Dr Oliver Mathematics
Mathematics
Discriminant
Past Examination Questions

This booklet consists of 22 questions across a variety of examination topics.
The total number of marks available is 150.

1. Given that the equation (4)

$$kx^2 + 12x + k = 0,$$

where k is a positive constant, has equal roots, find the value of k.

2. The equation (4)

$$2x^2 - 3x - (k + 1) = 0,$$

where k is a constant, has no real roots. Find the set of possible values of k.

3. The equation $x^2 + 3px + p = 0$, where p is an non-zero constant, has equal roots. Find the value of p. (4)

4. Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,
   (a) show that $q^2 + 8q < 0$. (2)
   (b) Hence find the set of possible values of q. (3)

5. The equation

$$x^2 + kx + (k + 3) = 0,$$

where k is a constant, has different real roots.
   (a) Show that $k^2 - 4k - 12 > 0$. (2)
   (b) Find the set of possible values of k. (4)

6.

$$x^2 - 8x - 29 \equiv (x + a)^2 + b,$$

where a and b are constants.
   (a) Find the value of a and find the value of b. (3)
   (b) Hence, or otherwise, show that the roots of (3)

$$x^2 - 8x - 29 = 0$$

   are $c \pm \sqrt{5}$, where c and d are constants.

7. The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

(a) Find the value of p. (4)

(b) For this value of p, solve the equation $x^2 + 2px + (3p + 4) = 0$. (2)

8. (a) Show that $x^2 + 6x + 11$ can be written as (2)

$$(x + p)^2 + q,$$

where p and q are constants.

(b) Sketch the graph of $y = x^2 + 6x + 11$, indicating clearly the coordinates of any intersections with the coordinate axes. (2)

(c) Find the value of the discriminant of $x^2 + 6x + 11$. (2)

9.

$$\mathrm{f}(x) = x^2 + (k + 3)x + k,$$

where k is a real constant.

(a) Find the discriminant of $\mathrm{f}(x)$ in terms of k. (2)

(b) Show that the discriminant of $\mathrm{f}(x)$ can be expressed in the form (2)

$$(k + a)^2 + b,$$

where a and b are integers to be found.

(c) Show that, for all values of k, the equation $\mathrm{f}(x) = 0$ has real roots. (2)

10. The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x.

(a) Show that k satisfies $k^2 + 4k - 32 < 0$. (3)

(b) Hence find the set of possible values of k. (4)

11. The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies $k^2 - 5k + 4 > 0$. (3)

(b) Hence find the set of possible values of k. (4)

12. The equation

$$x^2 + (k - 3)x + (3 - 2k) = 0,$$

where k is a constant, has two distinct real roots.

(a) Show that k satisfies (3)

$$k^2 + 2k - 3 > 0.$$

(b) Hence find the set of possible values of k. (4)

13. Given the simultaneous equations

$$2x + y = 1$$
$$x^2 - 4ky + 5k = 0,$$

where k is a non-zero constant,

(a) show that (2)

$$x^2 + 8kx + k = 0.$$

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k. (3)

(c) For this value of k, find the solution of the simultaneous equations. (3)

14.

$$x^2 + 2x + 3 \equiv (x + a)^2 + b,$$

where a and b are constants.

(a) Find the value of a and find the value of b. (2)

(b) Sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes. (3)

(c) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (b). (2)

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(d) Find the set of possible values of k, giving your answer in surd form. (4)

15. The equation

$$(k + 3)x^2 + 6x + k = 6,$$

where k is a constant, has two distinct real solutions for x.

(a) Show that k satisfies (4)

$$k^2 - 2k - 24 < 0.$$

(b) Hence find the set of possible values of k. (3)

16. The equation

$$(p - 1)x^2 + 4x + (p - 5) = 0,$$

where p is a constant, has no real roots.

(a) Show that p satisfies $p^2 - 6p + 1 > 0$. (3)

(b) Hence find the set of possible values of p. (4)

17.

$$4x - 5 - x^2 = q - (x + p)^2,$$

where p and q are integers.

(a) Find the value of p and the value of q. (3)

(b) Calculate the discriminate of $4x - 5 - x^2$. (2)

(c) Sketch the graph of $y = 4x - 5 - x^2$, showing the coordinates of any point at which the graph crosses the coordinate axes. (3)

18. The straight line with equation $y = 3x - 7$ does not cross or touch the curve with equation $y = 2px^2 - 6px + 4p$, where p is a constant.

(a) Show that $4p^2 - 20p + 9 < 0$. (4)

(b) Hence find the set of possible values of p. (4)

19.

$$\mathrm{f}(x) = x^2 + 4kx + (3 + 11k), \text{ where } k \text{ is a constant.}$$

(a) Express $\mathrm{f}(x)$ in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k. (3)

Given that the equation $\mathrm{f}(x) = 0$ has no real roots,

(b) find the set of possible values of k. (4)

Given that $k = 1$,

(c) sketch the graph of $y = \mathrm{f}(x)$, showing the coordinates of any point at which the graph crosses a coordinate axis. (3)

20. Given that

$$\mathrm{f}(x) = 2x^2 + 8x + 3,$$

(a) find the value of the discriminant of $\mathrm{f}(x)$. (2)

(b) Express $\mathrm{f}(x)$ in the form $p(x + q)^2 + r$, where p, q, and r are integers to be found. (3)

The line $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = \mathrm{f}(x)$.

(c) Calculate the value of c. (5)

21. (a) On separate axes sketch the graphs of (4)

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \dfrac{1}{x} + 5$.

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \frac{1}{x} + 5$ at two distinct points,

(b) show that $(5 - c)^2 > 12$. (3)

(c) Hence find the range of possible values for c. (4)

22. (2)

$$\mathrm{f}(x) = x^2 - 8x + 13.$$

Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b$, where a and b are constants.