Mechanics 2

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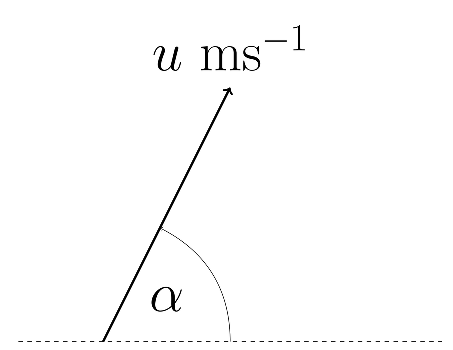
Mathematics

Modelling assumptions

All of the modelling assumptions — when they are appropriate and what the consequences of using them are — that you covered in chapter 1 of *Mechanics 1* need to be known for this module as well.

Components of velocity

Suppose that a particle is projected at in initial velocity u ms^{-1} at an angle $\alpha°$ to the horizontal, as shown below.



In the absence of air resistance, the horizontal component of the velocity, $u \cos \alpha$ will remain constant; the the initial vertical component of the velocity, $u \sin \alpha$ will change across time (just as in Mechanics 1) due to the influence of gravity.

Displacement, velocity, and acceleration

If $\mathbf{x}(t)$, $\mathbf{v}(t)$, and $\mathbf{a}(t)$ are vector-valued functions with respect to time, t, that represent displacement, velocity, and acceleration then

$$\mathbf{v} = \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}$$
$$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2},$$

and

$$\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t$$
$$\mathbf{x} = \int \mathbf{v}\,\mathrm{d}t;$$

when integrating you will generate an arbitrary vector, $\mathbf{c}$, and you will generally be able to work out what its components are from other information in the question.

CoM: point masses

If point masses m_1, m_2, $\ldots$, and m_n are at coordinates (x_1, y_1), (x_2, y_2), $\ldots$, and (x_n, y_n) then the centre of mass of the system has coordinates $(\bar{x}, \bar{y})$ where

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + \ldots + m_n x_n}{m_1 + m_2 + \ldots + m_n}$$

and

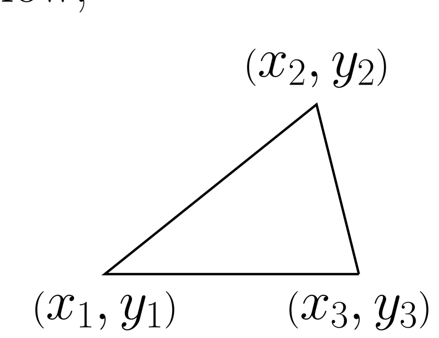
$$\bar{y} = \frac{m_1 y_1 + m_2 y_2 + \ldots + m_n y_n}{m_1 + m_2 + \ldots + m_n}.$$

CoM: uniform lamina

If a uniform lamina has an axis of symmetry then the centre of mass will lie on that axis. If a uniform lamina has more than one axis or symmetry then the centre of mass will lie on the intersection of those axes. Thus, the centres of mass of circles, squares, rectangles, equilateral triangles, and isosceles triangles should all be easy to find.

CoM: triangles

The centre of mass of a uniform triangular lamina lies at the *centroid* of the triangle, i.e., at the point where the three median lines (that is, the lines that join a vertex to the middle of the opposite side) meet. The centroid is two-thirds of the way along each median line, starting from the vertex. If you know the coordinates of the three vertices, such as shown in the figure below,



then

$$(\bar{x}, \bar{y}) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$

CoM: uniform sector of a circle

If the sector has radius r and centre angle 2α (measured in radians) then the centre of mass lies on the axis of symmetry of the sector is at a distance

$$\frac{2r \sin \alpha}{3\alpha}$$

from the centre of the circle.

CoM: uniform circular arc

If the arc has radius r and centre angle 2α (measured in radians) then the centre of mass lies on the axis of symmetry of the sector is at a distance

$$\frac{r \sin \alpha}{\alpha}$$

from the centre of the circle.

Laminae in equilibrium

If a lamina is suspended freely from a fixed point then it will rest in equilibrium in such a way that the centre of mass is vertically below the point of suspension.

Laminae on inclined planes

Suppose a lamina is on a rough plane and this plane is made to incline ever more steeply to the horizontal. What will happen first: will the lamina slide down the plane or will it topple over? This question requires to you calculate two things: the angle of inclination for which the component of the lamina's weight down the slope is greater than the frictional force opposing motion (this will tell you when it will slide) and the angle of inclination for which the line of action of the lamina's weight no longer passes through the side of the lamina that is contact with the plane (this will tell you when it will topple) and the smaller of the two angles provides you with your answer.

Work

The term ***work*** has a specific meaning in mechanics: for a constant force,

$$\text{work} = \text{force} \times \text{distance moved in force's direction},$$

where the force is measured in newtons and the distance is measured in metres.

Energy

$$\text{Kinetic energy} = \tfrac{1}{2}mv^2$$
$$\text{Gravitational potential energy} = mgh$$

where m (kg) is the mass of the object, v (ms^{-1}) is the object's velocity, and h (m) is the height gained from the initial level; both types of energy have as their unit the joule (J).

Power

Power is the rate at which work is done: it is measure in watts (W) and one watt is one joule per second. If a particle is moving then

$$P = Fv,$$

where P is power, F is the driving force (in newtons) and v is the speed (in ms^{-1}).

Newton's coefficient of restitution

Newton's coefficient of restitution, e, is a pure number and is calculated by

$$e = \frac{\text{speed of approach}}{\text{speed of rebound}}.$$

For problems involving collisions (and particularly those involving multiple collisions) you are strongly advised to draw clear diagrams and ensure that all of your initial and final velocities are clearly labelled. In the absence of air resistance, if an object falls from rest at height h before rebounding off a surface with coefficient of restitution e then the object will rise to a height e^2h.

Limiting equilibrium

A body is in *limiting equilibrium* when it is on the verge of moving. In those cases, friction will assume its maximum possible value of μR where μ is the coefficient of friction and R is the normal reaction at the point of contact.

Statics of rigid bodies

The good news here is that there is no additional theory to that of Mechanics 1. The bad news is that you will be expected to apply the two conditions for equilibrium (no net force and no net moment) in situations of considerably greater complexity than in the previous module. Questions involving ladders leaning against walls, rods hinged at one end and held in position by a string connected elsewhere, rods hinged at one end and supported by a strut from below, and so on are all standard questions and you should ensure that you can both quickly and accurately calculate both forces and moments in a wide variety of cases.