Further Pure Mathematics 2: Part 2

Dr Oliver
Further Mathematics

Circles

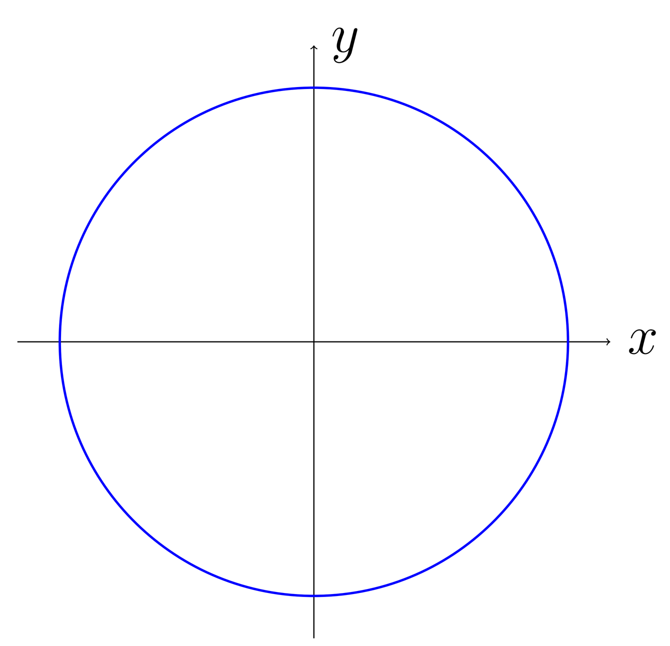

Figure 1: $r = a$: circle, centre $(0, 0)$, radius a

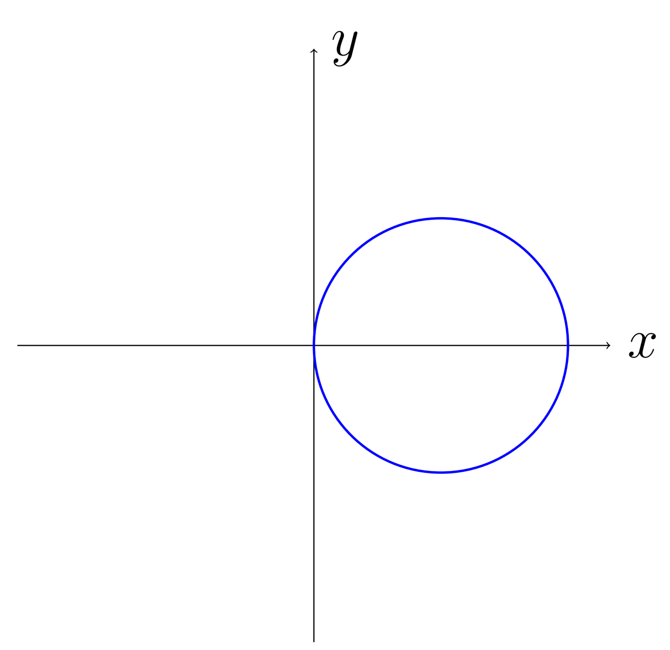

Figure 2: $r = a\cos\theta$: circle, centre $(\frac{1}{2}a, 0)$, radius $\frac{1}{2}a$

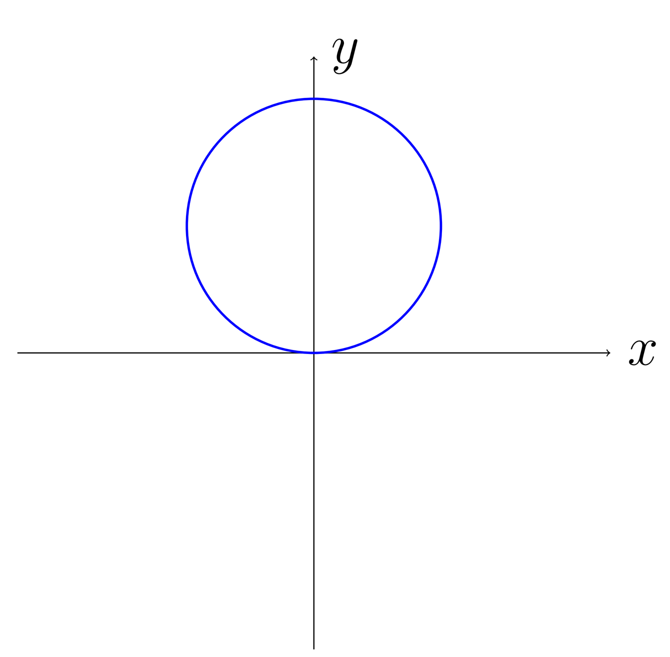

Figure 3: $r = a\sin\theta$: circle, centre $(0, \frac{1}{2}a)$, radius $\frac{1}{2}a$

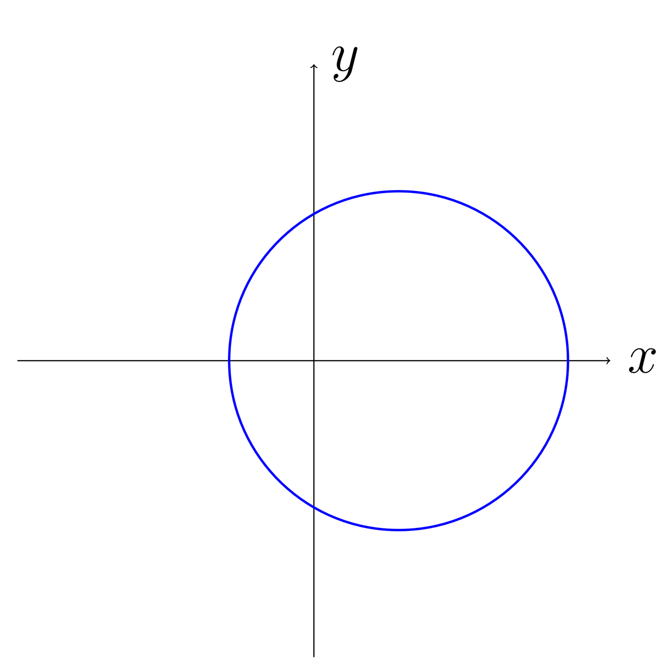

Figure 4: $a^2 = r^2 + b^2 - 2br\cos\theta$: circle, centre $(b, 0)$, radius a

Petal curves

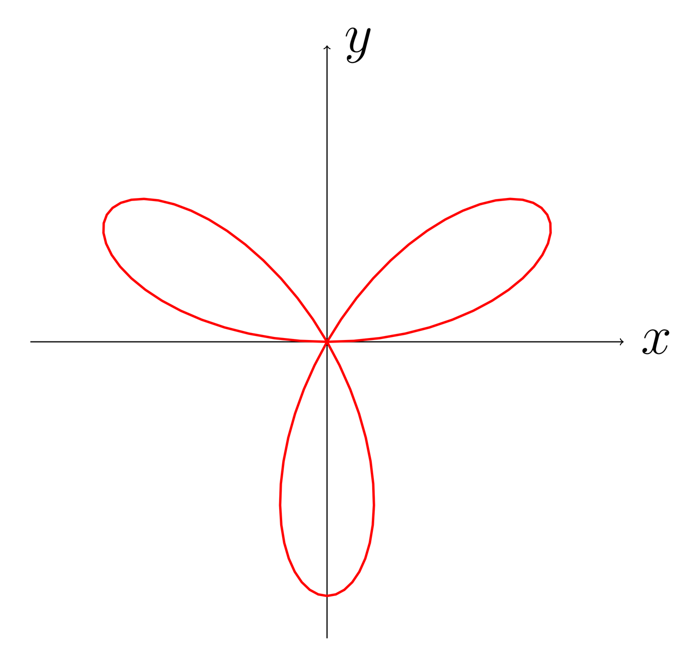

Figure 5: $r = a\sin 3\theta$

Note that there are six radial lines ($\theta = 0$, $\theta = \frac{\pi}{3}$, $\theta = \frac{2\pi}{3}$, $\theta = \pi$, $\theta = \frac{4\pi}{3}$, and $\theta = \frac{5\pi}{3}$) which should be marked in on a sketch.

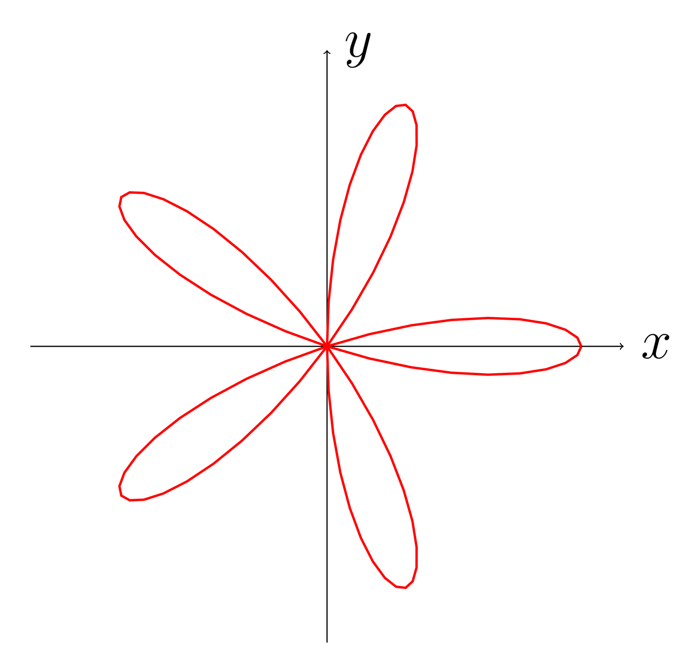

Figure 6: $r = a\cos 5\theta$

There are ten radial lines which should be marked on in this sketch: can you find the equation of all ten?

Half-lines

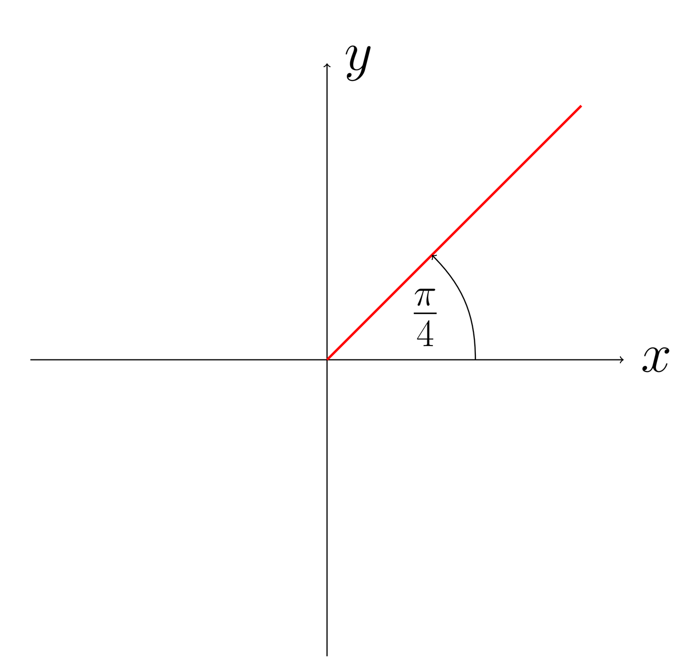

Figure 7: $\theta = \frac{\pi}{4}$

This half-line has the Cartesian equation

$$y = x, x > 0.$$

Horizontal and vertical lines

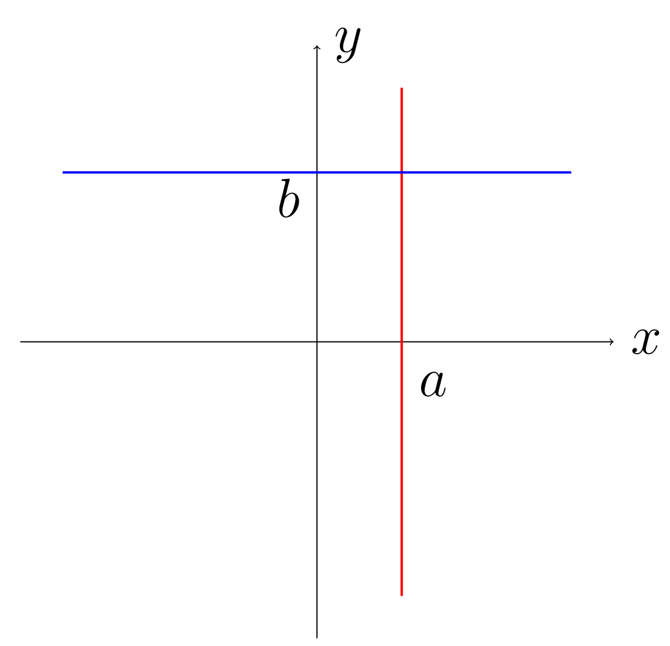

Figure 8: $r\cos\theta = a$ and $r\sin\theta = b$

Diagonal lines

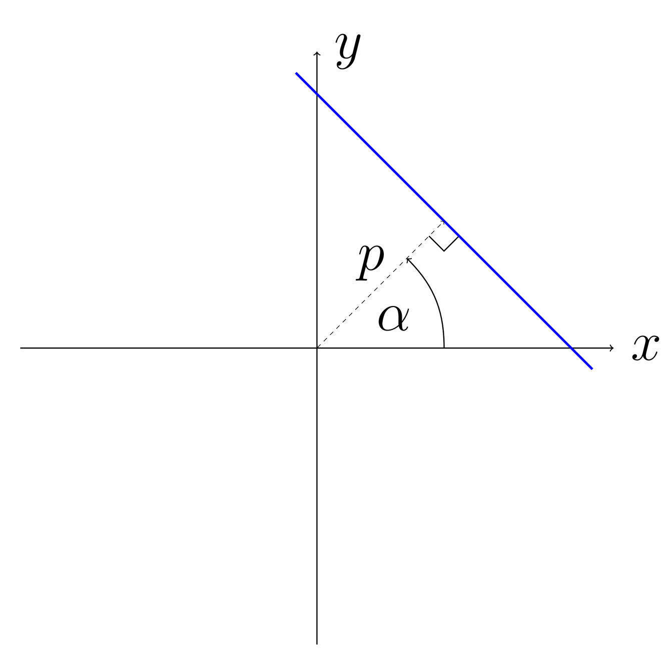

Figure 9: $p = r\cos(\alpha - \theta)$

The line is perpendicular to a line of length p at an angle α to the x-axis.

Cardioid

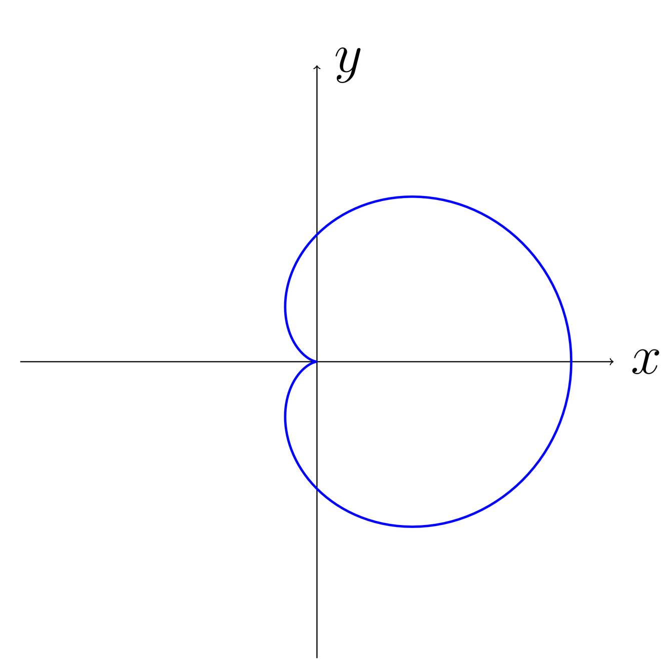

Figure 10: $r = a(1 + \cos\theta)$

Note that the initial line is a tangent to the curve as it approaches the pole.

Limacons

These are curves of the form $r = a + b\cos\theta$:

- if $a < b$ then there will be values of θ where $r < 0$ and so no points are plotted;
- if $a = b$ then we have the cardioid;
- if $b < a < 2b$ then the curve has a 'dimple';
- if $a \geq 2b$ then the curve loses the 'dimple' and becomes more rounded the greater the difference between a and $2b$.

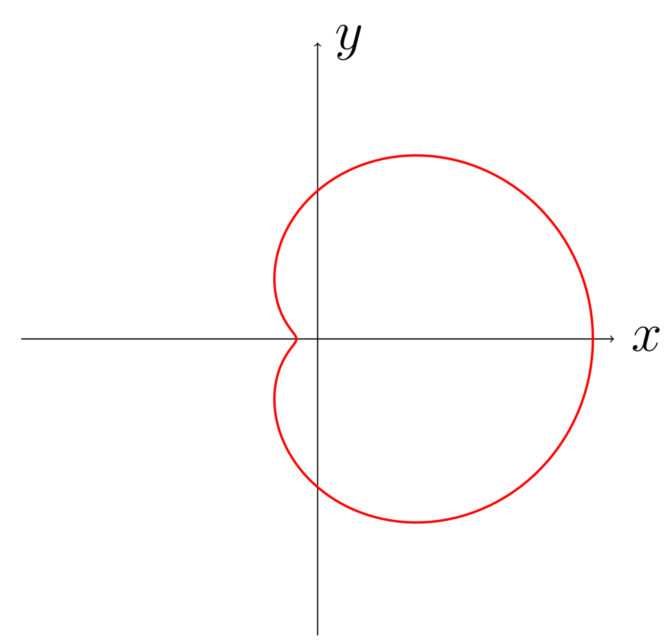

Figure 11: $r = a + b\cos\theta$ where $b < a < 2b$

Spirals

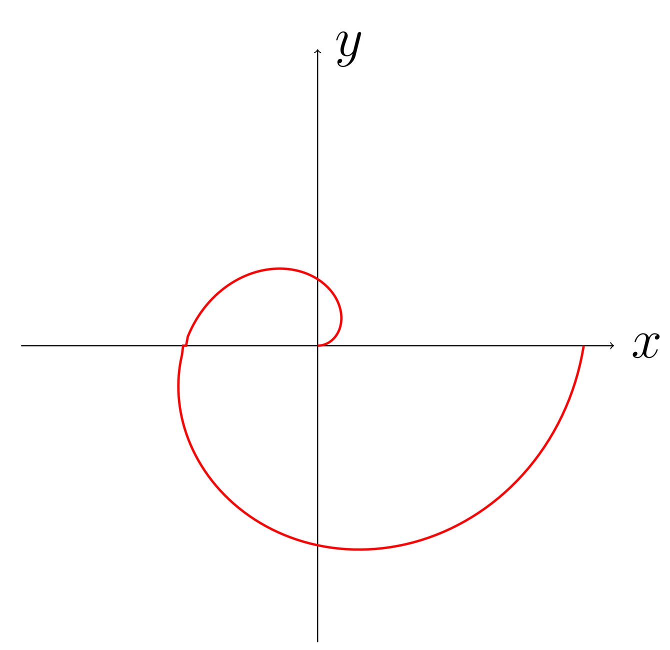

Figure 12: $r = a\theta$: the spiral of Archimedes

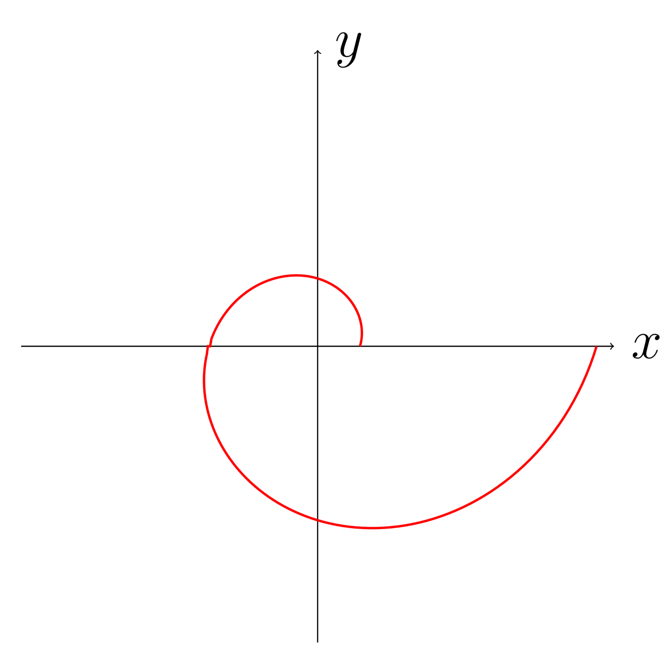

Figure 13: $r = e^{k\theta}$: the equiangular spiral