Dr Oliver Mathematics
Mathematics Standard Grade: Credit Level
2012 Paper 1: Non-Calculator
55 minutes

The total number of marks available is 37.
You must write down all the stages in your working.

1. Evaluate (2)

$$7.2 - 0.161 \times 30.$$

Solution

$$\begin{aligned} 7.2 - 0.161 \times 30 &= 7.2 - 1.61 \times 3 \\ &= 7.2 - 4.83 \\ &= \underline{\underline{2.37}} \end{aligned}$$

2. Expand and simplify (3)

$$(3x - 2)(2x^2 + x + 5).$$

Solution

$\times$	$2x^2$	$+x$	$+5$
$3x$	$6x^3$	$+3x^2$	$+15x$
-2	$-4x^2$	$-2x$	-20

$$(3x - 2)(2x^2 + x + 5) = \underline{\underline{6x^3 - x^2 + 13x - 20}}.$$

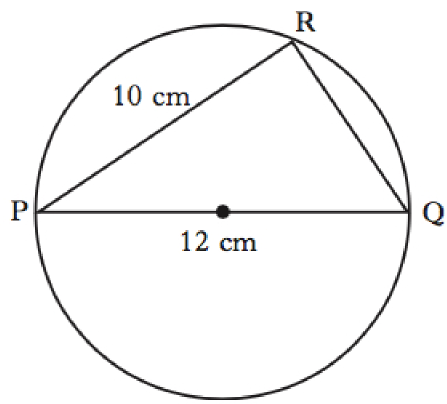
3. Change the subject of the formula to m: (2)

$$L = \frac{\sqrt{m}}{k}.$$

Solution

$$L = \frac{\sqrt{m}}{k} \Rightarrow \sqrt{m} = kL$$
$$\Rightarrow \underline{\underline{m = (kL)^2}}.$$

4. In the diagram, (4)
PQ is the diameter of the circle
$PQ = 12$ centimetres, and
$PR = 10$ centimetres.



Calculate the length of QR.
Give your answer as a surd in its simplest form.

Solution

Pythagoras' theorem (why?):

$$\begin{aligned} PQ^2 = PR^2 + QR^2 &\Rightarrow 12^2 = 10^2 + QR^2 \\ &\Rightarrow 144 = 100 + QR^2 \\ &\Rightarrow QR^2 = 44 \\ &\Rightarrow QR = \sqrt{4 \times 11} \\ &\Rightarrow QR = \sqrt{4} \times \sqrt{11} \\ &\Rightarrow \underline{\underline{QR = 2\sqrt{11} \text{ cm}}}. \end{aligned}$$

5. Mike is practising his penalty kicks. (3)
Last week, Mike scored 18 out of 30.
This week, he scored 16 out of 25.
Has his scoring rate improved?
Give a reason for your answer.

Solution

Last week:

$$\frac{18}{30} \times 100\% = \tfrac{3}{5} \times 100\% = 60\%.$$

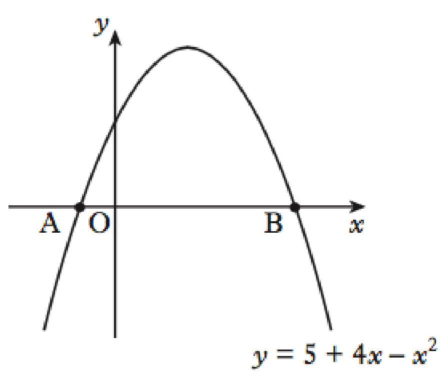
This week:

$$\frac{16}{25} \times 100\% = 16 \times 4\% = 64\%.$$

Yes, his his scoring rate has improved from 60% to 64%.

6. The diagram shows part of the graph of

$$y = 5 + 4x - x^2.$$



A is the point $(-1, 0)$.
B is the point $(5, 0)$.

(a) State the equation of the axis of symmetry of the graph. (2)

Solution

$$x = \frac{-1+5}{2} \Rightarrow \underline{\underline{x = 2}}.$$

(b) Hence, find the maximum value of $y = 5 + 4x - x^2$. (2)

Solution

$$x = 2 \Rightarrow y = 5 + 8 - 4 = \underline{\underline{9}}.$$

7. Given (4)

$$2x^2 - 2x - 1 = 0,$$

show that

$$x = \frac{1 \pm \sqrt{3}}{2}.$$

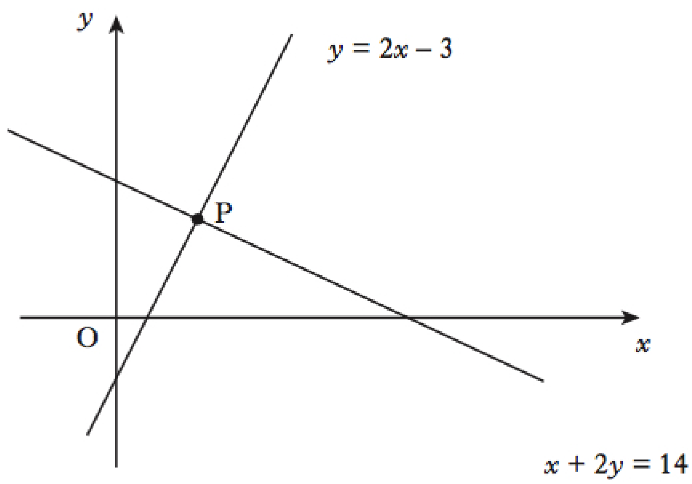
Solution

$$\begin{aligned} 2x^2 - 2x - 1 = 0 &\Rightarrow 2x^2 - 2x = 1 \\ &\Rightarrow x^2 - x = \tfrac{1}{2} \\ &\Rightarrow x^2 - x + \tfrac{1}{4} = \tfrac{1}{2} + \tfrac{1}{4} \\ &\Rightarrow (x - \tfrac{1}{2})^2 = \tfrac{3}{4} \\ &\Rightarrow x - \tfrac{1}{2} = \pm\tfrac{\sqrt{3}}{2} \\ &\Rightarrow \underline{\underline{x = \frac{1 \pm \sqrt{3}}{2}}}, \end{aligned}$$

as required.

8. The graph below shows two straight lines: (4)

$$y = 2x - 3$$
$$x + 2y = 14.$$



The lines intersect at the point P.
Find, **algebraically**, the coordinates of P.

Solution

$$\begin{aligned} x + 2y = 14 &\Rightarrow x + 2(2x - 3) = 14 \\ &\Rightarrow x + (4x - 6) = 14 \\ &\Rightarrow 5x = 20 \\ &\Rightarrow x = 4 \\ &\Rightarrow y = 5; \end{aligned}$$

hence, $\underline{\underline{P(4, 5)}}$.

9. Each day, Marissa drives 40 kilometres to work.

(a) On Monday, she drives at a speed of x kilometres per hour. (1)
Find the time taken, in terms of x, for her journey.

Solution
$\underline{\underline{\dfrac{40}{x}}}$.

(b) On Tuesday, she drives 5 kilometres per hour **faster**. (1)
Find the time taken, in terms of x, for this journey.

Solution

$\underline{\underline{\dfrac{40}{x+5}}}$.

(c) Hence find an expression, in terms of x, for the difference in times of the two journeys. (3)
Give this expression **in its simplest form**.

Solution

$$\begin{aligned}\text{Difference} &= \frac{40}{x} - \frac{40}{x+5} \\ &= \frac{40(x+5)}{x(x+5)} - \frac{40x}{x(x+5)} \\ &= \frac{40(x+5) - 40x}{x(x+5)} \\ &= \frac{(40x+200) - 40x}{x(x+5)} \\ &= \underline{\underline{\frac{200}{x(x+5)}}}.\end{aligned}$$

10. (a) Evaluate $(2^3)^2$. (1)

Solution

$$(2^3)^2 = 2^6 = \underline{\underline{64}}.$$

(b) Hence find n, when (1)

$$(2^3)^n = \tfrac{1}{64}.$$

Solution

$$\begin{aligned}(2^3)^n = \tfrac{1}{64} &\Rightarrow 2^{3n} = \frac{1}{2^6} \\ &\Rightarrow 2^{3n} = 2^{-6} \\ &\Rightarrow 3n = -6 \\ &\Rightarrow \underline{\underline{n = -2}}.\end{aligned}$$

11. The sum of consecutive even numbers can be calculated using the following number pattern:

$$\begin{aligned} 2+4+6 &= 3\times 4 = 12 \\ 2+4+6+8 &= 4\times 5 = 20 \\ 2+4+6+8+10 &= 5\times 6 = 30. \end{aligned}$$

(a) Calculate (1)

$$2+4+\ldots+20.$$

Solution

The line is the eighth of the series:

$$10\times 11 = \underline{\underline{110}}.$$

(b) Write down an expression for (1)

$$2+4+\ldots+n.$$

Solution

$$2+4+\ldots+n = \underline{\underline{\tfrac{1}{2}n(\tfrac{1}{2}n+1)}}.$$

(c) Hence, or otherwise, calculate (2)

$$10+12+\ldots+100.$$

Solution

$$\begin{aligned} 10+12+\ldots+100 &= (2+4+\ldots+100)-(2+4+\ldots+8) \\ &= 50\times 51 - 20 \\ &= 2\,550 - 20 \\ &= \underline{\underline{2\,530}} \end{aligned}$$