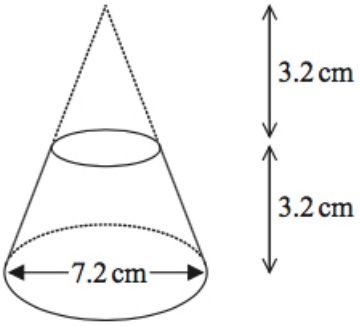
Dr Oliver Mathematics
Worked Examples
Mass, Density, and Volume 1

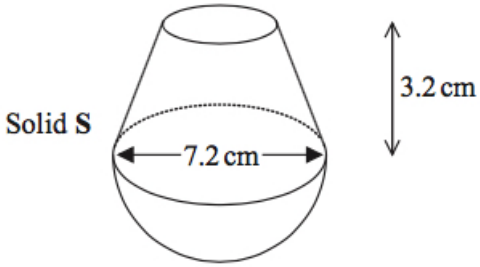
From: Edexcel 2018 November Paper 3H (Calculator)

1. Here is a frustum of a cone. (5)



The diagram shows that the frustum is made by removing a cone with height 3.2 cm from a solid cone with height 6.4 cm and base diameter 7.2 cm.

The frustum is joined to a solid hemisphere of diameter 7.2 cm to form the solid **S** shown below.



The density of the frustum is 2.4 g/cm^3.
The density of the hemisphere is 4.8 g/cm^3.

Calculate the average density of solid **S**.

Solution

Frustum:
We will cut the cone in half: radii are 1.8 cm (top) and 3.6 cm (bottom). Now,

$$\begin{aligned}\text{volume of the frustum} &= \text{volume of the whole cone} - \text{volume of the bit removed}\\ &= \tfrac{1}{3}\pi(3.6^2)(6.4) - \tfrac{1}{3}\pi(1.8^2)(3.2)\\ &= 27.648\pi - 3.456\pi\\ &= 24.192\pi\end{aligned}$$

and

$$\begin{aligned}\text{mass} &= \text{density} \times \text{volume}\\ &= 2.4 \times 24.192\pi\\ &= 58.060\,8\pi.\end{aligned}$$

Hemisphere:
Well,

$$\begin{aligned}\text{volume of the hemisphere} &= \frac{\text{volume of the sphere}}{2}\\ &= \frac{\frac{4}{3}\pi(3.6^3)}{2}\\ &= \tfrac{2}{3}\pi(3.6^3)\\ &= 31.104\pi\end{aligned}$$

and

$$\begin{aligned}\text{mass} &= \text{density} \times \text{volume}\\ &= 4.8 \times 31.104\pi\\ &= 149.299\,2\pi.\end{aligned}$$

In total:
The overall mass is

$$58.060\,8\pi + 149.299\,2\pi = 207.36\pi$$

and the overall volume is

$$24.192\pi + 31.104\pi = 55.296\pi.$$

Hence,

$$\begin{aligned}\text{average density} &= \frac{207.36\pi}{55.296\pi}\\ &= \underline{\underline{3.75\ \text{g/cm}^3}}.\end{aligned}$$