## Dr Oliver Mathematics GCSE Mathematics 2012 November Paper 2H: Calculator 1 hour 45 minutes

The total number of marks available is 100 .
You must write down all the stages in your working.

1. Use a calculator to work out

$$
\begin{equation*}
\frac{\sqrt{20.4}}{6.2 \times 0.48} \tag{2}
\end{equation*}
$$

Write down all the figures on your calculator display.
Give your answer as a decimal.

$$
\begin{aligned}
& \text { Solution } \\
& \qquad \begin{aligned}
\frac{\sqrt{20.4}}{6.2 \times 0.48} & =\frac{\sqrt{20.4}}{2.976} \\
& =\underline{\underline{1.5176868(\mathrm{FCD})} .}
\end{aligned}
\end{aligned}
$$

2. (a) Reflect shape $\mathbf{P}$ in the line $y=x$.


Solution

Mantrematics

(b) Describe fully the single transformation that maps triangle $\mathbf{A}$ onto triangle $\mathbf{B}$.


## Solution



3. A company sells boxes to factories.

Fred buys boxes.
The boxes are sold in packs of 1000 .
Each pack costs £193.86.
Fred orders 3 packs of boxes.
He gets a discount on his total order.
The table shows the discount he will get.

| Total Order | Discount |
| :---: | :---: |
| $£ 100-£ 300$ | $5 \%$ |
| $£ 301-£ 400$ | $10 \%$ |
| $£ 401$ and above | $15 \%$ |

Work out the total cost of the order after the discount.
You must show your working.

## Solution

$$
3 \times 193.86=581.58
$$

and the

$$
\begin{aligned}
\text { total cost } & =581.58 \times(1-0.15) \\
& =581.58 \times 0.85 \\
& =494.343
\end{aligned}
$$

the order will cost Fred $£ 494.34$ or $£ 494.35$.
4. The table gives some information about the birds Paula sees in her garden one day.

| Bird | Frequency |
| :---: | :---: |
| Magpie | 15 |
| Thrush | 10 |
| Starling | 20 |
| Sparrow | 27 |

Complete the accurate pie chart.


## Solution

The total number of birds is

$$
15+10+20+27=72
$$

Magpie:

$$
\frac{15}{72} \times 360=75^{\circ}
$$

Thrush:

$$
\frac{10}{72} \times 360=50^{\circ}
$$

Starling:

$$
\frac{20}{72} \times 360=100^{\circ}
$$

Sparrow:

$$
\frac{27}{72} \times 360=135^{\circ}
$$

E.g.,
5. $A, B$, and $C$ are three service stations on a motorway.

$A B=25$ miles.
$B C=25$ miles.
Aysha drives along the motorway from $A$ to $C$.
Aysha drives at an average speed of 50 mph from $A$ to $B$.
She drives at an average speed of 60 mph from $B$ to $C$.
Work out the difference in the time Aysha takes to drive from $A$ to $B$ and the time Aysha takes to drive from $B$ to $C$.
Give your answer in minutes.

## Solution

From $A$ to $B$, the time is

$$
\frac{25}{50}=0.5 \text { hours }=30 \text { minutes } .
$$

From $B$ to $C$, the time is

$$
\frac{25}{60}=0.41 \dot{6} \text { hours }=25 \text { minutes } .
$$

Hence, the difference is

$$
30-25=\underline{\underline{5 \text { minutes }}}
$$

6. $A B C D$ is a parallelogram.


Angle $A D B=38^{\circ}$.
Angle $B E C=41^{\circ}$.
Angle $D A B=120^{\circ}$.
Calculate the size of angle $x$.
You must give reasons for your answer.

## Solution

$\angle C D E=180-120-38=22^{\circ}$ (interior angles).
$\angle C E D=180-41=139^{\circ}$ (supplementary angles).
$\angle D C E=180-139-22=\underline{\underline{19^{\circ}}}$ (completing the triangle).
7. 160 cm of gold wire has a weight of 17.8 grams.

Work out the weight of 210 cm of the gold wire.

## Solution

$$
\frac{210}{160} \times 17.8=\underline{\underline{23.3625 \text { grams }}}
$$

8. (a) $n$ is an integer.
$-1 \leqslant n<4$.
List the possible values of $n$.

## Solution

$\underline{\underline{-1,0,1,2,3}}$.
(b) Write down the inequality shown in the diagram.


## Solution

$$
-4<x \leqslant 3 \text {. }
$$

(c) Solve

$$
3 y-2>5
$$

## Solution

$$
\begin{aligned}
3 y-2>5 & \Rightarrow 3 y>7 \\
& \Rightarrow y>2 \frac{1}{3}
\end{aligned}
$$

9. The stem and leaf diagram gives information about the numbers of tomatoes on 31 tomato plants.

| 0 | 8 | 8 | 9 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 1 | 5 | 5 |  |  |  |
| 2 | 1 | 2 | 2 | 6 | 7 | 8 | 8 |
| 3 | 0 | 2 | 5 | 5 | 7 | 9 |  |
| 4 | 2 | 2 | 3 | 5 | 8 | 8 |  |
| 5 | 1 | 1 | 3 | 4 | 7 |  |  |

(a) Work out the median.

## Solution

The median is the

$$
\frac{31+1}{2}=16 \text { th entry }
$$

and the median is $\underline{\underline{32}}$.
(b) Work out the interquartile range.

Solution

$$
\mathrm{IQR}=45-21=\underline{\underline{24}} .
$$

10. In the UK, petrol cost $£ 1.24$ per litre.

In the USA, petrol cost 3.15 dollars per US gallon.
1 US gallon $=3.79$ litres.
$£ 1=1.47$ dollars .
Was petrol cheaper in the UK or in the USA?

## Solution

In the USA, petrol costs

$$
\frac{3.15}{3.79 \times 1.47}=0.565397663(\mathrm{FCD})
$$

hence, it is cheaper in the USA.
11. The diagram shows a cube and a cuboid.


All the measurements are in cm .
The volume of the cube is $100 \mathrm{~cm}^{3}$ more than the volume of the cuboid.
(a) Show that

$$
\begin{equation*}
x^{3}-10 x=100 . \tag{2}
\end{equation*}
$$

## Solution

The volume of the cube is

$$
x \times x \times x=x^{3}
$$

and the volume of the cuboid is

$$
2 \times 5 \times x=10 x
$$

Finally, the is a difference of 100 :

$$
x^{3}-10 x=100
$$

(b) Use a trial and improvement method to find the value of $x$.

Give your answer correct to 1 decimal place.
You must show all your working.

## Solution

| $x$ | $x^{3}-10 x$ | Comment |
| :--- | :--- | :--- |
| 3 | -3 | too low |
| 4 | 24 | too low |
| 5 | 75 | too low |
| 6 | 156 | too high |

So, the answer is between 5 and 6 .
You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.
$\mathbf{F}(\mathbf{X})=$ and you type in $\mathrm{X}^{3}-10 \mathrm{X}$; then you press $\equiv$.
Start? and you enter 5; then you press $\equiv$.
End? and you enter 6; then you press $\equiv$.
Step? and enter 0.05 - 1 decimal place divided by 2 ; then you press $\equiv$.

| $x$ | $\mathrm{f}(x)$ | Comment |
| :--- | :--- | :--- |
| 5.35 | 99.63 | too low |
| 5.4 | 103.46 | too high |

Clearly,

$$
5.35<x<5.4
$$

and the answer is

$$
x=5.4(1 \mathrm{dp}) .
$$

12. The frequency table gives information about the times it took some office workers to get to the office one day.

| Time $(t$ minutes $)$ | Frequency |
| :---: | :---: |
| $0<t \leqslant 10$ | 4 |
| $10<t \leqslant 20$ | 8 |
| $20<t \leqslant 30$ | 14 |
| $30<t \leqslant 40$ | 16 |
| $40<t \leqslant 50$ | 6 |
| $50<t \leqslant 60$ | 2 |

(a) Draw a frequency polygon for this information.


## Solution

Mathematics

[^0]
(b) Write down the modal class interval.

## Solution

$30<t \leqslant 40$.

One of the office workers is chosen at random.
(c) Work out the probability that this office worker took more than 40 minutes to get to the office.

## Solution

$$
4+8+14+16+6+2=50
$$

and the probability that this office worker took more than 40 minutes to get to the office is

$$
\frac{8}{50}=\frac{4}{\underline{\underline{25}}} .
$$

13. The diagram shows a solid triangular prism.


The prism is made from metal.
The density of the metal is 6.6 grams per $\mathrm{cm}^{3}$.
Calculate the mass of the prism.

## Solution

$$
\begin{aligned}
\text { Cross-sectional area } & =\frac{1}{2} \times 5 \times 12 \\
& =30 \mathrm{~cm}^{2}
\end{aligned}
$$

and the

$$
\begin{aligned}
\text { volume } & =30 \times 15 \\
& =450 \mathrm{~cm}^{3} .
\end{aligned}
$$

Finally,

$$
\begin{aligned}
\text { mass } & =450 \times 6.6 \\
& =\underline{\underline{2970 \mathrm{~g}}}
\end{aligned}
$$

14. (a) Factorise

$$
\begin{equation*}
x^{2}+7 x \tag{1}
\end{equation*}
$$

## Solution

$$
x^{2}+7 x=x(x+7) .
$$

(b) Factorise

$$
\begin{equation*}
y^{2}-10 y+16 \tag{2}
\end{equation*}
$$

## Solution

$$
\begin{array}{cc}
\left.\begin{array}{ll}
\text { add to: } & -10 \\
\text { multiply to: } & +16
\end{array}\right\}-2,-8 \\
y^{2}-10 y+16 & =\underline{\underline{(y-2)(y-8)}}
\end{array}
$$

(c) (i) Factorise

$$
2 t^{2}+5 t+2
$$

## Solution

$$
\left.\begin{array}{lc}
\text { add to: } & +5 \\
\text { multiply to: } & (+2) \times(+2)=+4
\end{array}\right\}+1,+4
$$

E.g.,

$$
\begin{aligned}
2 t^{2}+5 t+2 & =2 t^{2}+4 t+t+2 \\
& =2 t(t+2)+1(t+2) \\
& =\underline{\underline{(2 t+1)(t+2)}} .
\end{aligned}
$$

(ii) $t$ is a positive whole number.

The expression

$$
2 t^{2}+5 t+2
$$

can never have a value that is a prime number.
Explain why.

## Solution

E.g., this is always a product of two whole numbers each of which is greater than 1.
15. $A B C D$ is a trapezium.

$A D=10 \mathrm{~cm}$.
$A B=9 \mathrm{~cm}$.
$D C=3 \mathrm{~cm}$.
Angle $A B C=$ angle $B C D=90^{\circ}$.
Calculate the length of $A C$.
Give your answer correct to 3 significant figures.

## Solution

$$
9-3=6 \mathrm{~cm}
$$

and

$$
\begin{aligned}
B C & =\sqrt{10^{2}-6^{2}} \\
& =\sqrt{64} \\
& =8 .
\end{aligned}
$$

Hence,

$$
\begin{aligned}
A C & =\sqrt{9^{2}+8^{2}} \\
& =\sqrt{145} \\
& =12.04159458(\mathrm{FCD}) \\
& =12.0 \mathrm{~cm}(3 \mathrm{sf}) .
\end{aligned}
$$

16. Bill's weight decreases from 64.8 kg to 59.3 kg .

Calculate the percentage decrease in Bill's weight. Give your answer correct to 3 significant figures.

## Solution

$$
\begin{aligned}
\text { Percentage decrease } & =\left(\frac{64.8-59.3}{64.8}\right) \times 100 \% \\
& =\frac{5.5}{64.8} \times 100 \% \\
& =8.487654321(\mathrm{FCD}) \\
& =\underline{\underline{8.49 \%(3 \mathrm{sf})}} .
\end{aligned}
$$

17. Here is a diagram.


Calculate the value of $x$.
Give your answer correct to 3 significant figures.

| Solution |  |
| :--- | :--- |
|  | $\mathrm{opp}=\mathrm{hyp} \times \sin$ $\Rightarrow x=32 \sin 60^{\circ}$ <br>  $\Rightarrow x=27.71281292(\mathrm{FCD})$ <br>  $=\underline{\underline{27.7 \mathrm{~cm}(3 \mathrm{sf})} .}$ |

18. (a) Complete the table of values for

$$
\begin{equation*}
y=\frac{6}{x} \tag{2}
\end{equation*}
$$

| $x$ | 0.5 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ |  | 6 | 3 |  | 1.5 |  | 1 |

## Solution

| $x$ | 0.5 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $\underline{12}$ | 6 | 3 | $\underline{\underline{2}}$ | 1.5 | $\underline{\underline{1.2}}$ | 1 |

(b) On the grid, draw the graph of

$$
y=\frac{6}{x}
$$

for $0.5 \leqslant x \leqslant 6$.


## Solution


19. Rob is learning about the planets.

Rob makes a model of the Sun.
He also makes a model of the planet Jupiter.
Rob is going to hang the two models in the school hall.
Rob wants a distance of 16 m between the two models.
The real distance between the planet Jupiter and the Sun is $8 \times 10^{8} \mathrm{~km}$.
Work out the scale Rob should use.
Give your answer in the form $1: n$.

## Solution

$$
\begin{aligned}
16 \mathrm{~m}: 8 \times 10^{8} \mathrm{~km} & \Rightarrow 16 \mathrm{~m}: 8000 \times 10^{8} \mathrm{~m} \\
& \Rightarrow 1: 500 \times 10^{8} \\
& \Rightarrow \underline{\underline{1: 5 \times 10^{10}}} .
\end{aligned}
$$

20. Simplify

$$
\begin{equation*}
\frac{x+1}{2}+\frac{x+3}{3} . \tag{3}
\end{equation*}
$$

## Solution

$$
\begin{aligned}
\frac{x+1}{2}+\frac{x+3}{3} & =\frac{3(x+1)}{6}+\frac{2(x+3)}{6} \\
& =\frac{3(x+1)+2(x+3)}{6} \\
& =\frac{3 x+3+2 x+6}{6} \\
& =\underline{=\frac{5 x+9}{6}} .
\end{aligned}
$$

21. Here are seven tiles.


Jim takes at random a tile.
He does not replace the tile.
Jim then takes at random a second tile.
(a) Calculate the probability that both the tiles Jim takes have the number 1 on them.

## Solution

| $\mathrm{P}(1,1)$ | $=\frac{2}{7} \times \frac{1}{6}$ |
| ---: | :--- |
|  | $=\frac{2}{42}$ |
|  | $=\frac{1}{\underline{21}}$. |

(b) Calculate the probability that the number on the second tile Jim takes is greater than the number on the first tile he takes.

## Solution

$$
\begin{aligned}
\mathrm{P}(\text { greater }) & =\left(\frac{2}{7} \times \frac{5}{6}\right)+\left(\frac{3}{7} \times \frac{2}{6}\right) \\
& =\frac{10}{42}+\frac{6}{42} \\
& =\frac{16}{42} \\
& =\frac{4}{21} .
\end{aligned}
$$

22. (a) Solve

$$
2 x^{2}+9 x-7=0 .
$$

Give your solutions correct to 3 significant figures.

## Solution

$a=2, b=9$, and $c=-7$ :

$$
\begin{aligned}
x & =\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
& =\frac{-9 \pm \sqrt{9^{2}-4 \times 2 \times(-7)}}{2 \times 2} \\
& =\frac{-9 \pm \sqrt{137}}{4} \\
& =-5.176174978 \text { or } 0.6761749777(\mathrm{FCD}) \\
& =-5.18 \text { or } 0.676(3 \mathrm{sf}) .
\end{aligned}
$$

(b) Solve

$$
\frac{2}{y^{2}}+\frac{9}{y}-7=0
$$

Give your solutions correct to 3 significant figures.

## Solution

Let

$$
y=\frac{1}{x}:
$$

then

$$
\begin{aligned}
y & =-0.1931928508 \text { or } 1.478907136(\mathrm{FCD}) \\
& =\underline{\underline{-0.193 ~ o r ~} 1.48(3 \mathrm{sf}) .}
\end{aligned}
$$

23. The diagram shows a pyramid.

$B C D E$ is a square with sides of length 10 cm .
The other faces of the pyramid are equilateral triangles with sides of length 10 cm .
(a) Calculate the volume of the pyramid.

Give your answer correct to 3 significant figures.

## Solution

Let $F$ be the midpoint of the square. Now,

$$
\begin{aligned}
B D & =\sqrt{10^{2}+10^{2}} \\
& =\sqrt{200} \\
& =10 \sqrt{2}
\end{aligned}
$$

and

$$
F D=5 \sqrt{2}
$$

Next,

$$
\begin{aligned}
A F & =\sqrt{10^{2}-(5 \sqrt{2})^{2}} \\
& =\sqrt{50} \\
& =5 \sqrt{2} .
\end{aligned}
$$

Finally,

$$
\begin{aligned}
\text { volume } & =\frac{1}{3} \times 10 \times 10 \times 5 \sqrt{2} \\
& =235.7022604(\mathrm{FCD}) \\
& =\underline{\underline{236} \mathrm{~cm}^{3}(3 \mathrm{sf})} .
\end{aligned}
$$

(b) Find the size of angle $D A B$.

## Solution

Let the size of the angle be $x^{\circ}$. Then,

$$
\begin{aligned}
& \cos x^{\circ}=\frac{A B^{2}+A D^{2}-B D^{2}}{2 \times A B \times A D} \\
\Rightarrow & \cos x^{\circ}=\frac{10^{2}+10^{2}-(10 \sqrt{2})^{2}}{2 \times 10 \times 10} \\
\Rightarrow & \cos x^{\circ}=0 \\
\Rightarrow & \underline{x^{\circ}=90^{\circ}} .
\end{aligned}
$$

24. The table gives information about the heights, $h$ metres, of trees in a wood.

| Height $(h$ metres $)$ | Frequency |
| :---: | :---: |
| $0<h \leqslant 2$ | 7 |
| $2<h \leqslant 4$ | 14 |
| $4<h \leqslant 8$ | 18 |
| $8<h \leqslant 16$ | 24 |
| $16<h \leqslant 20$ | 10 |

Draw a histogram to show this information.


## Solution

| Height $(h$ metres $)$ | Frequency | Width | Frequency Density |
| :---: | :---: | :---: | :---: |
| $0<h \leqslant 2$ | 7 | 2 | $\frac{7}{2}=3.5$ |
| $2<h \leqslant 4$ | 14 | 2 | $\frac{14}{2}=7$ |
| $4<h \leqslant 8$ | 18 | 4 | $\frac{18}{4}=4.5$ |
| $8<h \leqslant 16$ | 24 | 8 | $\frac{24}{8}=3$ |
| $16<h \leqslant 20$ | 10 | 4 | $\frac{10}{4}=2.5$ |


25. The diagram shows the triangle PQR .

$P Q=x \mathrm{~cm}$.
$P R=2 x \mathrm{~cm}$.
Angle $Q P R=30^{\circ}$.
The area of triangle $P Q R=A \mathrm{~cm}^{2}$.
Show that $x=\sqrt{2 A}$.
$\square$
Solution

$$
\begin{aligned}
& A=\frac{1}{2} \times x \times 2 x \times \sin 30^{\circ} \\
\Rightarrow \quad A & =\frac{1}{2} x^{2} \\
\Rightarrow \quad & x^{2}=2 A \\
\Rightarrow \quad & x=\sqrt{2 A},
\end{aligned}
$$

as required.


[^0]:    On $\qquad$

