Dr Oliver Mathematics GCSE Mathematics 2012 November Paper 2H: Calculator 1 hour 45 minutes

The total number of marks available is 100. You must write down all the stages in your working.

1. Use a calculator to work out

 $\frac{\sqrt{20.4}}{6.2 \times 0.48}$

Write down all the figures on your calculator display. Give your answer as a decimal.

Solution	On Oliver	
	$\frac{\sqrt{20.4}}{6.2 \times 0.48} = \frac{\sqrt{20.4}}{2.076}$	
	= 1.5176868 (FCD).	

2. (a) Reflect shape **P** in the line y = x.

Mathematics

(2)





(b) Describe fully the single transformation that maps triangle **A** onto triangle **B**.







3. A company sells boxes to factories. Fred buys boxes.

The boxes are sold in packs of $1\,000$.

Each pack costs £193.86.

Fred orders 3 packs of boxes.

He gets a discount on his total order.

The table shows the discount he will get.

Total Order	Discount
£100-£300	5%
$\pounds 301 - \pounds 400$	10%
$\pounds401$ and above	15%

Work out the total cost of the order after the discount. You must show your working. (5)

Solution	
$3 \times 193.86 = 581.58$	
and the	
total cost = $581.58 \times (1 - 0.15)$	
$= 581.58 \times 0.85$	
= 494.343;	
the order will cost Fred $\underline{\pounds494.34}$ or $\underline{\pounds494.35}$.	

4. The table gives some information about the birds Paula sees in her garden one day.

(3)

Bird	Frequency
Magpie	15
Thrush	10
Starling	20
Sparrow	27

Complete the accurate pie chart.





5. A, B, and C are three service stations on a motorway.



AB = 25 miles. BC = 25 miles. Aysha drives along the motorway from A to C. Aysha drives at an average speed of 50 mph from A to B. She drives at an average speed of 60 mph from B to C. Work out the difference in the time Aysha takes to drive from A to B and the time Aysha takes to drive from B to C. Give your answer in minutes.



(4)

6. ABCD is a parallelogram.



Angle $ADB = 38^{\circ}$. Angle $BEC = 41^{\circ}$. Angle $DAB = 120^{\circ}$. Calculate the size of angle x. You must give reasons for your answer.

- 7. 160 cm of gold wire has a weight of 17.8 grams. Work out the weight of 210 cm of the gold wire.
- 8. (a) n is an integer. $-1 \le n < 4.$

Solution

List the possible values of n.

- Solution -1, 0, 1, 2, 3.
- (b) Write down the inequality shown in the diagram.





(c) Solve

3y - 2 > 5.



Solution

$$3y - 2 > 5 \Rightarrow 3y > 7$$
$$\Rightarrow \underline{y > 2\frac{1}{3}}.$$

9. The stem and leaf diagram gives information about the numbers of tomatoes on 31 tomato plants.

0	8	8	9					
1	1	1	5	5				
2	1	2	2	6	7	8	8	Key: $5 \mid 7 = 57$ tomatoes
3	0	2	5	5	7	9		Key. 5 / 7 57 tollatoes
4	2	2	3	5	8	8		
5	1	1	3	4	7			

(a) Work out the median.

Solution



(b) Work out the interquartile range.

$$IQR = 45 - 21 = 24$$

10. In the UK, petrol cost £1.24 per litre.
In the USA, petrol cost 3.15 dollars per US gallon.
1 US gallon = 3.79 litres.
£1 = 1.47 dollars.

Was petrol cheaper in the UK or in the USA?

(2)

(4)

(1)

Solution

In the USA, petrol costs

$$\frac{3.15}{3.79 \times 1.47} = 0.565\,397\,663 \text{ (FCD)};$$

hence, it is cheaper in the \underline{USA} .

11. The diagram shows a cube and a cuboid.



All the measurements are in cm.

The volume of the cube is 100 cm^3 more than the volume of the cuboid.

(a) Show that

$$x^3 - 10x = 100.$$

Solution

The volume of the cube is

$$x \times x \times x = x^3$$

and the volume of the cuboid is

$$2 \times 5 \times x = 10x.$$

Finally, the is a difference of 100:

$$x^3 - 10x = 100.$$

(b) Use a trial and improvement method to find the value of x. Give your answer correct to 1 decimal place.You must show all your working.

(4)

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Solution								
	x	$x^3 - 10x$	Comment					
	3	-3	too low	-				
	4	24	too low					
	5	75	too low					
	6	156	too high					
F(X) = and you type in . Start? and you enter 5;	$X^3 - 1$ then y	10X; then vou press	you press = = .	=].	(-991	L) IT	15 1010	
F(X) = and you type in Start? and you enter 5; End? and you enter 6; th Step? and enter $0.05 - 1$	$X^3 - 1$ then y hen yc decir	10X; then you press ou press [= nal place	you press = =]. =]. divided by 2	=]. 2; the	n yo	u pr	ess =].
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F(X)= and you type in Start? and you enter 5; End? and you enter 6; th Step? and enter 0.05 - 1	$X^{3} - 1$ then y hen y decir x 5.35 5.4	10X; then you press $[=]$ nal place f(x) 99.63 103.46	you press = =. divided by 2 Comment too low too high	2; the -	n yo	u pr	ess [=	
F(X)= and you type in Start? and you enter 5; End? and you enter 6; ti Step? and enter 0.05 – 1 Clearly, and the answer is	$X^{3} - 1$ then y hen y decir x 5.35 5.4	10X; then you press ou press [= nal place f(x) 99.63 103.46 5.35 < x x = 5.4 (you press = =]. =]. divided by 2 Comment too low too high = < 5.4 (1 dp)	=]. 2; the - -	n yo	u pr	ess =	

12. The frequency table gives information about the times it took some office workers to get to the office one day.

Time (t minutes)	Frequency
$0 < t \leqslant 10$	4
$10 < t \leq 20$	8
$20 < t \leqslant 30$	14
$30 < t \leq 40$	16
$40 < t \leqslant 50$	6
$50 < t \leqslant 60$	2

(a) Draw a frequency polygon for this information.



Mathematics

Dr Oliver Mathematics





(b) Write down the modal class interval.



One of the office workers is chosen at random.

(c) Work out the probability that this office worker took more than 40 minutes to get (2) to the office.

Solution

$$4 + 8 + 14 + 16 + 6 + 2 = 50$$

and the probability that this office worker took more than 40 minutes to get to the office is

$$\frac{8}{50} = \frac{4}{25}$$

13. The diagram shows a solid triangular prism.

(3)

(1)



The prism is made from metal. The density of the metal is 6.6 grams per cm³. Calculate the mass of the prism.



14. (a) Factorise

 $x^2 + 7x.$

(1)

Solution $x^2 + 7x = \underline{x(x+7)}.$ (2)

(b) Factorise

 $y^2 - 10y + 16.$

Dr Oliver

Solution	
	add to: $-10 \rangle - 2 - 8$
	multiply to: $+16 \int -7$, \circ
	$y^2 - 10y + 16 = \underline{(y-2)(y-8)}.$

(c) (i) Factorise

$$2t^2 + 5t + 2.$$

(3)

Solution	add to: $+5$
E.g.,	multiply to: $(+2) \times (+2) = +4$ $\left.\right\} + 1, +4$
	$2t^{2} + 5t + 2 = 2t^{2} + 4t + t + 2$ = 2t(t + 2) + 1(t + 2) = <u>(2t + 1)(t + 2)</u> .

(ii) t is a positive whole number.The expression

$$2t^2 + 5t + 2$$

can never have a value that is a prime number. Explain why.

Solution

E.g., this is always a product of two whole numbers each of which is greater than 1.

15. ABCD is a trapezium.

(5)





AD = 10 cm. AB = 9 cm. DC = 3 cm. Angle ABC = angle $BCD = 90^{\circ}$. Calculate the length of AC. Give your answer correct to 3 significant figures.



16. Bill's weight decreases from 64.8 kg to 59.3 kg.Calculate the percentage decrease in Bill's weight.Give your answer correct to 3 significant figures.



17. Here is a diagram.



Calculate the value of x. Give your answer correct to 3 significant figures.

Solution

opp = hyp × sin
$$\Rightarrow x = 32 \sin 60^{\circ}$$

 $\Rightarrow x = 27.712\,812\,92 \text{ (FCD)}$
 $= \underline{27.7 \text{ cm } (3 \text{ sf})}.$

18. (a) Complete the table of values for

$$y = \frac{6}{x}$$
.

(2)

	7)	7		2ℓ	in	297	
x	0.5	1	2	3	4	5	6
y		6	3		1.5	te	1

Solution							
	$x \mid 0.5$	1	2	3	4	5	6
	$y \mid \underline{12}$	6	3	$\underline{\underline{2}}$	1.5	<u>1.2</u>	1
	1101		2				

(b) On the grid, draw the graph of

$$y = \frac{6}{x}$$

for $0.5 \leq x \leq 6$.





(3)

19. Rob is learning about the planets. Rob makes a model of the Sun. He also makes a model of the planet Jupiter. Rob is going to hang the two models in the school hall. Rob wants a distance of 16 m between the two models. The real distance between the planet Jupiter and the Sun is 8×10^8 km. Work out the scale Rob should use. Give your answer in the form 1: n.

Solution

$$\begin{array}{l} 16 \ \mathrm{m}: 8 \times 10^8 \ \mathrm{km} \Rightarrow 16 \ \mathrm{m}: 8\,000 \times 10^8 \ \mathrm{m} \\ \Rightarrow 1: 500 \times 10^8 \\ \Rightarrow \underline{1:5 \times 10^{10}}. \end{array}$$

20. Simplify

$$\frac{x+1}{2} + \frac{x+3}{3}.$$

Solution

$$\frac{x+1}{2} + \frac{x+3}{3} = \frac{3(x+1)}{6} + \frac{2(x+3)}{6}$$

$$= \frac{3(x+1) + 2(x+3)}{6}$$

$$= \frac{3x+3+2x+6}{6}$$

$$= \frac{5x+9}{6}.$$

21. Here are seven tiles.

Jim takes at random a tile.

He does **not** replace the tile.

Jim then takes at random a second tile.

(a) Calculate the probability that both the tiles Jim takes have the number 1 on them.

(2)



$$P(1,1) = \frac{2}{7} \times \frac{1}{6}$$
$$= \frac{2}{42}$$
$$= \frac{1}{21}.$$

(b) Calculate the probability that the number on the second tile Jim takes is greater (3) than the number on the first tile he takes.

Solution

$$P(\text{greater}) = (\frac{2}{7} \times \frac{5}{6}) + (\frac{3}{7} \times \frac{2}{6}) \\
= \frac{10}{42} + \frac{6}{42} \\
= \frac{16}{42} \\
= \frac{4}{21}.$$

22. (a) Solve

 $2x^2 + 9x - 7 = 0.$

Give your solutions correct to 3 significant figures.

Solution a = 2, b = 9, and c = -7: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-9 \pm \sqrt{9^2 - 4 \times 2 \times (-7)}}{2 \times 2}$ $= \frac{-9 \pm \sqrt{137}}{4}$ $= -5.176 \, 174 \, 978 \text{ or } 0.676 \, 174 \, 977 \, 7 \text{ (FCD)}$ $= -5.18 \text{ or } 0.676 \, (3 \text{ sf}).$

(b) Solve

 $\frac{2}{y^2} + \frac{9}{y} - 7 = 0.$

Give your solutions correct to 3 significant figures.

(2)



23. The diagram shows a pyramid.



BCDE is a square with sides of length 10 cm.

The other faces of the pyramid are equilateral triangles with sides of length 10 cm.

(a) Calculate the volume of the pyramid.Give your answer correct to 3 significant figures.

Solution

Let F be the midpoint of the square. Now,

$$BD = \sqrt{10^2 + 10^2}$$
$$= \sqrt{200}$$
$$= 10\sqrt{2}$$
and
$$FD = 5\sqrt{2}.$$
Next,
$$AF = \sqrt{10^2 - (5\sqrt{2})^2}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}.$$

(4)

Finally,

Solution

volume =
$$\frac{1}{3} \times 10 \times 10 \times 5\sqrt{2}$$

= 235.702 260 4 (FCD)
= $\underline{236 \text{ cm}^3 (3 \text{ sf})}$.

(b) Find the size of angle DAB.

Let the size of the angle be x° . Then,

$$\cos x^{\circ} = \frac{AB^2 + AD^2 - BD^2}{2 \times AB \times AD}$$

$$\Rightarrow \quad \cos x^{\circ} = \frac{10^2 + 10^2 - (10\sqrt{2})^2}{2 \times 10 \times 10}$$

$$\Rightarrow \quad \cos x^{\circ} = 0$$

$$\Rightarrow \quad \underline{x^{\circ} = 90^{\circ}}.$$

24. The table gives information about the heights, h metres, of trees in a wood.

Height $(h \text{ metres})$	Frequency
$0 < h \leqslant 2$	7
$2 < h \leqslant 4$	14
$4 < h \leqslant 8$	18
$8 < h \leqslant 16$	24
$16 < h \leqslant 20$	10

Draw a histogram to show this information.



(2)



Solution				
	Height $(h \text{ metres})$	Frequency	Width	Frequency Density
	$0 < h \leqslant 2$	7	2	$\frac{7}{2} = 3.5$
	$2 < h \leqslant 4$	14	2	$\frac{14}{2} = 7$
	$4 < h \leqslant 8$	18	4	$\frac{18}{4} = 4.5$
	$8 < h \leqslant 16$	24	8	$\frac{24}{8} = 3$
	$16 < h \leq 20$	10	4	$\frac{10}{4} = 2.5$





25. The diagram shows the triangle PQR.



PQ = x cm. PR = 2x cm. Angle $QPR = 30^{\circ}$. The area of triangle PQR = A cm². Show that $x = \sqrt{2A}$.

Solution



$$A = \frac{1}{2} \times x \times 2x \times \sin 30^{\circ}$$

$$\Rightarrow A = \frac{1}{2}x^{2}$$

$$\Rightarrow x^{2} = 2A$$

$$\Rightarrow \underline{x} = \sqrt{2A},$$

as required.

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