Dr Oliver Mathematics
Second Order Differential Equations: Part 1

1. If
$$f''(x) - f'(x) - 2\,f(x) = 0 \text{ with } f(0) = 2 \text{ and } f'(0) = -2,$$
evaluate $f(1)$.

Solution

Complementary Function:

$$m^2 - m - 2 \Rightarrow (m-2)(m+1) = 0$$
$$\Rightarrow m = -1 \text{ or } m = 2.$$

So
$$f(x) = Ae^{-x} + Be^{2x}$$
for some constants A and B. Now,
$$f(x) = Ae^{-x} + Be^{2x} \Rightarrow f'(x) = -Ae^{-x} + 2Be^{2x}.$$

Next,
$$f(0) = 2 \Rightarrow A + B = 2 \quad (1)$$
and
$$f'(0) = -2 \Rightarrow -A + 2B = -2 \quad (2).$$

Add $(1) + (2)$:
$$3B = 0 \Rightarrow B = 0$$
$$\Rightarrow A = 2$$

and hence
$$f(x) = 2e^{-x}.$$

Finally,
$$f(1) = \underline{\underline{2e^{-1}}}.$$