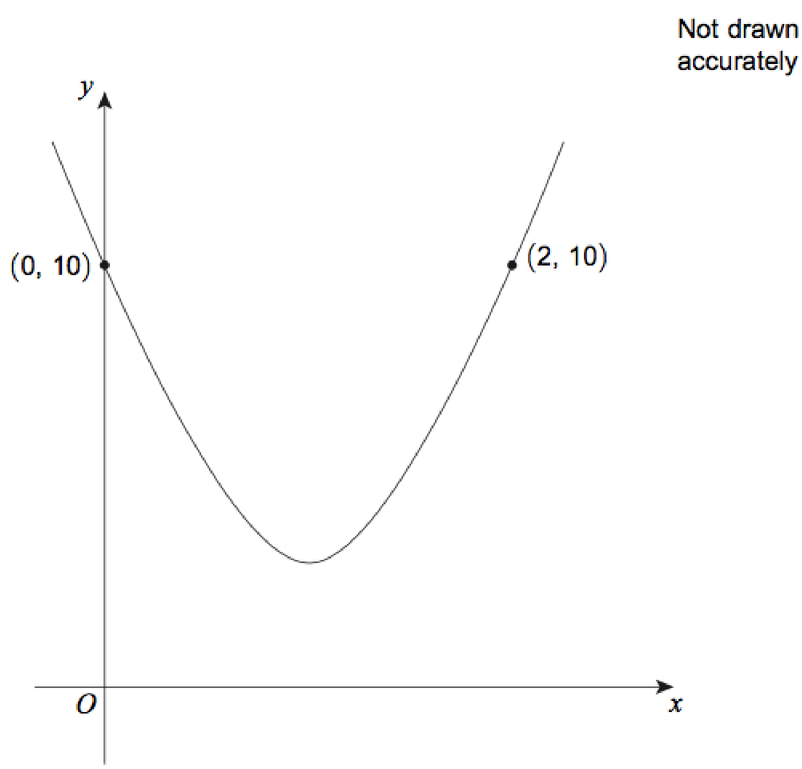
Solution

$$\sin = \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin \text{angle} = \tfrac{16}{34}$$
$$\Rightarrow \text{angle} = 28.072\,486\,94 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\text{angle} = 28.1^\circ \text{ (3 sf)}}}.$$

18. The sketch shows the quadratic curve

$$y = 4(x - a)^2 + b.$$

The curve passes through $(0, 10)$ and $(2, 10)$.



(a) Give reasons why the value of a is 1. (2)

Solution
Draw in the line of symmetry and that is

$$x = \frac{0+2}{2} = 1.$$

Hence, $\underline{\underline{a = 1}}$.

(b) Work out the value of b. (2)

Solution

$$\begin{aligned} 4(0-1)^2 + b = 10 &\Rightarrow 4 + b = 10 \\ &\Rightarrow \underline{\underline{b = 6}}. \end{aligned}$$

(c) Write the equation of the curve in the form $y = px^2 + qx + r$. (2)

Solution

$\times$	x	-1
x	x^2	$-x$
-1	$-x$	$+1$

$$\begin{aligned} y = 4(x-1)^2 + 6 &\Rightarrow y = 4(x^2 - 2x + 1) + 6 \\ &\Rightarrow y = (4x^2 - 8x + 4) + 6 \\ &\Rightarrow \underline{\underline{y = 4x^2 - 8x + 10}}. \end{aligned}$$

19. Use the factor theorem to show that $(x-3)$ is not a factor of (2)

$$x^3 - 10x - 3.$$

Solution

Let

$$f(x) = x^3 - 10x - 3.$$

Then

$$f(3) = 3^3 - 10(3) - 3 = -6$$

and, hence, $(x-3)$ is not a factor of this cubic.

20. The transformation matrix $\mathbf{P}$ represents a 90° anti-clockwise rotation about the origin.

(a) Describe fully the **single** transformation represented by the matrix $\mathbf{P}^3$. (2)

Solution

The transformation is a 270° anti-clockwise rotation about the origin (or 90° clockwise rotation about the origin).

The transformation matrix $\mathbf{Q}$ is

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The transformation matrix $\mathbf{R}$ is

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$$

(b) Describe fully the **single** transformation represented by the matrix $\mathbf{QR}$. (2)

Solution

$$\begin{aligned} \mathbf{QR} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned}$$

and this is a rotation, by 180°, about the origin.

21. A cubic curve has (3)

- a maximum point at $A(-4, 10)$.
- a minimum point at $B(2, -26)$.

The tangent to the curve at A and the normal to the curve at B intersect at point C.

Work out the area of triangle ABC.
You may sketch a diagram to help you.

Solution

Well, the tangent to the curve at A has gradient 0 (why?) and the normal to the curve at B has infinite gradient (why?) so $C(2, 10)$. Finally,

$$\begin{aligned} \text{area} &= \tfrac{1}{2} \times AC \times BC \\ &= \tfrac{1}{2} \times [2 - (-4)] \times [10 - (-26)] \\ &= \tfrac{1}{2} \times 6 \times 36 \\ &= \underline{\underline{108}}. \end{aligned}$$

22. A quadratic sequence starts

$$302 \qquad 600 \qquad 894 \qquad 1\,184$$

(a) Work out an expression for the nth term. (3)

Solution

Let nth term be

$$an^2 + bn + c.$$

Write down the sequence:	302		600		894		1 184
First line of differences:		298		294		290	
Second line of differences:			-4		-4		

Sequence:	$a + b + c$		$4a + 2b + c$		$9a + 3b + c$
First line:		$3a + b$		$5a + b$	
Second line:			$2a$		

We compare terms:

$$2a = -4 \Rightarrow a = -2,$$

$$\begin{aligned} 3a + b = 298 &\Rightarrow 3 \times (-2) + b = 298 \\ &\Rightarrow b = 304, \end{aligned}$$

and

$$\begin{aligned} a + b + c = 302 &\Rightarrow -2 + 304 + c = 302 \\ &\Rightarrow c = 0; \end{aligned}$$

hence,

$$n\text{th term} = \underline{\underline{-2n^2 + 304n}}.$$

A term in the sequence has value 0.

(b) Find the position of this term. (2)

Solution

$$\begin{aligned} -2n^2 + 304n = 0 &\Rightarrow -2n(n - 152) = 0 \\ &\Rightarrow n = 0 \text{ or } \underline{\underline{n = 152}}. \end{aligned}$$

23. The continuous curve $y = \mathrm{f}(x)$ has exactly two stationary points. (1)
P is a maximum point when $x = a$.
Q is a stationary point of inflection when $x = b$.
$a < b$.
Which of these is correct?
Tick one box only.

When $a < x < b$, $\frac{dy}{dx}$ is positive

and

when $x > b$, $\frac{dy}{dx}$ is positive

☐

When $a < x < b$, $\frac{dy}{dx}$ is positive

and

when $x > b$, $\frac{dy}{dx}$ is negative

☐

When $a < x < b$, $\frac{dy}{dx}$ is negative

and

when $x > b$, $\frac{dy}{dx}$ is positive

☐

When $a < x < b$, $\frac{dy}{dx}$ is negative

and

when $x > b$, $\frac{dy}{dx}$ is negative

☐

Solution

We draw a table:

	$x = a^-$	$x = a$	$x = a^+$	$x = b^-$	$x = a$	$x = B^+$
Slope	+ve	0	−ve	−ve	?	−ve

The fourth one.

24. $$a^2 < 4 \text{ and } a + 2b = 8.$$ (4)

Work out the range of possible values of b.
Give your answer as an inequality.

Solution

Well,

$$a^2 < 4 \Rightarrow -2 < a < 2$$

and

$$\begin{aligned} a + 2b = 8 &\Rightarrow a = 8 - 2b \\ &\Rightarrow -2 < 8 - 2b < 2 \\ &\Rightarrow -10 < -2b < -6 \\ &\Rightarrow -5 < -b < -3 \\ &\Rightarrow \underline{\underline{3 < b < 5}}. \end{aligned}$$

25. Work out the values of x between 0° and 360° for which (4)

$$25\cos^2 x = 9.$$

Give your answers to 1 decimal place.

Solution

$$\begin{aligned} 25\cos^2 x = 9 &\Rightarrow \cos^2 x = \tfrac{9}{25} \\ &\Rightarrow \cos x = \pm\tfrac{3}{5}. \end{aligned}$$

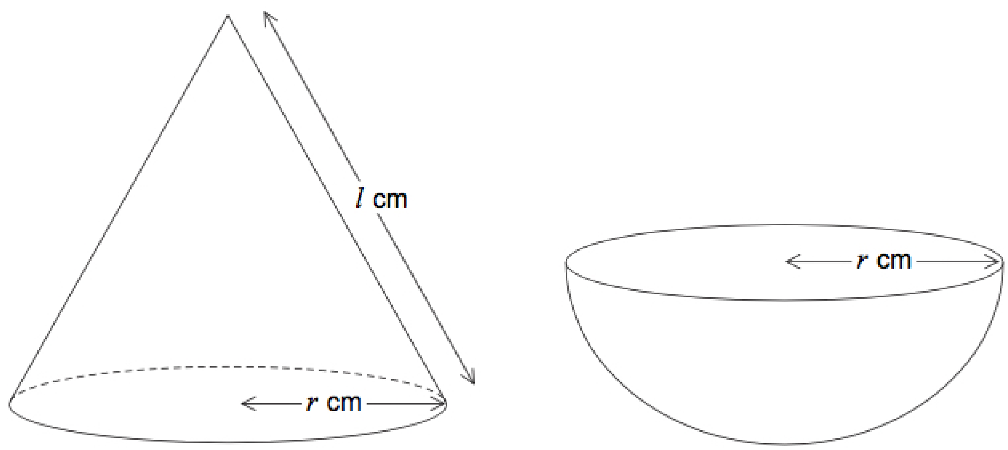
$\underline{\cos x = \tfrac{3}{5}}$:

$$\begin{aligned} \cos x = \tfrac{3}{5} &\Rightarrow x = 53.130\,102\,35,\ 306.869\,897\,6 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 53.1^\circ,\ 306.9^\circ \text{ (1 dp)}}}. \end{aligned}$$

$\underline{\cos x = -\tfrac{3}{5}}$:

$$\begin{aligned} \cos x = -\tfrac{3}{5} &\Rightarrow x = 126.869\,897\,6,\ 233.130\,102\,35 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 126.9^\circ,\ 233.1^\circ \text{ (1 dp)}}}. \end{aligned}$$

26. A cone has base radius r cm and slant height l cm.
A hemisphere has radius r cm.



The curved surface area of the cone equals the curved surface area of the hemisphere.

(a) Show that $l = 2r$. (1)

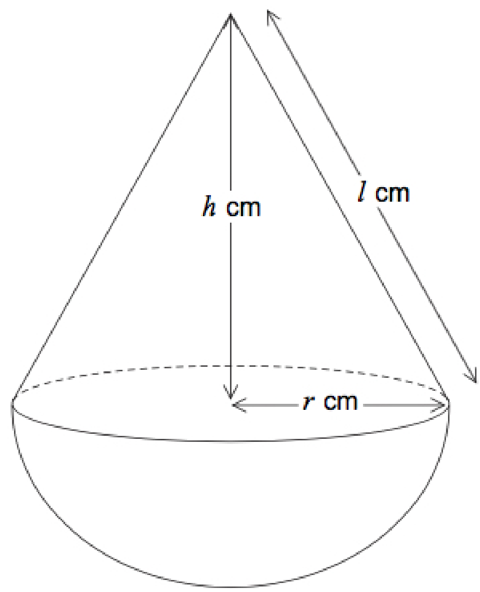
Solution

Well, since the areas are equal,

$$\frac{1}{2} \times 4\pi r^2 = \pi r l \Rightarrow \underline{\underline{2r = l}},$$

as required.

The cone has vertical height h cm.
The cone and hemisphere are joined to make the shape shown below.



(b) Show that the volume of the shape can be written as (4)

$$\tfrac{1}{3}\pi r^3(a + \sqrt{b}) \text{ cm}^3,$$

where a and b are integers.

Solution

$$\begin{aligned}
\text{Volume} &= \text{hemisphere} + \text{cone} \\
&= (\tfrac{1}{2} \times \tfrac{4}{3}\pi r^3) + (\tfrac{1}{3}\pi r^2 h) \\
&= \tfrac{2}{3}\pi r^3 + \tfrac{1}{3}\pi r^2(\sqrt{l^2 - r^2}) \\
&= \tfrac{2}{3}\pi r^3 + \tfrac{1}{3}\pi r^2(\sqrt{(2r)^2 - r^2}) \\
&= \tfrac{2}{3}\pi r^3 + \tfrac{1}{3}\pi r^2(\sqrt{4r^2 - r^2}) \\
&= \tfrac{2}{3}\pi r^3 + \tfrac{1}{3}\pi r^2(\sqrt{3r^2}) \\
&= \tfrac{2}{3}\pi r^3 + \tfrac{1}{3}\pi r^3\sqrt{3} \\
&= \underline{\underline{\tfrac{1}{3}\pi r^3(2 + \sqrt{3})}};
\end{aligned}$$

hence, $\underline{\underline{a = 2}}$ and $\underline{\underline{b = 3}}$.

27. Work out the values of a when (4)

$$2^{a^2} = 8^a \times 16.$$

Do **not** use trial and improvement.
You must show your working.

Solution

$$\begin{aligned} 2^{a^2} = 8^a \times 16 &\Rightarrow 2^{a^2} = (2^3)^a \times 2^4 \\ &\Rightarrow 2^{a^2} = 2^{3a} \times 2^4 \\ &\Rightarrow 2^{a^2} = 2^{3a+4} \\ &\Rightarrow a^2 = 3a + 4 \\ &\Rightarrow a^2 - 3a - 4 = 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & -3 \\ \text{multiply to:} & -4 \end{array}\right\} -4,\ +1$$

$$\begin{aligned} &\Rightarrow (a-4)(a+1) = 0 \\ &\Rightarrow \underline{\underline{a = -1 \text{ or } a = 4}}. \end{aligned}$$