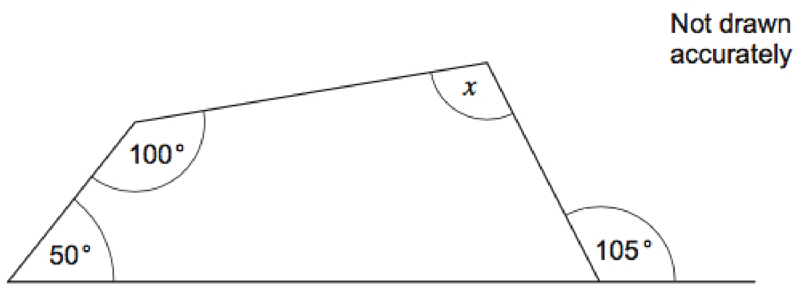
Dr Oliver Mathematics
AQA GCSE Mathematics
2012 November Paper 2: Calculator
2 hours

The total number of marks available is 105.
You must write down all the stages in your working.

1. Work out the value of x. (3)



Solution

Well, the supplementary angle to the 105° is

$$180 - 105 = 75^\circ$$

and

$$\begin{aligned} 50 + 100 + x + 75 = 360 &\Rightarrow x + 225 = 360 \\ &\Rightarrow \underline{\underline{x = 135^\circ}}. \end{aligned}$$

2. Here are Jon's marks in two tests. (3)

Test A	18 out of 25
Test B	30 out of 40

Which test gives the higher percentage mark?
You **must** show your working.

Solution

Test A	18 out of 25	$\frac{18}{25} \times 100\% = 72\%$
Test B	30 out of 40	$\frac{30}{40} \times 100\% = 75\%$

Hence, he scored more marks in Test B.

3. Solve (4)

$$3(2x + 4) + 8 = 50.$$

Solution

$$\begin{aligned} 3(2x+4)+8=50 &\Rightarrow 6x+12+8=50 \\ &\Rightarrow 6x = 30 \\ &\Rightarrow \underline{\underline{x=5}}. \end{aligned}$$

4. (a) Put each of these numbers into the correct box. (3)

27 2 8 11 64

	Square Number	Odd Number	Even Number
Cube Number			
Prime Number	———		

Solution

	Square Number	Odd Number	Even Number
Cube Number	$\underline{\underline{64}}$	$\underline{\underline{27}}$	$\underline{\underline{8}}$
Prime Number	———	$\underline{\underline{11}}$	$\underline{\underline{2}}$

(b) Why is it **never** possible to put any number in the Prime Number & Square Number? (1)

Solution
E.g., any square number is divisible by its square root so cannot be prime.

5. (3)

$$A = \frac{4x + 3y}{x - y}.$$

Work out the value of A when $x = 6$ and $y = -1$.

Solution

$$\begin{aligned} A &= \frac{4(6) + 3(-1)}{6 - (-1)} \\ &= \frac{24 - 3}{6 + 1} \\ &= \frac{21}{7} \\ &= \underline{\underline{3}}. \end{aligned}$$

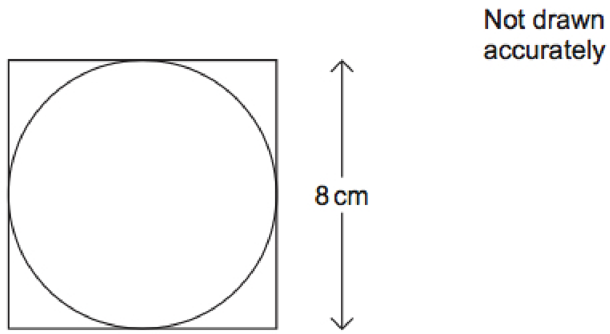
6. Circle the **two** equations that are equivalent to (2)

$$2y = 3x + 4.$$

A	$2x = 3y + 4$
B	$y - \frac{3}{2} = 2$
C	$y = \frac{3}{2} + 4$
D	$3x - 2y + 4 = 0$

Solution
A – no: there is a mis-match between the coeffient of x and the coeffient of y.
B – $2y = 3x + 4 \Rightarrow 2y - 3x = 4 \Rightarrow \frac{3}{2} = 2$ so $\underline{\underline{\text{yes}}}$.
C – no: $2y = 3x + 4 \Rightarrow y = \frac{3}{2} + 2$.
D – $2y = 3x + 4 \Rightarrow 3x - 2y + 4 = 0$ so $\underline{\underline{\text{yes}}}$.

7. The diagram shows a circle inside a square. (3)



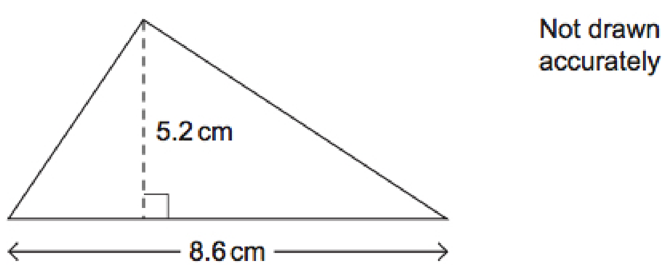
Work out the area of the circle.

Solution

Well, clearly $r = 4$ (why?) so

$$\begin{aligned} \text{area} &= \pi \times 4^2 \\ &= \underline{\underline{16\pi \text{ cm}^2 \text{ or } 50.7 \text{ cm}^2 \text{ (3 sf)}}}. \end{aligned}$$

8. Work out the area of the triangle. (3)



Give your answer to 1 decimal place.

Solution

$$
\begin{aligned}
\text{Area} &= \tfrac{1}{2}bh \\
&= \tfrac{1}{2} \times 8.6 \times 5.2 \\
&= 22.36 \text{ (exact!)} \\
&= \underline{\underline{22.4 \text{ cm}^2 \text{ (1 dp)}}}.
\end{aligned}
$$

9. Show that the equation (2)

$$x^3 + 8x = 30$$

has a solution between $x = 2.2$ and $x = 2.3$.

Solution

When $x = 2.2$,

$$(2.2)^3 + 8(2.2) = 28.248$$

and, when $x = 2.3$,

$$(2.3)^3 + 8(2.3) = 30.567.$$

Clearly, $y = x^3 + 8x - 30$ crosses the real line (why?) and, hence, has a $\underline{\underline{\text{solution}}}$ between $x = 2.2$ and $x = 2.3$.

10. (a) A drink is made from 1.5 litres of orange juice and 7.5 litres of lemonade. (3)

What fraction of the drink is orange juice?
Give your answer in its simplest form.

Solution

$$
\begin{aligned}
\text{Fraction} &= \frac{1.5}{1.5 + 7.5} \\
&= \frac{1.5}{9} \\
&= \underline{\underline{\tfrac{1}{6}}}.
\end{aligned}
$$

(b) A different drink is made from 2 litres of blackcurrant juice and 12 litres of water. (3)

How much more blackcurrant juice should be added so that 25% of the drink is blackcurrant juice?

Solution

Let x be the number of litres of blackcurrant juice. Then

$$\frac{x}{x+12} = \frac{25}{100} \Rightarrow \frac{x}{x+12} = \frac{1}{4}$$
$$\Rightarrow 4x = x + 12$$
$$\Rightarrow 3x = 12$$
$$\Rightarrow x = 4.$$

Hence, since 2 litres were added to it,

$$4 - 2 = \underline{\underline{2 \text{ litres}}}$$

must now be added.

11. Mark went fishing on four Saturdays.

	Week 1	Week 2	Week 3	Week 4
Number of fish caught	4	1	6	3
Time fishing	2.5 hours	1.5 hours	5 hours	2.5 hours
Mean weight of fish caught	1.2 kg	2.3 kg	0.8 kg	1.9 kg

(a) Work out the **mean** number of fish caught **per hour** in **Week 1**. (2)

Solution

$$\text{Mean} = \frac{4}{2.5}$$
$$= \underline{\underline{1.6 \text{ fish/hour}}}.$$

(b) Mark says, "One of the fish I caught weighed 5 kg." (2)

In which week did this happen?
Give a reason for your answer.

Solution

$$\begin{aligned} \text{Week 1}: &\quad 4 \times 1.2 = 4.8 \text{ kg} \\ \text{Week 2}: &\quad 1 \times 2.3 = 2.3 \text{ kg} \\ \text{Week 3}: &\quad 6 \times 0.8 = 4.8 \text{ kg} \\ \text{Week 1}: &\quad 3 \times 1.9 = 5.7 \text{ kg} \end{aligned}$$

So, it must have been in Week 4.

12. (a) Expand and simplify (2)

$$(x+6)^2.$$

Solution

$\times$	x	$+6$
x	x^2	$+6x$
$+6$	$+6x$	$+36$

$$(x+6)^2 = x^2 + 12x + 36.$$

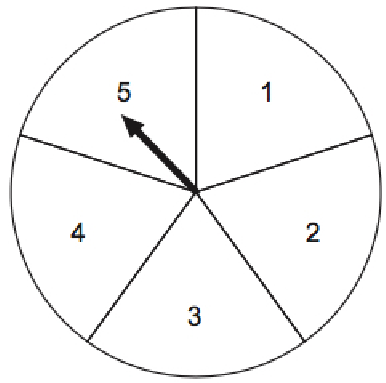
(b) Expand and simplify (4)

$$9w(3x-4y) - 5w(x+y).$$

Solution

$$\begin{aligned} 9w(3x-4y) - 5w(x+y) &= 27wx - 36wy - 5wx - 5wy \\ &= 27wx - 5wx - 36wy - 5wy \\ &= 22wx - 41wy \text{ or } w(22x - 41y). \end{aligned}$$

13. Matt made this spinner.
He spins the arrow 200 times.



(a) How many times would you expect the arrow to stop on the number 5 if the spinner is fair? (2)

Solution

$$\frac{200}{5} = \underline{\underline{40 \text{ times}}}.$$

(b) The table shows the number of times the arrow stops on each number. (2)

Stops On	1	2	3	4	5
Number of times	32	41	65	27	35

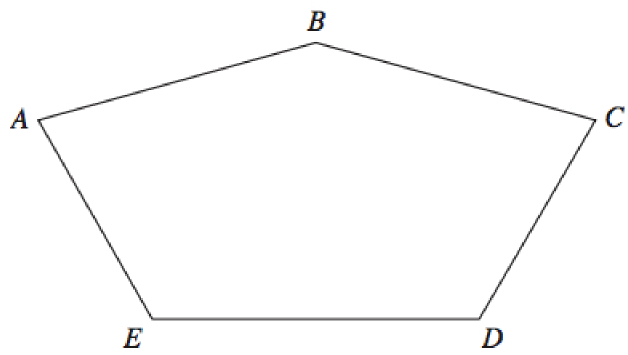
Do you think the spinner is fair?
Give a reason for your answer.

Solution

No: e.g., I expect the spinner to stop 40 times on each number if it is fair but 65 times on number 3 seems far too high, biased towards 3, the outcome 3 is (more than) double 4.

14. This pentagon has a vertical line of symmetry. (5)
The ratio of angles

$$B : C : D = 6 : 3 : 4.$$



Not drawn accurately

Work out the size of angle B.

Solution

Well,

$$A : B : C : D : E = 3 : 6 : 3 : 4 : 4.$$

Now,

$$3 + 6 + 3 + 4 + 4 = 20$$

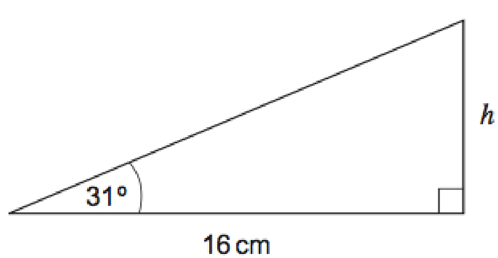
and

$$\frac{540}{20} = 27.$$

(Why 540?) Hence,

$$B = 6 \times 27 = \underline{\underline{162^\circ}}.$$

15. Work out the height h. (3)



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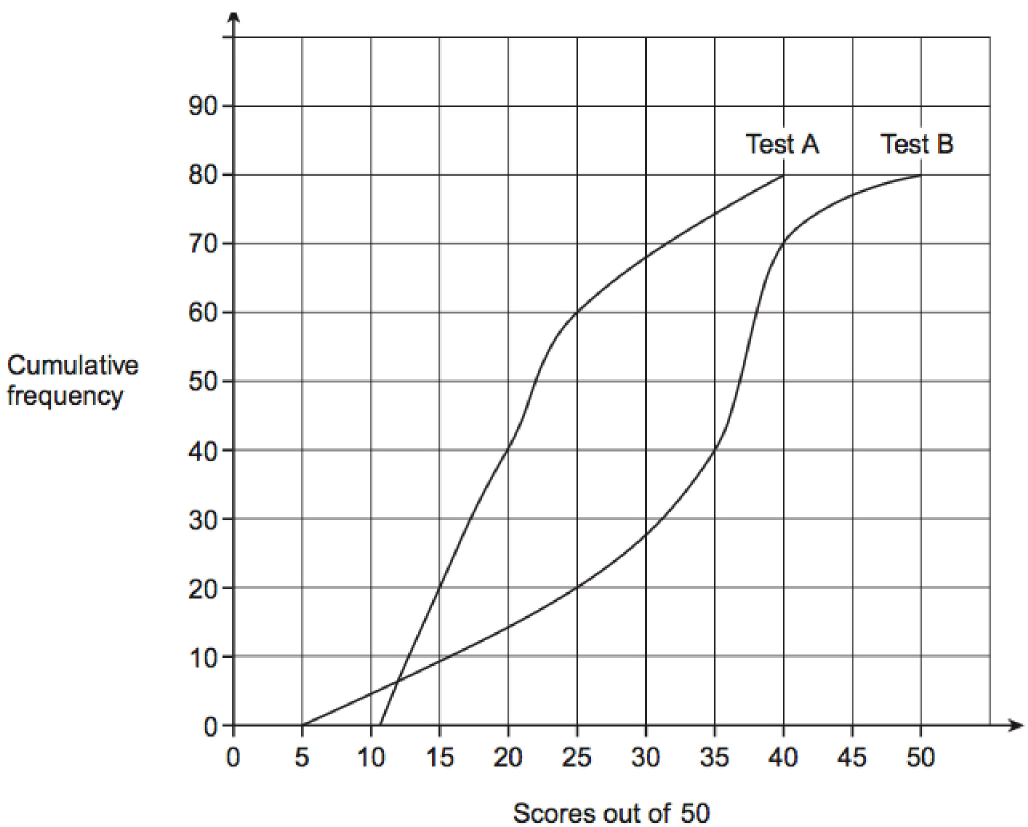
Solution

$$\begin{aligned}\tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan 31^\circ = \frac{h}{16}\\ &\Rightarrow h = 16\tan 31^\circ\\ &\Rightarrow h = 9.613\,769\,904 \text{ (FCD)}\\ &\Rightarrow \underline{\underline{h = 9.61 \text{ cm (3 sf)}}}.\end{aligned}$$

16. The same students take two tests.
The scores out of 50 are represented on the cumulative frequency graphs.



(a) How many students took each test? (1)

Solution
80 students.

(b) Work out the median score for each test. (2)

Solution

Test A: $\underline{\underline{20 \text{ marks}}}$.

Test B: $\underline{\underline{35 \text{ marks}}}$.

The interquartile range for test B is 13.

(c) Work out the interquartile range for test A. (2)

Solution

$$\begin{aligned} \text{IQR} &= 60 \text{ th piece of data} - 20 \text{ th piece of data} \\ &= 25 - 15 \\ &= \underline{\underline{10 \text{ marks}}}. \end{aligned}$$

(d) Which test is more difficult?
Give one reason to support your answer. (1)

Solution

$\underline{\underline{\text{Test A}}}$, e.g., lower median for test A, lower on average for test A, lower and upper quartiles are less for test A.

17. These expressions represent three numbers. (3)

$$x \qquad x+3 \qquad 4x$$

Work out the mean in terms of x.
Give your answer in its simplest form.

Solution

$$\begin{aligned} \text{Mean} &= \frac{x + (x+3) + 4x}{3} \\ &= \frac{6x+3}{3} \\ &= \frac{3(2x+1)}{3} \\ &= \underline{\underline{2x+1}}. \end{aligned}$$

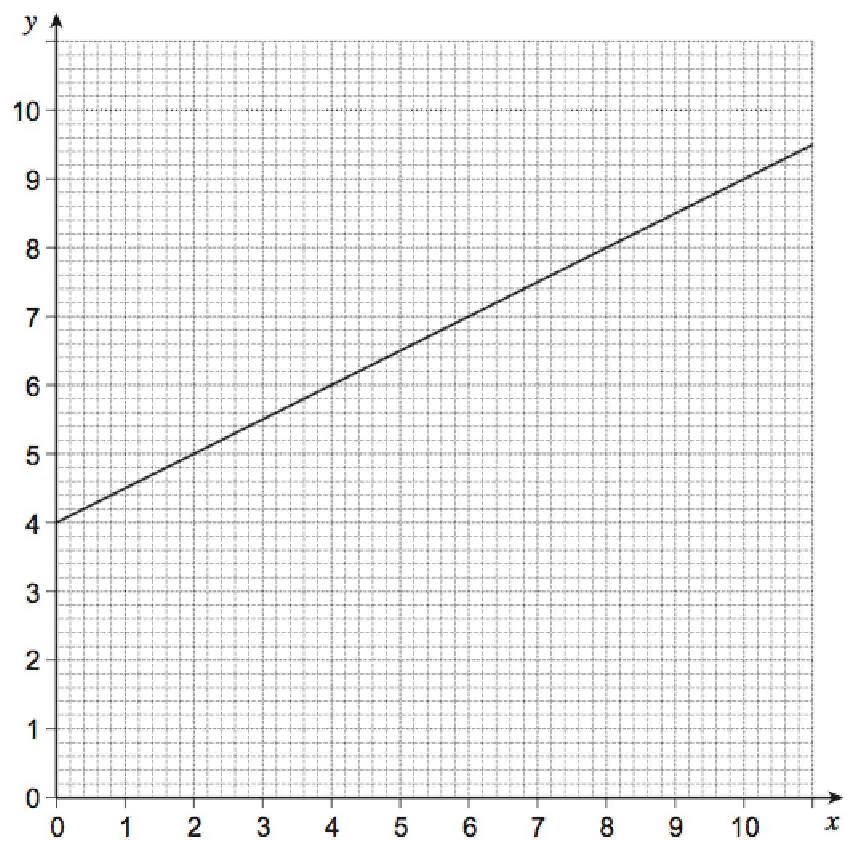
18. Solve (4)

$$\frac{18+5x}{3} = 10 - x.$$

Solution

$$\begin{aligned}\frac{18+5x}{3} = 10 - x &\Rightarrow 18 + 5x = 3(10 - x)\\ &\Rightarrow 18 + 5x = 30 - 3x\\ &\Rightarrow 8x = 12\\ &\Rightarrow 8\underline{\underline{x = 1\tfrac{1}{2}}}.\end{aligned}$$

19. Work out the equation of the line shown. (3)



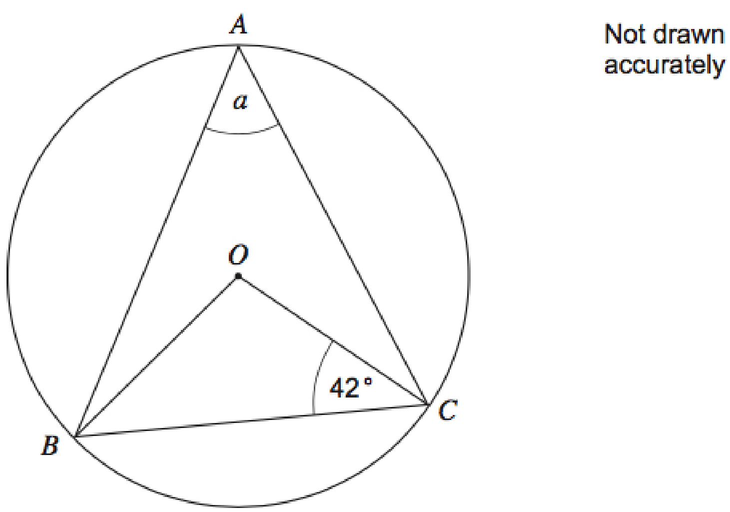
Solution

$$\begin{aligned}\text{Gradient} &= \frac{9-4}{10-0} \\ &= \frac{5}{10} \\ &= \tfrac{1}{2}\end{aligned}$$

and, when $x = 0$, $y = 4$. Hence, the equation of the line shown is

$$\underline{\underline{y = \tfrac{1}{2}x + 4}}.$$

20. The diagram shows a circle, centre O. (3)

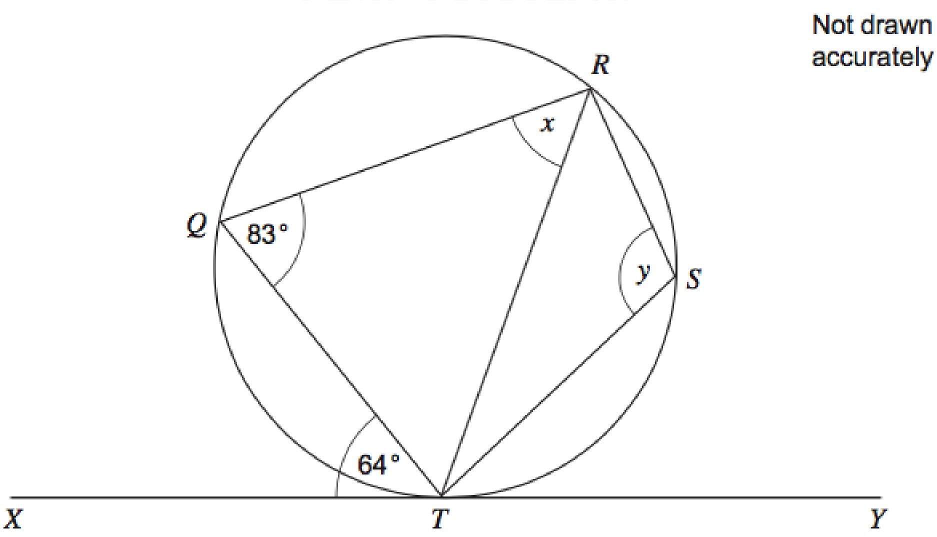


Work out the value of a.

Solution

$\angle OBC = 42^\circ$ (base angles)
$\angle BOC = 180 - 42 - 42 = 96^\circ$ (complete the triangle)
$\angle BAC = \frac{1}{2}\angle BOC = \frac{1}{2} \times 96 = \underline{\underline{48^\circ}}$ (angle at the centre is twice the angle at the circumference)

21. XTY is a tangent to the circle.



(a) Write down the value of x. (2)
Give a reason for your answer.

Solution
$x = \underline{\underline{64^\circ}}$, alternating sement theorem.

(b) Work out the value of y. (1)

Solution
$y = 180 - 83 = \underline{\underline{97^\circ}}$ (opposite angle in a cyclic quadrilateral are supplementary)

22. The table shows the probabilities that I am on time or late for work each day. (5)

It also shows the amount of pay deducted for being late each day.

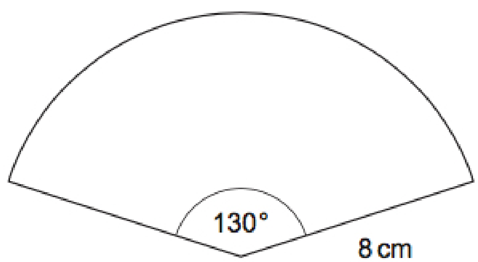
	On Time	Up to 30 minutes late	30 minutes to 1 hour late
Probability	0.6	0.3	0.1
Amount Deducted	—	£8	£16

Work out the probability that I have exactly £16 deducted **over two days**.

Solution

$$\begin{aligned}
\text{P}(\pounds 16) &= \text{P(on time, 30+)} + \text{P(less than 30, less than 30)} + \text{P(30+, on time)} \\
&= (0.6 \times 0.1) + (0.3 \times 0.3) + (0.1 \times 0.6) \\
&= 0.06 + 0.09 + 0.06 \\
&= \underline{\underline{0.21}}.
\end{aligned}$$

23. The diagram shows a sector of a circle. (4)



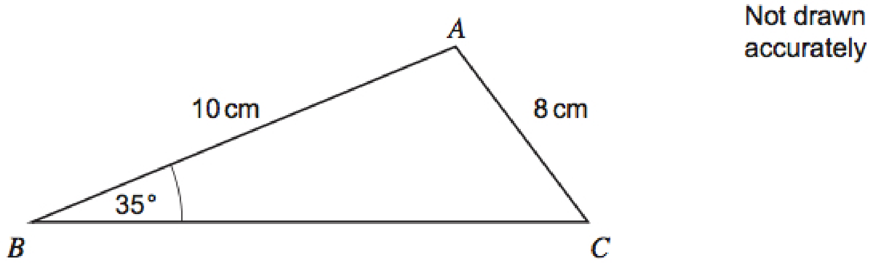
Not drawn accurately

Work out the area of the sector.
Give your answer to a suitable degree of accuracy.

Solution

$$\begin{aligned}
\text{Area} &= \frac{130}{360} \times \pi \times 8^2 \\
&= 72.605\,696\,88 \text{ (FCD)} \\
&= \underline{\underline{72.6 \text{ cm}^2 \text{ (3 sf)}}}.
\end{aligned}$$

24. In the diagram, angle A is obtuse. (4)



Work out the size of angle A.

Solution

$$\frac{\sin\angle BCA}{AB} = \frac{\sin\angle ABC}{AC} \Rightarrow \frac{\sin\angle BCA}{10} = \frac{\sin 35^\circ}{8}$$
$$\Rightarrow \sin\angle BCA = \frac{10\sin 35^\circ}{8}$$
$$\Rightarrow \angle BCA = 45.804\,926,614 \text{ (FCD) (because } A \text{ is obtuse)}$$
$$\Rightarrow \angle BAC = 180 - 35 - 45.804\ldots$$
$$\Rightarrow \angle BAC = 99.195\,073\,86 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle BAC = 99.2^\circ \text{ (1 dp)}}}.$$

25. n is a positive integer. (4)

Prove that

$$n^2 + 3n + 2$$

must be a multiple of 2.

Solution

$$\left.\begin{array}{ll} \text{add to:} & +3 \\ \text{multiply to:} & +2 \end{array}\right\} +1,\ +2$$

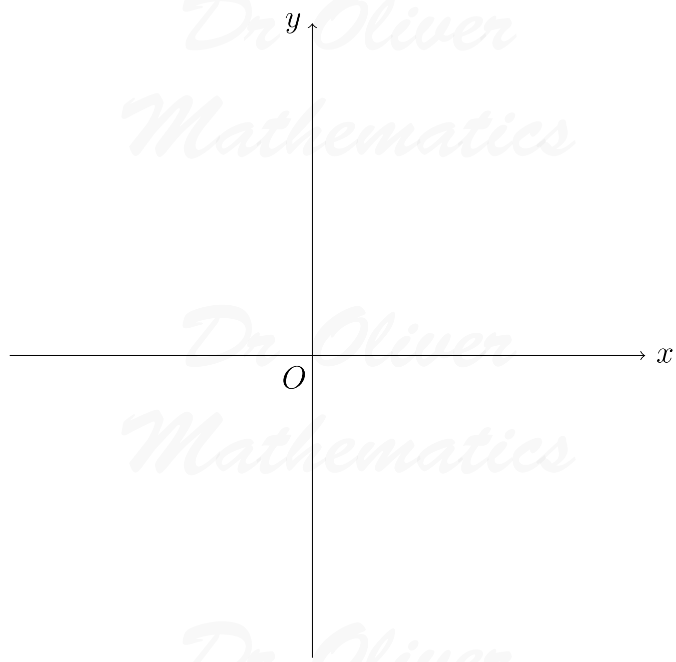
So

$$\begin{aligned} & n^2 + 3n + 2 \\ = \ & (n+1)(n+2) \\ = \ & \text{one even number} \times \text{one odd number OR one odd number} \times \text{one even number} \\ = \ & \text{one even number;} \end{aligned}$$

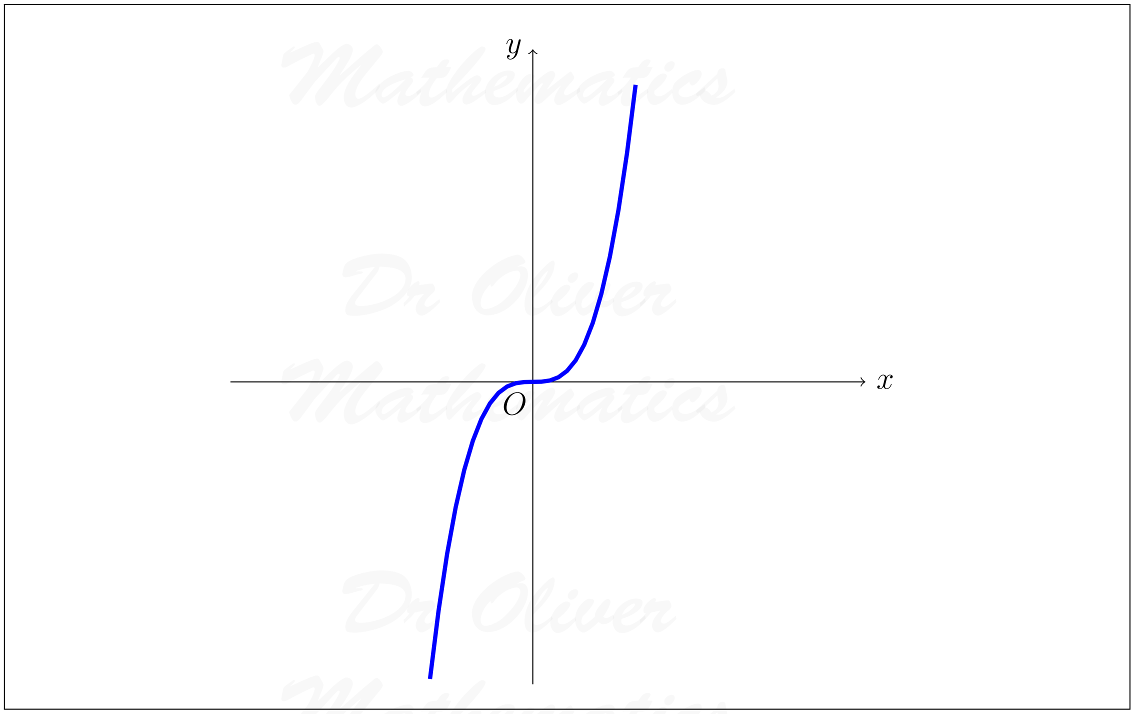
hence, $n^2 + 3n + 2$ must be a multiple of $\underline{\underline{2}}$.

26. (a) On the axes, make a sketch of (1)

$$y = x^3.$$



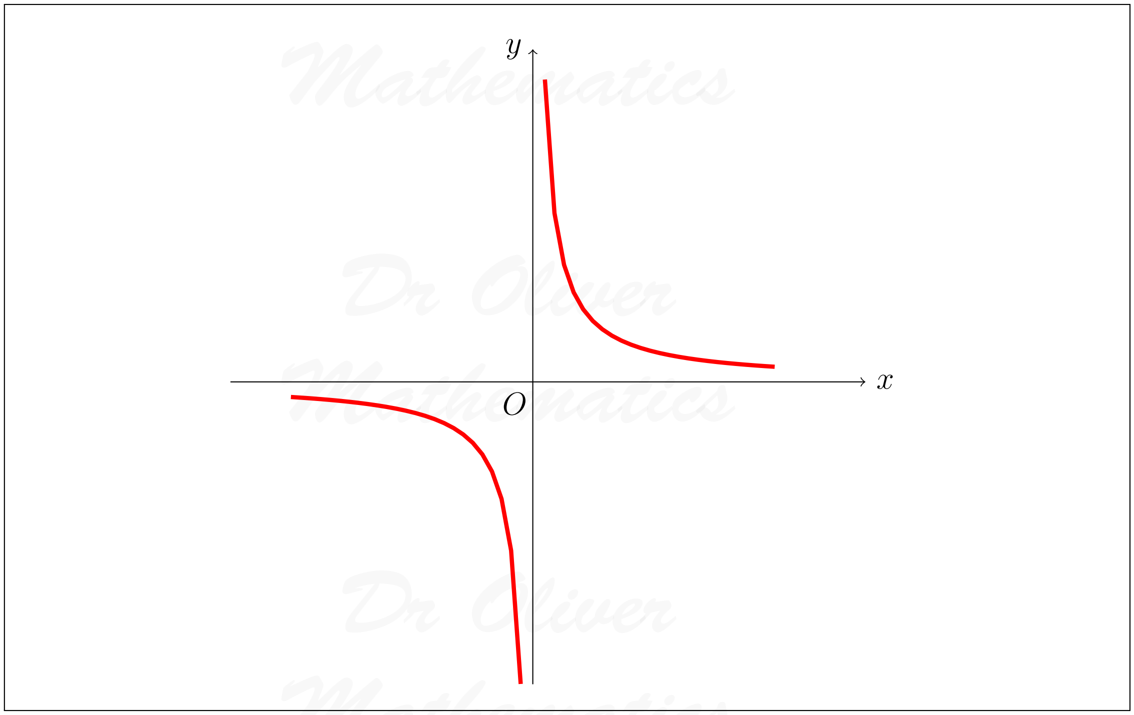
Solution



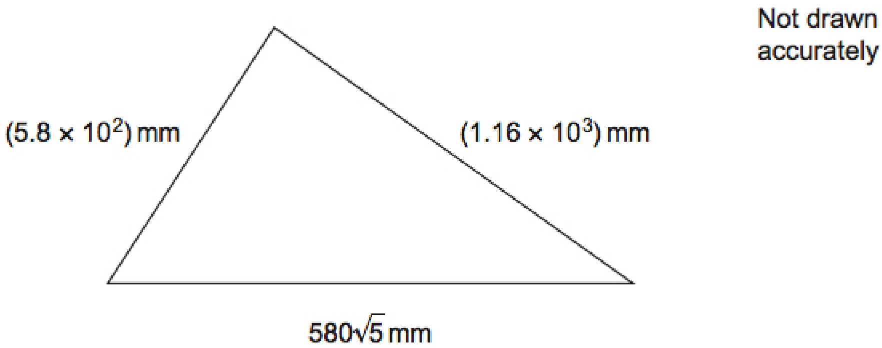
(b) On the axes, make a sketch of (1)

$$y = \frac{1}{x}.$$

Solution



27. Is this a right-angled triangle? (4)



You **must** justify your answer.

Solution

$$\begin{aligned} \text{`opp'}^2 + \text{`adj'}^2 &= (5.8 \times 10^2)^2 + (1.16 \times 10^3)^2 \\ &= 1\,682\,000 \end{aligned}$$

and

$$\begin{aligned}\text{`hyp'}^2 &= (580\sqrt{5})^2 \\ &= 1\,682\,000.\end{aligned}$$

So, $\underline{\underline{\text{yes}}}$, this is a right-angled triangle because for it follows Pythagoras' theorem.

28. Solve the simultaneous equations (5)

$$y = 10 - xy \qquad\qquad\qquad\qquad = 2x^2 + 4.$$

Solution

Compare the two expressions for y:

$$2x^2 + 4 = 10 - x \Rightarrow 2x^2 + x - 6 = 0$$

$$\left.\begin{array}{ll}\text{add to:} & \qquad\qquad +1 \\ \text{multiply to:} & (+2) \times (-6) = -12\end{array}\right\} -3,\ +4$$

$$\begin{aligned}&\Rightarrow 2x^2 + 4x - 3x - 6 = 0 \\ &\Rightarrow 2x(x+2) - 3(x+2) = 0 \\ &\Rightarrow (2x-3)(x+2) = 0 \\ &\Rightarrow 2x - 3 = 0 \text{ or } x + 2 = 0 \\ &\Rightarrow x = 1\tfrac{1}{2} \text{ or } x = -2 \\ &\Rightarrow y = 8\tfrac{1}{2} \text{ or } y = 12;\end{aligned}$$

hence, the solutions are

$$\underline{\underline{x = 1\tfrac{1}{2},\ y = 8\tfrac{1}{2} \text{ or } x = -2,\ y = 12.}}$$