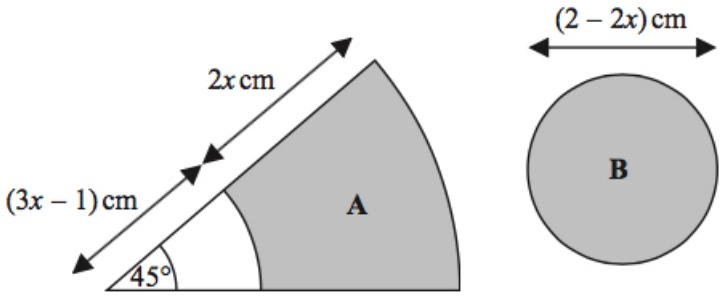
Dr Oliver Mathematics
Worked Examples
Circles and Sectors 1

From: Edexcel 2020 June Paper 1H (Non-Calculator)

1. The diagram shows two shaded shapes, **A** and **B**. (5)

   Shape **A** is formed by removing a sector of a circle with radius $(3x-1)$ cm from a sector of the circle with radius $(5x-1)$ cm.
   Shape **B** is a circle of diameter $(2-2x)$ cm.



   The area of shape **A** is equal to the area of shape **B**.

   Find the value of x.
   You must show all your working.

Solution

Shape **A**:

$$\text{Shape } \mathbf{A} = \frac{45}{360} \times \pi \times [(5x-1)^2 - (3x-1)^2]$$

×	$5x$	-1
$5x$	$25x^2$	$-5x$
-1	$-5x$	$+1$

$\times$	$3x$	-1
$3x$	$9x^2$	$-3x$
-1	$-3x$	$+1$

$$= \frac{45}{360} \times \pi \times [(25x^2 - 10x + 1) - (9x^2 - 6x + 1)]$$
$$= \frac{45}{360} \times \pi \times (16x^2 - 4x)$$
$$= \tfrac{1}{8}(16x^2 - 4x)\pi$$

Shape **B**:

$$\text{Shape } \mathbf{B} = \pi(1 - x)^2.$$

Comparing:
Now,

$$\tfrac{1}{8}(16x^2 - 4x)\pi = (1 - x)^2\pi \Rightarrow 16x^2 - 4x = 8(1 - 2x + x^2)$$
$$\Rightarrow 16x^2 - 4x = 8 - 16x + 8x^2$$
$$\Rightarrow 8x^2 + 12x - 8 = 0$$
$$\Rightarrow 4(2x^2 + 3x - 2) = 0$$

$$\left.\begin{array}{ll} \text{add to:} & +3 \\ \text{multiply to:} & (+2) \times (-2) = -4 \end{array}\right\} +4, -1$$

$$\Rightarrow 4[2x^2 + 4x - x - 2] = 0$$
$$\Rightarrow 4[2x(x + 2) - 1(x + 2)] = 0$$
$$\Rightarrow 4(2x - 1)(x + 2) = 0$$
$$\Rightarrow 2x - 1 = 0 \text{ or } x + 2 = 0$$
$$\Rightarrow x = \tfrac{1}{2} \text{ or } x = -2;$$

but $x \neq -2$ (why?). Hence, $\underline{\underline{x = \frac{1}{2}}}$.