Dr Oliver Mathematics
GCSE Mathematics
2017 November Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Write 36 as a product of its prime factors. (2)

Solution

$$\begin{array}{r|l} & 36 \\ \hline 2 & 18 \\ \hline 2 & 9 \\ \hline 3 & 3 \\ \hline 3 & 1 \\ \hline \end{array}$$

So

$$36 = 2 \times 2 \times 3 \times 3 = \underline{\underline{2^2 \times 3^2}}.$$

2. Kiaria is 7 years older than Jay. (4)
Martha is twice as old as Kiaria.
The sum of their three ages is 77.

Find the ratio of Jay's age to Kiaria's age to Martha's age.

Solution

Let K, J, and M be the ages of Kiaria, Jay, and Martha respectively. Then

$$K = J + 7 \Rightarrow J = K - 7,$$

$$M = 2K,$$

and

$$K + J + M = 77.$$

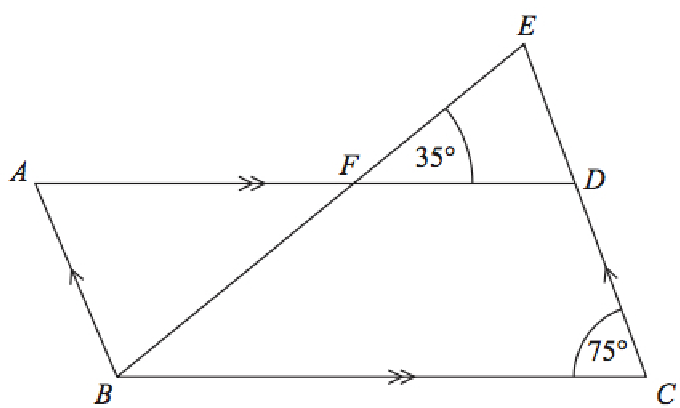
Now,

$$
\begin{aligned}
K + J + M = 77 &\Rightarrow K + (K-7) + (2K) = 77 \\
&\Rightarrow 4K - 7 = 77 \\
&\Rightarrow 4K = 84 \\
&\Rightarrow K = 21 \\
&\Rightarrow J = 14 \\
&\Rightarrow M = 42.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
J : K : M &= \underline{\underline{14 : 21 : 42}} \\
&= 7 \times 2 : 7 \times 3 : 7 \times 6 \\
&= \underline{\underline{2 : 3 : 6}}.
\end{aligned}
$$

3. $ABCD$ is a parallelogram. (4)



EDC is a straight line.
F is the point on AD so that BFE is a straight line.
Angle $EFD = 35°$.
Angle $DCB = 75°$.

Show that angle $ABF = 70°$.
Give a reason for each stage of your working.

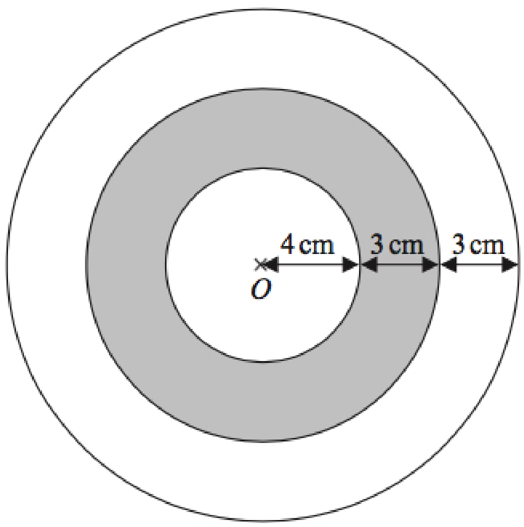
Solution

$\angle AFB = 35^\circ$ (vertically opposite)
$\angle BAF = 75^\circ$ ($ABCD$ is a parallelogram)
Now, the angles in $\triangle ABF$ all add up to 180°:

$$\begin{aligned}\angle ABF + \angle BFA + \angle FAB = 180 &\Rightarrow \angle ABF + 35 + 75 = 180 \\ &\Rightarrow \angle ABF + 110 = 180 \\ &\Rightarrow \underline{\underline{\angle ABF = 70^\circ}},\end{aligned}$$

as required.

4. The diagram shows a logo made from three circles. (4)



Each circle has centre, O.
Daisy says that exactly $\frac{1}{3}$ of the logo is shaded.

Is Daisy correct?
You must show all your working.

Solution

$$\begin{aligned}\text{Total area} &= \pi \times (4 + 3 + 3)^2 \\ &= \pi \times 10^2 \\ &= 100\pi\end{aligned}$$

and

$$\begin{aligned} \text{shaded area} &= \left[\pi \times (4+3)^2\right] - \left[\pi \times 4^2\right] \\ &= \left[\pi \times 7^2\right] - \left[\pi \times 4^2\right] \\ &= 49\pi - 16\pi \\ &= 33\pi. \end{aligned}$$

Daisy is wrong: she needs $33\frac{1}{3}\pi$, rather than 33π.

5. The table shows information about the weekly earnings of 20 people who work in a shop.

Weekly earnings (£x)	Frequency
$150 < x \leqslant 250$	1
$250 < x \leqslant 350$	11
$350 < x \leqslant 450$	5
$450 < x \leqslant 550$	0
$550 < x \leqslant 650$	3

(a) Work out an estimate for the mean of the weekly earnings. (3)

Solution

Weekly earnings (£x)	Frequency	Midpoint	Mid× Frequency
$150 < x \leqslant 250$	1	200	200
$250 < x \leqslant 350$	11	300	3 300
$350 < x \leqslant 450$	5	400	2 000
$450 < x \leqslant 550$	0	500	0
$550 < x \leqslant 650$	3	600	1 800
Total	20		7 300

Now,

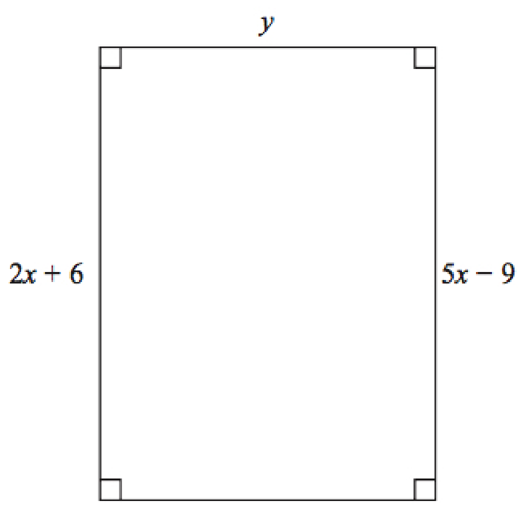
$$\begin{aligned} \text{Mean} &\approx \frac{\sum fx}{\sum f} \\ &= \frac{7\,300}{20} \\ &= \underline{\underline{£365}}. \end{aligned}$$

Nadiya says, "The mean may not be the best average to use to represent this information."

(b) Do you agree with Nadiya? (1)
You must justify your answer.

> **Solution**
> Yes: the mean is pulled upwards by the fact there are three items on $550 < x \leqslant 650$.

6. Here is a rectangle. (4)



All measurements are in centimetres.
The area of the rectangle is 48 cm^2.

Show that $y = 3$.

> **Solution**
> Since it is a rectangle,
>
> $$\begin{aligned} 2x + 6 = 5x - 9 &\Rightarrow 3x = 15 \\ &\Rightarrow x = 5 \\ &\Rightarrow 2x + 6 = 2(5) + 6 = 16. \end{aligned}$$

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Finally,

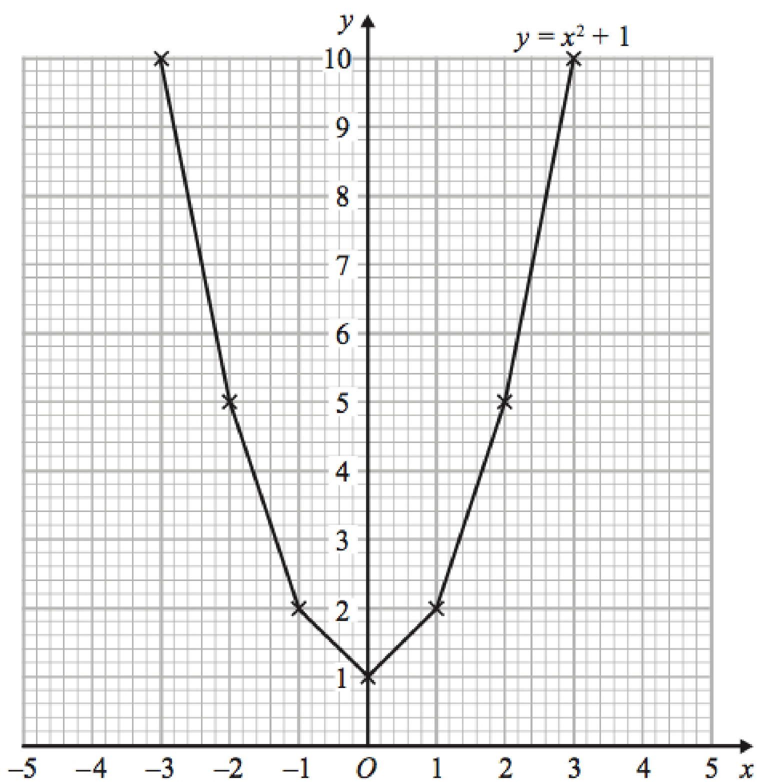
$$(2x+6)y = 48 \Rightarrow 16y = 48$$
$$\Rightarrow \underline{\underline{y = 3}},$$

as required.

7. Brogan needs to draw the graph of (1)

$$y = x^2 + 1.$$

Here is her graph.



Write down one thing that is wrong with Brogan's graph.

Solution

E.g., Brogan needs a <u>curve</u>, not a series of straight lines.

8. Write these numbers in order of size. (2)
Start with the smallest number.

$$0.2\dot{4}\dot{6} \qquad 0.24\dot{6} \qquad 0.\dot{2}4\dot{6} \qquad 0.246$$

Solution

$$\begin{aligned}
0.2\dot{4}\dot{6} &= 0.246\,464\,\ldots \\
0.24\dot{6} &= 0.246\,666\,\ldots \\
0.\dot{2}4\dot{6} &= 0.246\,246\,\ldots \\
0.246 &
\end{aligned}$$

Hence,

$$\underline{\underline{0.246,\ 0.\dot{2}4\dot{6},\ 0.2\dot{4}\dot{6},\ \text{and } 0.24\dot{6}}}.$$

9. James and Peter cycled along the same 50 km route. (5)
James took $2\frac{1}{2}$ hours to cycle the 50 km.

Peter started to cycle 5 minutes after James started to cycle.
Peter caught up with James when they had both cycled 15 km.

James and Peter both cycled at constant speeds.

Work out Peter's speed.

Solution

$$\begin{aligned}
\text{James's average speed} &= \frac{50}{2\frac{1}{2}} \\
&= 20 \text{ km/h}.
\end{aligned}$$

Peter caught up with James when they had both cycled 15 km:

$$\begin{aligned}
\text{time} &= \frac{15}{20} \\
&= \tfrac{3}{4} \text{ hours} \\
&= 45 \text{ minutes}.
\end{aligned}$$

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So Peter's time was $45 - 5 = 40$ minutes $= \frac{2}{3}$ hours. Hence,

$$\begin{aligned} \text{Peter's average speed} &= \frac{15}{\frac{2}{3}} \\ &= 15 \times \tfrac{3}{2} \\ &= \underline{\underline{\underline{22\tfrac{1}{2} \text{ km/h}}}}. \end{aligned}$$

10. (a) Write down the value of $100^{\frac{1}{2}}$. (1)

Solution

$$100^{\frac{1}{2}} = \underline{\underline{\underline{10}}}.$$

(b) Find the value of $125^{\frac{2}{3}}$. (2)

Solution

$$\begin{aligned} 125^{\frac{2}{3}} &= \left(125^{\frac{1}{3}}\right)^2 \\ &= 5^2 \\ &= \underline{\underline{\underline{25}}}. \end{aligned}$$

11. 3 teas and 2 coffees have a total cost of £7.80. (4)
5 teas and 4 coffees have a total cost of £14.20.

Work out the cost of one tea and the cost of one coffee.

Solution

Let t and c be the price of tea and coffee respectively. Then

$$\begin{aligned} 3t + 2c &= 7.80 \quad (1) \\ 5t + 4c &= 14.20 \quad (2). \end{aligned}$$

E.g., multiply (1) by 2:

$$6c + 4c = 15.60 \quad (3)$$

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and do (3) − (2):

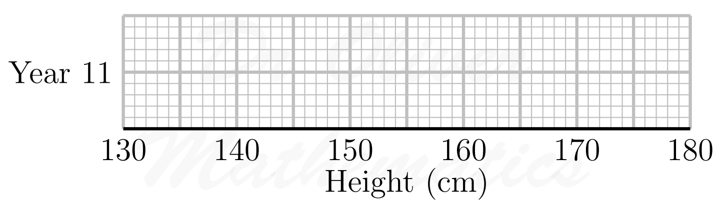
$$
\begin{aligned}
t = 1.40 \Rightarrow 3(1.40) + 2c &= 7.80 \\
\Rightarrow 4.20 + 2c &= 7.80 \\
\Rightarrow 2c &= 3.60 \\
\Rightarrow c &= 1.80.
\end{aligned}
$$

Hence, the cost of one tea is $\underline{\underline{\text{£}1.40}}$ and the cost of one coffee is $\underline{\underline{\text{£}1.80}}$.

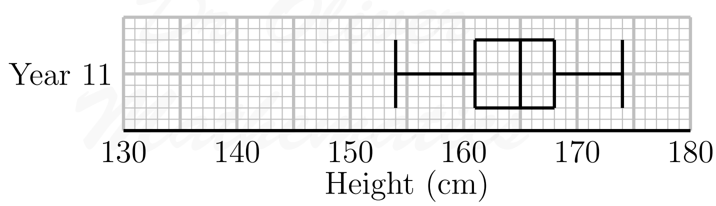
12. The table shows information about the heights, in cm, of a group of Year 11 girls.

Value	Height (cm)
Least height	154
Median	165
Lower Quartile	161
Interquartile Range	7
Range	20

(a) Draw a box plot for this information. (3)



Solution



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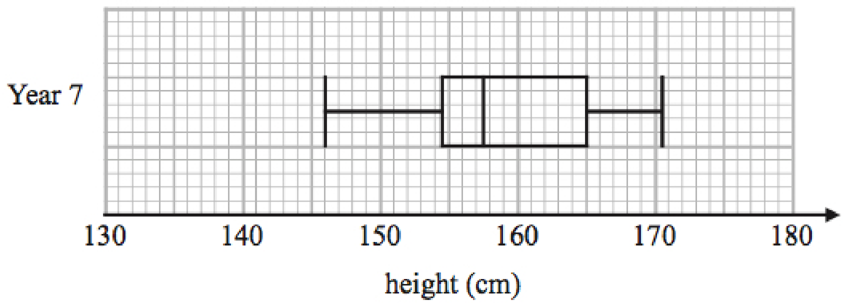
The box plot below shows information about the heights, in cm, of a group of Year 7 girls.



(b) Compare the distribution of heights of the Year 7 girls with the distribution of heights of the Year 11 girls. (2)

Solution

Average

Since the median for the Year 11 is higher than the median for the Year 7, the Year 11 have more height on average.

Spread

Since the range for the Year 11 (20) is smaller than the range for the Year 7 ($170\frac{1}{2} - 146 = 24\frac{1}{2}$), the heights were more consistent in the Year 11 girls.

OR

Since the IQR for the Year 11 (7) is smaller than the IQR for the Year 7 ($165 - 154\frac{1}{2} = 10\frac{1}{2}$), the heights were more consistent in the Year 11 girls.

13. A factory makes 450 pies every day.
The pies are chicken pies or steak pies.

Each day Milo takes a sample of 15 pies to check.

The proportion of the pies in his sample that are chicken is the same as the proportion of the pies made that day that are chicken.

On Monday Milo calculated that he needed exactly 4 chicken pies in his sample.

(a) Work out the total number of chicken pies that were made on Monday. (2)

Solution

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$$\begin{aligned}\text{Total number of chicken pies} &= \tfrac{4}{15} \times 450 \\ &= 4 \times 30 \\ &= \underline{\underline{120}}.\end{aligned}$$

On Tuesday, the number of steak pies Milo needs in his sample is 6 correct to the nearest whole number.

Milo takes at random a pie from the 450 pies made on Tuesday.

(b) Work out the lower bound of the probability that the pie is a steak pie. (2)

Solution

So,

$$5.5 \leqslant \text{steak pies} < 6.5.$$

Now,

$$\begin{aligned}\tfrac{5.5}{15} \times 450 &= 5.5 \times 30 \\ &= 165.\end{aligned}$$

Hence, the lower bound of the probability that the pie is a steak pie is

$$\underline{\underline{\tfrac{165}{450}}} = \tfrac{15\times 11}{15\times 30} = \underline{\underline{\tfrac{11}{15}}}.$$

14. The ratio (3)

$$(y + x) : (y - x)$$

is equivalent to $k : 1$.

Show that

$$y = \frac{x(k+1)}{k-1}.$$

Solution

$$
\begin{aligned}
(y+x):(y-x) = k:1 &\Rightarrow \frac{y+x}{y-x} = \frac{k}{1} \\
&\Rightarrow \frac{y+x}{y-x} = k \\
&\Rightarrow y + x = k(y-x) \\
&\Rightarrow y + x = ky - kx \\
&\Rightarrow y - ky = -kx - x \\
&\Rightarrow y(1-k) = -x(k+1) \\
&\Rightarrow y = \frac{-x(k+1)}{-(k-1)} \\
&\Rightarrow \underline{\underline{y = \frac{x(k+1)}{k-1}}},
\end{aligned}
$$

as required.

15. $x = 0.4\dot{3}\dot{6}$. (3)

Prove algebraically that x can be written as $\frac{24}{55}$.

Solution

$$
\begin{aligned}
10x &= 4.363\,636\,\ldots \quad (1) \\
1\,000x &= 436.363\,636\,\ldots \quad (2)
\end{aligned}
$$

Do (2) − (1):

$$
\begin{aligned}
990x = 432 &\Rightarrow 9 \times 110x = 9 \times 48 \\
&\Rightarrow 110x = 48 \\
&\Rightarrow 2 \times 55x = 2 \times 24 \\
&\Rightarrow 55x = 24 \\
&\Rightarrow \underline{\underline{x = \tfrac{24}{55}}},
\end{aligned}
$$

as required.

16. y is directly proportional to $\sqrt[3]{x}$. (3)

$y = 1\frac{1}{6}$ when $x = 8$.

Find the value of y when $x = 64$.

> **Solution**
>
> $y \propto \sqrt[3]{x}$ means $y = k\sqrt[3]{x}$, for some constant k. Now,
>
> $$\begin{aligned} 1\tfrac{1}{6} = k\sqrt[3]{8} &\Rightarrow \tfrac{7}{6} = 2k \\ &\Rightarrow k = \tfrac{7}{12} \end{aligned}$$
>
> and so
>
> $$y = \tfrac{7}{12}\sqrt[3]{x}.$$
>
> Finally,
>
> $$\begin{aligned} y &= \tfrac{7}{12}\sqrt[3]{64} \\ &= \tfrac{7}{12} \times 4 \\ &= \underline{\underline{\tfrac{7}{3} \text{ or } 2\tfrac{1}{3}}}. \end{aligned}$$

17. n is an integer. (2)

Prove algebraically that the sum of

$$\tfrac{1}{2}n(n+1)$$

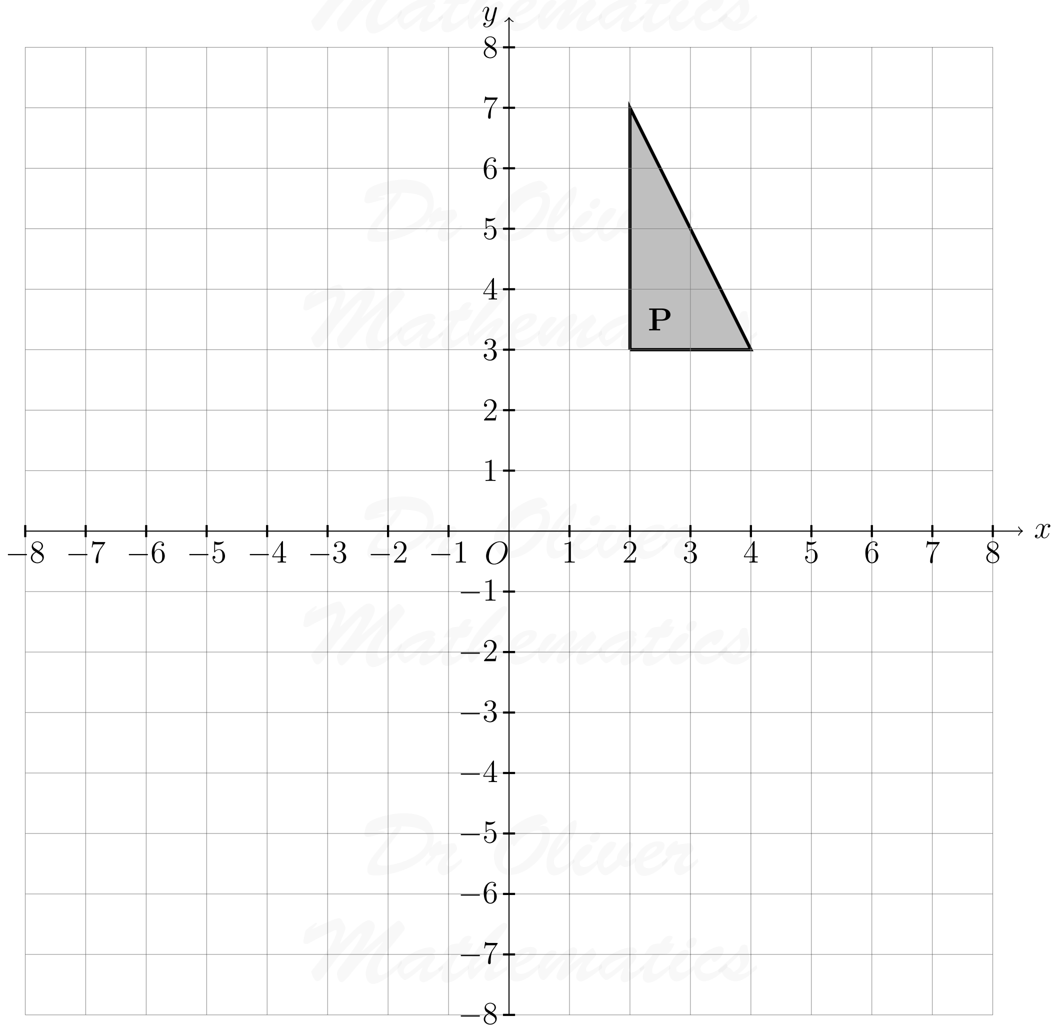
and

$$\tfrac{1}{2}(n+1)(n+2)$$

is always a square number.

> **Solution**
>
> $$\begin{aligned} \tfrac{1}{2}n(n+1) + \tfrac{1}{2}(n+1)(n+2) &= \tfrac{1}{2}(n+1)[n + (n+2)] \\ &= \tfrac{1}{2}(n+1)(2n+2) \\ &= \tfrac{1}{2}(n+1) \cdot 2(n+1) \\ &= (n+1)^2; \end{aligned}$$
>
> hence, that is always a $\underline{\underline{\text{square number}}}$.

18. Enlarge shape **P** by scale factor $-\frac{1}{2}$ with centre of enlargement $(0, 0)$. (2)



Label your image **Q**.

Solution

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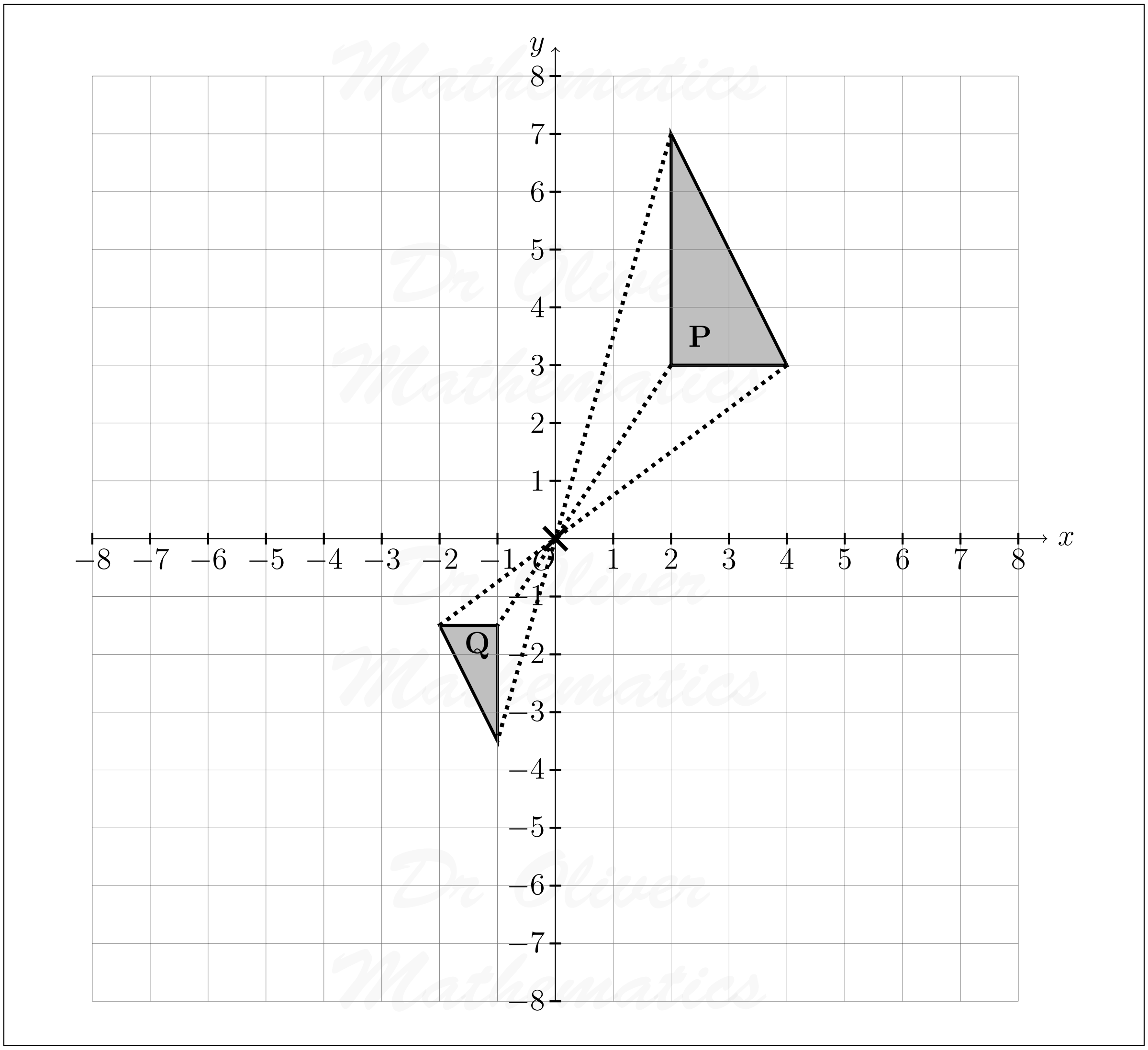

19. $ABCD$ is a rectangle. (4)

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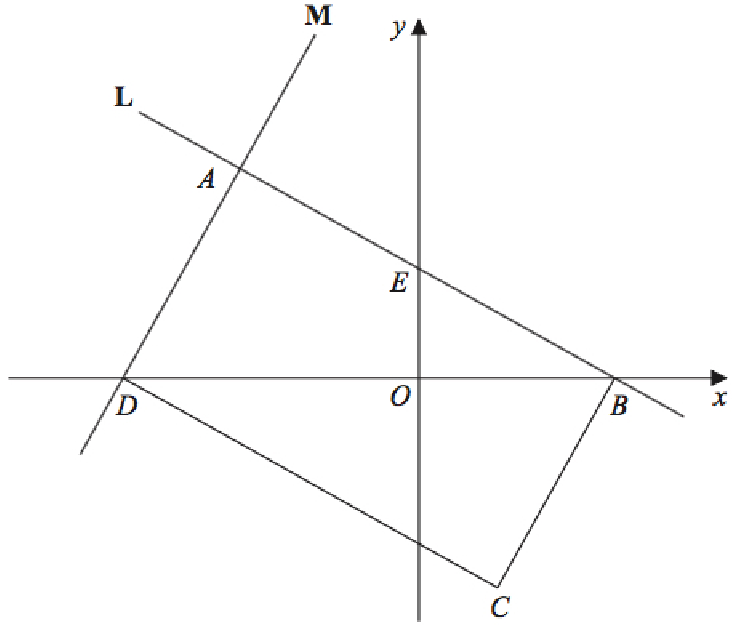


A, E, and B are points on the straight line **L** with equation

$$x + 2y = 12.$$

A and D are points on the straight line **M**.
$AE = EB$.

Find an equation for **M**.

Solution

$$\begin{aligned} x + 2y = 12 &\Rightarrow 2y = -x + 12 \\ &\Rightarrow y = -\tfrac{1}{2}x + 6. \end{aligned}$$

Now, the gradient of the line perpendicular to **L** is

$$-\frac{1}{-\frac{1}{2}} = 2$$

and the equation of **M** is

$$y = 2x + c,$$

for some constant c. Now,

$$y = 0 \Rightarrow x + 0 = 12$$

and $B(12, 0)$ and

$$x = 0 \Rightarrow 0 + 2y = 12 \Rightarrow y = 6$$

and $E(0, 6)$. Hence $A(-12, 12)$. Now,

$$\begin{aligned} 12 = 2(-12) + c &\Rightarrow 12 = -24 + c \\ &\Rightarrow c = 36 \end{aligned}$$

and the equation of $\mathbf{M}$ is

$$\underline{\underline{y = 2x + 36}}.$$

20. The table shows some values of x and y that satisfy the equation (4)

$$y = a \cos x^\circ + b.$$

x	0	30	60	90	120	150	180
y	3	$1 + \sqrt{3}$	2	1	0	$1 - \sqrt{3}$	-1

Find the value of y when $x = 45$.

Solution

$$\begin{aligned} x = 90, y = 1 \Rightarrow 1 &= a \cos 90^\circ + b \\ \Rightarrow 1 &= a(0) + b \\ \Rightarrow b &= 1 \\ x = 0, y = 3 \Rightarrow 3 &= a \cos 0^\circ + 1 \\ \Rightarrow 3 &= a(1) + 1 \\ \Rightarrow a &= 2; \end{aligned}$$

hence,

$$y = 2 \cos x^\circ + 1.$$

Finally,

$$\begin{aligned} y &= 2 \cos 45^\circ + 1 \\ &= 2(\tfrac{1}{\sqrt{2}}) + 1 \\ &= \underline{\underline{\sqrt{2} + 1}}. \end{aligned}$$

21. Show that (3)

$$\frac{6-\sqrt{8}}{\sqrt{2}-1}$$

can be written in the form $a + b\sqrt{2}$ where a and b are integers.

Solution

$$\begin{aligned}\frac{6-\sqrt{8}}{\sqrt{2}-1} &= \frac{6-\sqrt{4\times 2}}{\sqrt{2}-1} \\ &= \frac{6-\sqrt{4}\times\sqrt{2}}{\sqrt{2}-1} \\ &= \frac{6-2\sqrt{2}}{\sqrt{2}-1} \\ &= \frac{6-2\sqrt{2}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}\end{aligned}$$

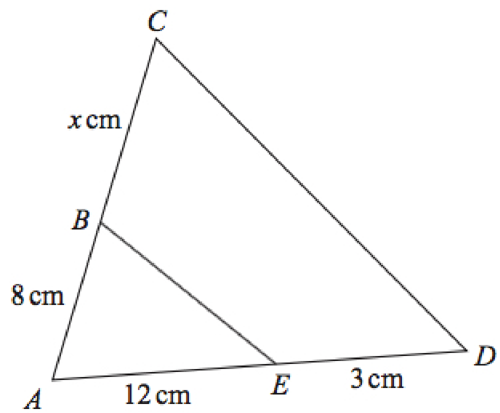
$\times$	6	$-2\sqrt{2}$
$\sqrt{2}$	$6\sqrt{2}$	-4
$+1$	$+6$	$-2\sqrt{2}$

$\times$	$\sqrt{2}$	-1
$\sqrt{2}$	2	$-\sqrt{2}$
$+1$	$+\sqrt{2}$	-1

$$\begin{aligned} &= \frac{4\sqrt{2}+2}{1} \\ &= \underline{\underline{4\sqrt{2}+2}};\end{aligned}$$

hence, $\underline{\underline{a = 4}}$ and $\underline{\underline{b = 2}}$.

22. The two triangles in the diagram are similar. (5)

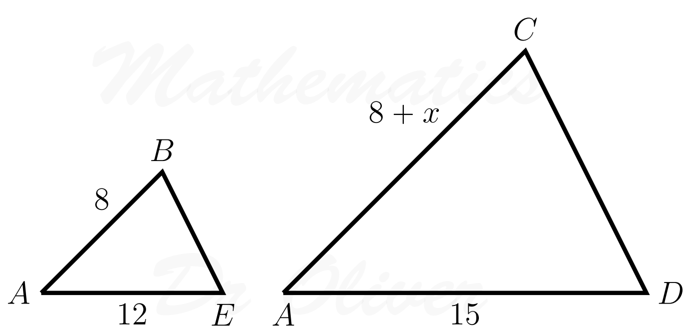


There are two possible values of x.

Work out each of these values.
State any assumptions you make in your working.

Solution

This is interesting - and I cannot think of a question that we have had like it! Assumptions? Clearly $\triangle ABE$ is similar to $\triangle ACD$ (going from AB to AC and going from AE to AD)? Well, yes it is - but $\triangle AEB$ is similar to $\triangle ADC$ (going from AE to AC and going from AD to AD)



OR

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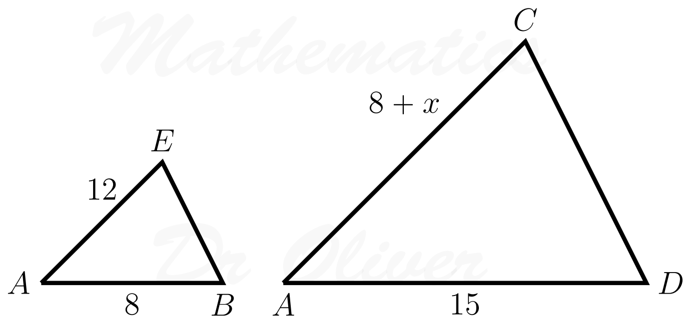


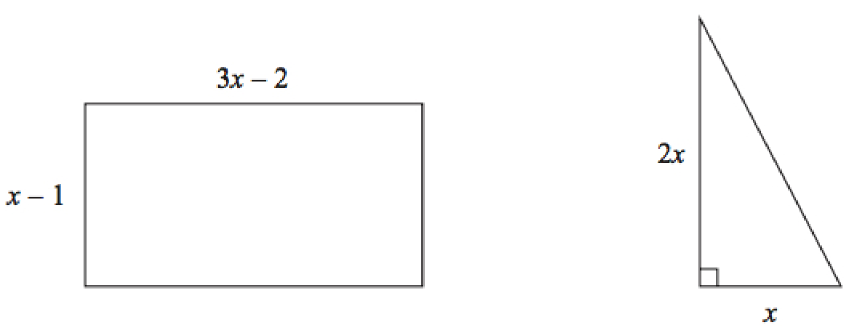
$\underline{\triangle ABE \text{ to } \triangle ACD \textit{ in that order}}$:

$$\begin{aligned}\frac{8+x}{8} = \frac{12+3}{12} &\Rightarrow \frac{8+x}{8} = \frac{15}{12}\\ &\Rightarrow \frac{8+x}{8} = \tfrac{5}{4}\\ &\Rightarrow 8+x = 8 \times \tfrac{5}{4}\\ &\Rightarrow 8+x = 10\\ &\Rightarrow \underline{\underline{x = 2}}.\end{aligned}$$

$\underline{\triangle AEB \text{ to } \triangle ACD \textit{ in that order}}$::

$$\begin{aligned}\frac{8+x}{12} = \frac{15}{8} &\Rightarrow 8+x = 12 \times \tfrac{15}{8}\\ &\Rightarrow 8+x = 3 \times \tfrac{15}{2}\\ &\Rightarrow 8+x = 22.5\\ &\Rightarrow \underline{\underline{x = 14.5}}.\end{aligned}$$

23. Here is a rectangle and a right-angled triangle. (5)



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All measurements are in centimetres.
The area of the rectangle is greater than the area of the triangle.

Find the set of possible values of x.

Solution

$$\text{Rectangle} > \text{triangle} \Rightarrow (x-1)(3x-2) > \tfrac{1}{2}x(2x)$$

$\times$	$3x$	-2
x	$3x^2$	$-2x$
-1	$-3x$	$+2$

$$\begin{aligned} &\Rightarrow 3x^2 - 5x + 2 > x^2 \\ &\Rightarrow 2x^2 - 5x + 2 > 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & -5 \\ \text{multiply to:} & (+2)\times(+2) = +4 \end{array}\right\} -4,\ -1$$

$$\begin{aligned} &\Rightarrow 2x^2 - 4x - x + 2 > 0 \\ &\Rightarrow 2x(x-2) - (x-2) > 0 \\ &\Rightarrow (2x-1)(x-2) > 0 \\ &\Rightarrow x < \tfrac{1}{2} \text{ or } x > 2. \end{aligned}$$

Now, $(x-1)$ is a length and so we cannot have $x < \frac{1}{2}$. Hence, $\underline{\underline{x > 2}}$.