

Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2004 June Paper 1: Calculator
2 hours

The total number of marks available is 80.

You must write down all the stages in your working.

1. Given that

$$y = \frac{3x - 2}{x^2 + 5},$$

find

- (a) an expression for $\frac{dy}{dx}$, (2)
- (b) the x -coordinates of the stationary points. (2)
2. Find the x -coordinates of the three points of intersection of the curve (5)

$$y = x^3$$

with the line

$$y = 5x - 2,$$

expressing non-integer values in the form

$$a \pm \sqrt{b},$$

where a and b are integers.

3. (a) Sketch on the same diagram the graphs of (3)

$$y = |2x + 3| \text{ and } y = 1 - x.$$

- (b) Find the values of x for which (3)

$$x + |2x + 3| = 1.$$

4. The function f is defined, for $0^\circ \leq x \leq 360^\circ$, by

$$f(x) = a \sin(bx) + c,$$

where a , b , and c are positive integers.

Given that the amplitude of f is 2 and the period of f is 120° ,

(a) state the value of a and of b . (2)

Given further that the minimum value of f is -1 ,

(b) state the value of c , (1)

(c) sketch the graph of f . (3)

5. The straight line (6)

$$5y + 2x = 1$$

meets the curve

$$xy + 24 = 0$$

at the points A and B .

Find the length of AB , correct to one decimal place.

6. The table below shows (6)

- the daily production, in kilograms, of two types, S_1 and S_2 , of sweets from a small company and
- the percentages of the ingredients A , B , and C required to produce S_1 and S_2 .

	A	B	C	Daily Production
Type S_1	60	30	10	300
Type S_2	50	40	10	240

Given that the costs, in dollars per kilogram, of A , B , and C are 4, 6, and 8 respectively, use matrix multiplication to calculate the total cost of daily production.

7. To a cyclist travelling due south on a straight horizontal road at 7 ms^{-1} , the wind appears to be blowing from the north-east. (5)

Given that the wind has a constant speed of 12 ms^{-1} , find the direction from which the wind is blowing.

8. A curve has the equation (7)

$$y = (ax + 3) \ln x,$$

where $x > 0$ and a is a positive constant.

The normal to the curve at the point where the curve crosses the x -axis is parallel to the line

$$5y + x = 2.$$

Find the value of a .

9. (a) Calculate the term independent of x in the binomial expansion of (3)

$$\left(x - \frac{1}{2x^5}\right)^{18}.$$

- (b) In the binomial expansion of (4)

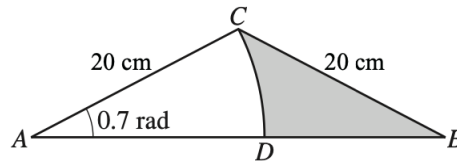
$$(1 + kx)^n,$$

where $n \geq 3$ and k is a constant, the coefficients of x^2 and x^3 are equal.

Express k in terms of n .

10. The diagram shows an isosceles triangle ABC in which

- $BC = AC = 20$ cm and
- angle $BAC = 0.7$ radians.

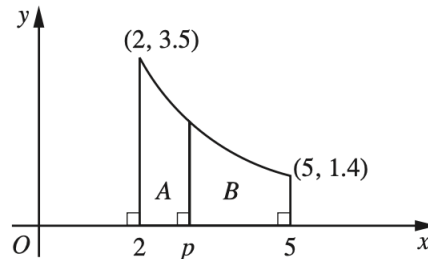


DC is an arc of a circle, centre A .

Find, correct to 1 decimal place,

- (a) the area of the shaded region, (4)
 (b) the perimeter of the shaded region. (4)

11. The diagram shows part of a curve, passing through the points $(2, 3.5)$ and $(5, 1.4)$.



The gradient of the curve at any point (x, y) is

$$-\frac{a}{x^3},$$

where a is a positive constant.

(a) how that $a = 20$ and obtain the equation of the curve. (5)

The diagram also shows lines perpendicular to the x -axis at $x = 2$, $x = p$, and $x = 5$.

Given that the areas of the regions A and B are equal,

(b) find the value of p . (5)

EITHER

12. An examination paper contains 12 different questions of which

- 3 are on trigonometry,
- 4 are on algebra, and
- 5 are on calculus.

Candidates are asked to answer 8 questions.

Calculate

- (a) (i) the number of different ways in which a candidate can select 8 questions if there is no restriction, (2)
- (ii) the number of these selections which contain questions on only 2 of the 3 topics, trigonometry, algebra, and calculus. (2)

A fashion magazine runs a competition, in which 8 photographs of dresses are shown, lettered A, B, C, D, E, F, G , and H .

Competitors are asked to submit an arrangement of 5 letters showing their choice of dresses in descending order of merit.

The winner is picked at random from those competitors whose arrangement of letters agrees with that chosen by a panel of experts.

(b) (i) Calculate the number of possible arrangements of 5 letters chosen from the 8. (2)

Calculate the number of these arrangements

- (ii) in which A is placed first, (2)
- (iii) which contain A . (2)

OR

13. The table shows experimental values of the variables x and y which are related by the equation

$$y = Ab^x,$$

where A and b are constants.

x	2	4	6	8	10
y	9.8	19.4	37.4	74.0	144.4

- (a) Use the data above in order to draw, on graph paper, the straight line graph of $\log_{10} y$ against x . (2)
- (b) Use your graph to estimate the value of A and of b . (5)
- (c) On the same diagram, draw the straight line representing $y = 2^x$ and hence find the value of x for which (3)

$$Ab^x = 2^x.$$